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CONTENTS

VOL. 122—YEAR 1959

Part 1, 1959

Part 2, 1959

Contents Vol. 122, *Year* 1959

Part 3, 1959

Part 4, 1959

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Vol. 122.] [Part I

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SERIES A (GENERAL)

PART I, 1959.

The Long-term Trend in the Size Distribution of Income

By H. F. Lydall

Oxford Institute of Statistics

[Read before the Royal Statistical Society on November 19th, 1958, the President, Sir Harry Campion, C.B., C.B.E., in the Chair]

1. Introduction

The distribution of income is a topic which has claimed the attention of many famous writers. Amongst modern writers we recall particularly the work of Stamp (1920, 1927), Bowley (1919, 1920, 1927, 1937, 1942), Clark (1932, 1937), Barna (1945) and Seers (1951). The early analysts of income distribution had to work with inadequate statistics, which were largely the by-products of enquiries made for other purposes, or derived from records kept in the course of administration. The major sources were the reports of the Commissioners of Inland Revenue, the censuses of population and production, the results of particular enquiries into earnings, and the statistics of the unemployment and health insurance schemes. The estimates of national income and its composition which were constructed from these sources were inevitably a patch-work, the seams of which are only too obvious in the works of Stamp, Clark or Bowley.

During the last war the Government took over the responsibility of preparing official estimates of the national income and its composition. In part this meant that the patching process was now done behind the scenes in Whitehall, instead of by individual investigators in the public view. But it also meant that steps could now be taken by the Government, in fulfilment of its new responsibility, to extract fuller information from official records (such as those of the Inland Revenue) and to set on foot special enquiries to fill gaps in the figures or to improve their quality. This process has continued steadily throughout the seventeen years that have elapsed since the publication of the first National Income White

Paper in 1941 ; and the modern investigator can now draw upon a wealth of material published each year in the Blue Book *National Income and Expenditure.*

Most of the effort of early investigators, and in more recent years of the official statisticians, was directed towards estimating aggregate national income, or its components on the income or expenditure side of the account. Some attention was given to the distribution of personal income by size, but on this aspect there was little that could be done without the help of the Inland Revenue. Since the British system of income tax under separate schedules does not normally require an assessment of the total income of each recipient, statistics of the size distribution of total income do not automatically come into existence in the course of administration.* But the Inland Revenue possess all the necessary information for constructing such a distribution—at least for taxpayers with incomes above the exemption limit—when required to do so. Their first attempts to produce estimates of this kind were made for the year 1918–19 on behalf of the Royal Commission on the Income Tax. Revised estimates for the year 1918–19 were published in the 63*rd Report of the Commissioners of His Majesty's Inland Revenue* (Cmd. 1083). Further estimates of the same sort were made for the year 1919–20 at the request of the Colwyn Committee. (*Report of the Committee on National Debt and Taxation* (1927). See Appendix XIV.) The estimates for the latter year cover about one-third of incomes and half of aggregate income in the United Kingdom. They are interesting so far as they go, but they relate to a somewhat abnormal period. Since profits under Schedule D were assessed at this time on the average of the previous three years, the profit incomes included must reflect to some extent the very high profits of the war years. A further difficulty of interpretation arises from the fact that at this date the United Kingdom included the whole of Ireland.

While these two estimates were the first attempts of the Inland Revenue to make use of the income tax records to construct a distribution of total income, the introduction of super-tax (later called surtax) in 1910 began to produce assessments of the total income of those in the higher ranges of income. The first statistics, which relate to the year 1911–12, showed only 12,000 incomes above the super-tax limit of £5,000. Ten years later, when prices and incomes had risen greatly and the super-tax limit had been reduced to £2,000, the number of incomes covered still did not exceed 100,000; and throughout the inter-war period the number of surtax payers was never much above this level. The surtax figures, therefore, can only be used to describe the shape of the very high tail of the distribution of income.

The first satisfactory estimate of the size distribution of incomes in this country was the result of a special enquiry undertaken by the Inland Revenue just before the last war. The results were published in the 83*rd Report of the Commissioners of His Majesty's Inland Revenue*. The data relate to assessments made in 1937–38, four-fifths of which refer to income received in the previous year. A complete count was made of incomes exceeding £200 and this revealed just over 4 million incomes above this limit with an aggregate income of nearly £2,000 million. With suitable additions for the incomes falling between

* The first British Income Tax Law, Pitt's Act of 1799, did require an assessment of total income; but it was repealed in 1802. When the income tax was temporarily revived in the following year by Addington it was put on a new basis, with the famous five schedules. The result was a large rise in the revenue but the loss of statistics of income distribution. Pitt's Act produced statistics of size distribution for a single year (1801); but since it seems that there was considerable under-assessment the statistics cannot be relied on very heavily.

£200 and the exemption limit of £125 the estimates can be made to cover over a third of total incomes and over half of aggregate personal income. Two further surveys of this kind have been made in the post-war years, the first relating to 1949–50 and the second to 1954–55. (94*th* and 99*th Reports* of the Inland Revenue.) In these two surveys the figures mostly refer to income received in the year of assessment. Only profits and professional earnings are for the preceding year. These were sample surveys of all incomes above the exemption limit, which was £135 in the first case and £155 in the second. The number of incomes represented was over 20 millions in each year, or nearly 80 per cent. of the number of incomes in the population, and the proportion of personal income represented was correspondingly high. Thus these two surveys give a very full picture of the distribution of income by size. For information about the distribution of incomes below the exemption limit in recent years reference can be made to a number of household surveys, especially the Savings Surveys. Details for the year 1951–52 are given in Lydall (1955) and for subsequent years in Erritt and Nicholson (1958).

The income surveys made by the Inland Revenue provide "benchmarks" from which estimates for intervening years, and extrapolations up to the latest year, can be constructed. The Central Statistical Office has published in the National Income Blue Books detailed estimates of the size distribution of income in 1938 and in various post-war years. Those for 1938, 1949 and 1954 are clearly based on the income surveys, while those for more recent years have doubtless been arrived at by intelligent extrapolations. For this purpose recourse can be had to the surtax figures, the aggregate tax receipts under PAYE and other materials in the hands of the Inland Revenue.

The purpose of this paper is to analyse the data on the size distribution of income which are now available in considerable detail for the period 1938–57. This period is long enough in itself to permit some assessment of the long-term trend in the distribution of income and of its causes. In a concluding section of the paper I shall consider how far the fragmentary evidence about changes in income distribution before 1938 enables us to judge the trend over a more extended period.

2. Definition of Income

The first problem that confronts anyone who wishes to discuss the distribution of income is to decide what meaning to attach to the word "income". The word is used in different senses by the Inland Revenue, by national income experts and by the man in the street. In the National Income Blue Book personal income is taken to consist of personal factor incomes—before depreciation and stock appreciation—plus government transfers, plus the investment income of life assurance and superannuation funds and other non-profit-making bodies, plus employers' contributions for national insurance and private superannuation. Not all of this, however, is allocated to particular ranges of income. The Blue Book figures of income allocated by ranges are derived fundamentally from Inland Revenue records. Their definition of income—which may be called *taxable* income—excludes depreciation (at Inland Revenue rates), part of government transfers, the investment income of life assurance and superannuation funds and non-profit-making bodies, employers' and most of employees' contributions for national insurance and private superannuation, most income in kind, part of the imputed rent of owner-occupied houses, and the interest accruing on National Savings Certificates. Against this, taxable income includes all retirement pensions. A further difference is that the estimate of profits of unincorporated

firms, including professional persons and farmers, which is used in the national income accounts is higher than the total of such profits assessed by the Inland Revenue. This difference is attributable, first, to the fact that the national income figure includes depreciation, secondly, to a difference in the method of estimating farmers' net incomes and, thirdly, to an allowance—which is necessarily arbitrary—for under-assessment of profits.

In the Blue Book tables of personal income by ranges, the government statisticians have adjusted the Inland Revenue distributions by adding in incomes below the exemption limit (down as far as £50), national assistance grants and those national insurance benefits and grants which are non-taxable, and the estimated income in kind of domestic servants and agricultural workers. These are comparatively straightforward adjustments; but they still leave a volume of unallocated income which seems, at first sight, formidably large. In 1938, out of total personal income of £5,078 million, £615 million (or 12 per cent.) has not been allocated; and in 1957, out of nearly £18,000 million, the unallocated portion amounts to £2,737 million (or 14 per cent.). On closer inspection, however, it appears that these sums include several items which can scarcely be regarded as forming part of personal income in any normal sense of the term. This applies particularly to employers' contributions for national insurance and to depreciation allowances of unincorporated businesses. Similarly, if we are only interested in personal income after tax, there is no need to add back that part of employees' national insurance contributions which is deductible for tax purposes. The investment incomes of non-profit-making bodies can also be excluded; and, finally, certain kinds of government grants in kind which the Central Statistical Office includes in personal income can just as well be treated as part of social service expenditure (and brought into account under that heading).

With these adjustments the unallocated part of personal income is considerably reduced, leaving only about £470 million in 1938 and £1,775 million in 1957. It consists broadly of two kinds of income: firstly, the income of life assurance funds from employers' contributions and investments, plus the income of superannuation funds from all contributions and investments, less current retirement pensions from these sources (the net amount being about £120 million in 1938 and £650 million in 1957); and secondly, various kinds of unassessed income, including interest accruing on National Savings Certificates, the imputed rent of owner-occupied homes (in excess of the Schedule A valuation), co-operative society dividends, post-war credits, and a residue of unassessed profit and investment income.

There is an obvious danger that if we concentrate our attention solely on allocated income we may be led to draw incorrect conclusions about the trend in the distribution of income as a whole. Some attempt should, therefore, be made to allocate the unallocated part of income. That this is not an easy task is clear from the fact that the government statisticians have not attempted it; and in the present state of knowledge it is really not possible to make a complete and detailed allocation of these items. Barna (1945) allocated some of the components of unallocated income for 1937, and Cartter (1955) allocated all of it for the year 1948–49, but used very unsatisfactory assumptions. In view of the great importance of the matter I have been bold enough to make an estimate of the share of the top 1 per cent. of the population in unallocated income. These estimates are presented in Section 6; and in the same section I have estimated the effects on the share of income received by this top group of the population of three other adjustments to personal income. These are: (1) the addition to income of benefits in kind derived from the social services, (2) the reduction in the purchasing power of income which results from indirect taxation,

after allowing for subsidies, and (3) the addition of imputed income from undistributed company profits.

3. Method of Analysis

The distribution of income does not conform exactly to any simple mathematical function. Although Pareto obtained surprisingly good results when he tested his hypothesis against data available to him on the distribution of higher incomes in various countries, a closer study of the matter reveals that the Pareto function is only an approximation to the shape of the upper tail of the distribution. The double-logarithmic cumulated frequency line (in this country at least) has a definite "wave" in it, and probably also a slight but steady downward curvature in the higher ranges. Various writers have suggested that the distribution of incomes can most aptly be described by means of a lognormal curve. Aitchison and Brown (1957) give an excellent summary of the history of these suggestions and a lengthy bibliography. The lognormal curve gives a better fit than most other simple functions but there are systematic deviations which prevent us from accepting it as an exact description. Moreover, despite the existence of a number of plausible hypotheses to explain why the distribution of income should be lognormal in character, none of them is entirely convincing. (See Aitchison and Brown (1957) for references.) We are, therefore, obliged to take a pragmatic approach to the task of describing the distribution.

There are several convenient graphic methods of comparing income size distributions, of which the Pareto and Lorenz curves are the best known. (See Bowman (1945) for a useful summary.) Some writers also use the Gini coefficient to summarize the shape of the Lorenz curve. My intention, however, is not so much to summarize the distribution as a whole as to consider changes in the shape of the distribution in its different parts. For this purpose the Lorenz curve tends to be too insensitive an instrument and I have used instead a number of different curves drawn on double-logarithmic paper. With their assistance it is possible to establish fairly accurately the position of each percentile in the top half of the income distribution, and, in some of the post-war years with somewhat less accuracy, down as far as the seventieth percentile from the top. In the same way we can measure the proportion of total income received by each inter-percentile group. Thus it is possible to follow the changes over the years in the absolute and relative incomes of particular percentiles, and in the proportions of total income received by particular percentage groups of the population. The method used here is similar in many aspects to that used by Paish (1957) in his stimulating paper.

Although the method of percentiles is a convenient way of comparing changes in the level of income of persons at equivalent positions in the income distribution, it is not intended to imply that the unit occupying a given percentile position in one year is the same unit as that which occupies the same percentile in a different year. If every unit maintained its rank in the social hierarchy from year to year the comparison would be exact; but there is, of course, some reshuffling of ranks as time passes. Some of this is caused by the rise or decline of the fortunes of particular economic or occupational groups; some is also attributable to the fluctuations of fortune, such as the rapid promotion of an individual, the loss of a job, retirement, changes in the profits of a business and the like. We shall be able to study the changes in the general shape (and size) of the top half or two-thirds of the income pyramid but it will not be possible, with the data at our disposal, to study the fluctuations in the fortunes of individuals. This is a subject about which very little is known in any country, although much emphasis is given to it by certain economists who aim at

explaining the differences in savings behaviour of different income groups by this means (Modigliani and Brumberg, 1954; and Friedman, 1957).

Since the data at our disposal are derived originally from the records of the Inland Revenue, the units whose income will be considered in this paper are income units as defined for tax purposes. According to Inland Revenue practice the income of a married man includes the income of his wife; and tax units, therefore, consist of single persons (with incomes above the exemption limit) and married couples, together with their dependants. It is arguable that a more satisfactory unit of account would be the household or the family, and some students of this subject have striven to allocate all income to individual persons, including children; but for practical purposes the income unit as defined in British tax practice is not an unreasonable unit of account. It should, however, be noted that one effect of treating a married couple as a single unit for income purposes is that, when two people who are both in receipt of an income get married, two units in the distribution of income are eliminated and are replaced by one unit at a higher income level. In other words, the shape of the income distribution can be affected by the marriage habits of the population and, more especially, by a change in the proportion of married women who go out to work.* We shall be obliged to make a more detailed study of the influence of changes of this sort on the distribution of income at an appropriate stage in the ensuing discussion.

The Blue Book tables show the estimated number of incomes falling within specified ranges of income before tax, and the amounts of income, of combined income tax and surtax, and of income after tax in each of these ranges. In order to identify the percentiles it is, of course, necessary to know the total number of incomes. For the post-war years the government statisticians have made estimates of the total number of income units receiving an income of £50 a year or more; but for 1938 there is no such official estimate. The method used for the post-war years gives results which are broadly equivalent to those which would be obtained by taking the total number of married couples and of single persons aged 18 and over, which is a possible alternative way of defining the number of incomes. I have estimated the total number of incomes in 1938 by the latter method, using the information collected in the *National Register* of September 1939 (*National Register*, 1944). In round numbers the figure comes to 24 millions, which is not very different from the estimates which have been made by other investigators. Seers (1950, p. 32) estimated the number at 23½ millions, using a different, but very rough, method.

The latest Blue Book (1958, Table 31) contains estimates of the distribution of allocated income for the years 1938, 1949, 1954, 1955, 1956 and 1957. In this paper I shall concentrate attention on the years 1938, 1949, 1954 and 1957. The first three of these are the years (for which Blue Book estimates are available) which are closest to the Inland Revenue's income surveys of 1937–38, 1949–50 and 1954–55; and it is clear that the Blue Book data for these years are likely to be better than for others. It is also possible to supplement the Blue Book information for these years by reference to the more detailed income survey tables published by the Inland Revenue. The year 1957 is included as being the most recent.

4. The Trend in the Distribution of Allocated Income

Estimates of the values of the percentiles of income before and after tax in the years 1938, 1949, 1954 and 1957 are given in Table 1. Since the figures have been obtained by

* A change in the proportion of women at work would, of course, change the shape of the distribution, irrespective of Inland Revenue practice.

graphic interpolation they may deviate by one or two per cent. from their true values, but the broad pattern of the changes in the relative position of the different percentiles would not be affected by any inaccuracies of this sort. This pattern is best observed when the figures in Table 1 are converted into index numbers, as is done in Table 2. From the top

TABLE 1

Percentiles of Allocated Income Before and After Tax

Percentile*	Before Tax 1938 £	Before Tax 1949 £	Before Tax 1954 £	Before Tax 1957 £	After Tax 1938 £	After Tax 1949 £	After Tax 1954 £	After Tax 1957 £
First	1,140	1,860	2,210	2,450	940	1,280	1,600	1,800
Fifth	393	765	995	1,180	380	655	890	1,020
Tenth	266	565	795	940	263	520	740	850
Twentieth	185	430	625	792	184	410	610	740
Fiftieth	(110)	261	382	512	(110)	250	(360)	(470)

* Numbered from the highest downwards.
Note: Less reliable estimates are put in brackets.

TABLE 2

Index Numbers of Percentiles of Allocated Income

1938 = 100

Percentile	Before Tax 1949	Before Tax 1954	Before Tax 1957	After Tax 1949	After Tax 1954	After Tax 1957
First	163	194	215	136	170	191
Fifth	194	252	299	172	234	268
Tenth	212	299	353	198	281	323
Twentieth	232	338	428	223	332	402
Fiftieth	(237)	(347)	(465)	(227)	(327)	(427)

Percentile	1949 = 100 Before Tax 1954	1949 = 100 Before Tax 1957	1949 = 100 After Tax 1954	1949 = 100 After Tax 1957	1954 = 100 Before Tax 1957	1954 = 100 After Tax 1957
First	119	132	125	141	111	113
Fifth	130	154	136	156	119	115
Tenth	141	166	142	163	118	115
Twentieth	145	184	149	180	127	121
Fiftieth	(146)	(196)	(144)	(188)	(134)	(131)

Note: Less reliable estimates are put in brackets.

half of that table it will be seen that, whereas the first percentile (from the top) in the pre-tax distribution rose a little more than twofold between 1938 and 1957, the fiftieth percentile (which is the median) increased to more than four and a half times its 1938 level. The fifth, tenth, and twentieth percentiles occupy intermediate positions and maintain a consistent pattern, the lower percentiles always showing a greater increase since 1938 than those above them. The percentiles in the post-tax distribution exhibit similar tendencies, although they have increased less rapidly than the pre-tax percentiles. The contrast between the rate of increase of the top percentile and the median remains very striking.

These figures lend strong support to the view that there has been a substantial reduction

in inequality of income since 1938. But a number of interesting details should also be noted. Firstly, the tendency towards reduced inequality of allocated income has persisted in each of the four subperiods. In the first subperiod (1938–49) the top percentile in the pre-tax distribution rose by slightly less than half the percentage increase of the median (63 per cent. compared with 137 per cent.); in the second subperiod (1949–54) the relative rate of increase of the median was slightly more rapid; and in the third subperiod (1954–57) the median appears to have increased by three times the percentage rate of increase of the top percentile. If anything, the tendency towards reduced inequality of pre-tax income seems to have been accelerating.*

A second noteworthy feature of these figures is that they show that the direct effects of taxation bear comparatively little responsibility for the reduction in inequality of post-tax income. If we take the ratio of the first percentile to the median as a measure of inequality of income, we can partition the total change in this ratio between any two years in the post-tax distribution into the part which is caused by a reduction in pre-tax inequality and a remainder, which is attributable to the tax system. Between 1938 and 1957 the ratio of the first post-tax percentile to the median fell from 8·55 to 3·83, or by 55 per cent.; but of this total change almost all is attributable to the change in the pre-tax ratio, which fell in the same period from 10·36 to 4·79, or by 54 per cent. The impact of taxation was greatest in the first subperiod, when the post-tax ratio fell by 40 per cent. and the pre-tax ratio by 31 per cent. But even in this case three-quarters of the reduction in inequality—as measured by this ratio—was caused by the change in the pre-tax distribution.

Since 1949 there have been substantial reductions in tax rates which have been sufficient to offset part of the trend towards greater equality of pre-tax income. Thus from 1949 to 1957 the pre-tax ratio of the first percentile to the median fell from 7·13 to 4·79, or by 33 per cent., but the post-tax ratio fell from 5·12 to 3·83, or by only 25 per cent. The same effect can be observed if we compare the change in the pre-tax value of each percentile with the change in its post-tax value. From 1938 to 1949 the pre-tax value of the first percentile rose by much more than its post-tax value (63 per cent. compared with 36 per cent.), while the pre-tax value of the median rose only slightly more than its post-tax value (137 per cent. compared with 127 per cent.). Between 1949 and 1957, on the other hand, and in both the subperiods within this period, the post-tax value of the first percentile rose more than its pre-tax value, while the post-tax value of the median rose slighly less than its pre-tax value. It would not be correct, however, to assume that the whole of this difference is attributable to the relaxations in tax rates, substantial though they have been. For if the pre-tax income of the first percentile had increased in the same proportion as the pre-tax income of the median, the effective tax rate on the first percentile would have been heavier than it actually was, with the consequence that the post-tax income of the first percentile would have increased less rapidly than its pre-tax income. Hence the peculiar inversion of relative movements noted above must be recognized as being a combined effect of the relaxation of tax rates themselves and of the persistent failure of pre-tax allocated incomes in the upper ranges to rise as fast as pre-tax allocated incomes in the middle of the distribution.

* There is some evidence that in the period 1949–54 some of the percentiles below the median did not increase so fast as those above it, at least as far up as the fifth percentile. For example, the seventieth percentile in the Inland Revenue's estimate of taxable income in 1949–50 stood at £170, and at £220 in 1954–55, an increase of only 29 per cent., compared with a 44 per cent. increase for the median. Since 1954, however, there is no evidence of any similar tendency for the percentiles below the median to lag behind.

The trend in the distribution of allocated income before tax over this period can be appreciated most vividly from an inspection of Fig. 1, where the percentiles of income before tax are plotted on a logarithmic scale against the passage of time on an arithmetic scale. The slopes of the lines connecting the percentile points on Fig. 1 measure the average annual rates of increase in the income of each percentile over each period. The slope of the tenth percentile has been almost constant over each of the three periods studied while the other percentiles seem to be converging towards it. The convergence of the first and

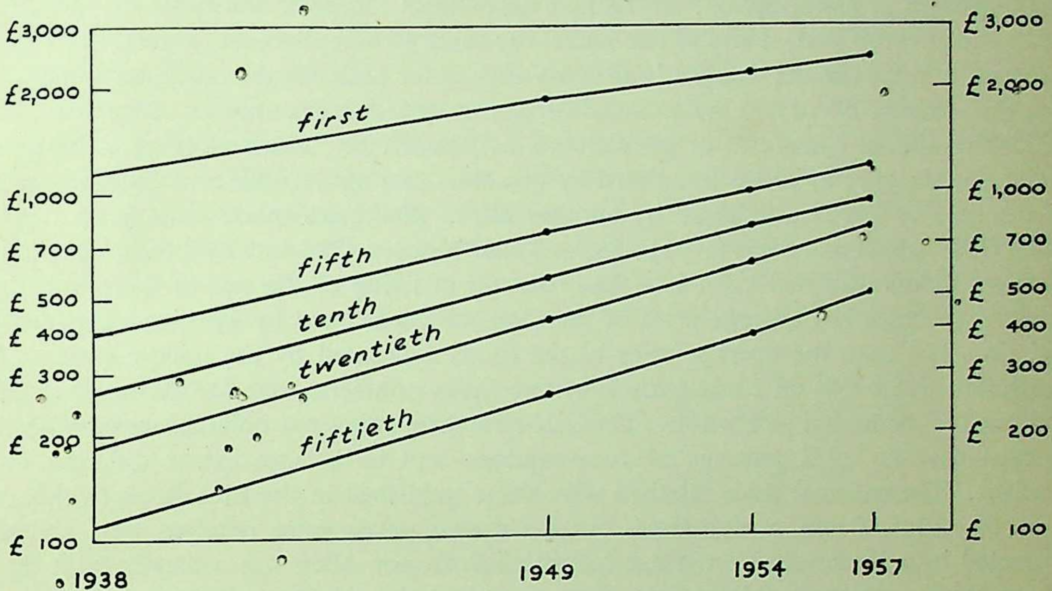

FIG. 1.—Percentiles of income before tax.

fifth percentiles is especially marked since 1949, and of the twentieth and fiftieth percentiles since 1954. A crude extrapolation of the trend since 1949 would bring the fifth, tenth and twentieth percentiles very close together in ten years from now and the distance between the first and fiftieth percentiles would be much diminished.

The Changes in Real Allocated Incomes

Up to this point we have been concerned with percentiles of income expressed in current values. But since prices have been rising continuously over this period, it is necessary to adjust for price changes if we wish to measure the changes in real incomes. Such a calculation can, of course, provide only a rough estimate of what we want to know, since the prices of individual commodities have changed in varying degrees since pre-war days and the pattern of consumption has also been changing, partly no doubt in response to relative price movements. The simplest procedure would be to divide current money values by a single generalized consumer price index, such as that which is published in the National Income Blue Book. This index measures the difference between the cost of the actual assortment of goods and services purchased by British consumers as a whole in a given year and the cost of buying the same assortment of goods and services in 1948.* For the interval 1938–48 this index is, in effect, a Laspeyres (base-weighted) index; but

* In the latest Blue Book an index based on the consumption pattern in 1954 is also published.

for later years it is a Paasche, or currently weighted, index. In normal circumstances, it might be expected to over-estimate the rise in prices from 1938 to 1948 and to underestimate it thereafter; but since rationing interfered with the freedom of consumer choice for the greater part of the period under review, it is impossible to say whether the normal expectations are likely to have been fulfilled in this case.

But the use of a single index to deflate the incomes of all classes of the community can be criticized on the ground that it pays no regard to possible differences between classes in the pattern of consumption. If, to take an extreme example, one class ate only bread and another only meat, it would not make any sense to compile a single food price index intended to represent the changes in the cost of food for both classes, unless it so happened that the prices of bread and meat moved exactly in step with one another. Moreover, even within traditional categories of goods, such as "meat", or "men's clothing", the prices of the specific type of goods purchased by one class may move differently from the prices of the type of goods purchased by another class. Estimates made some years ago by Allen (1949, 1952) and Seers (1951) suggested that between 1938 and 1947 the cost of living of the working class had risen less than the cost of living of the rest of the population, primarily because the average price of the foodstuffs consumed by working-class families had risen less than the average price of the foods consumed by the middle class. This conclusion was based on a comparison of two index numbers: one for the working class and one for the nation as a whole. The index numbers employed different weights—based in each case on 1938 patterns of consumption—and to a large extent different price relatives. The national price relatives were those published in the Blue Book (which was in fact a White Paper at that time) but the working-class price relatives were specially estimated by a Technical Committee (of which Professor Allen was a member) set up to advise on the construction of a new index to replace the old Cost-of-Living Index.

All work on estimating comparative changes in the cost of living of different classes has been seriously hampered up till recently by the lack of data on the different expenditure patterns of those classes. But with the publication of the results of the Ministry of Labour's 1953 household expenditure survey (*Report of an Enquiry into Household Expenditures in 1953–54* (1957)) it is now possible to make direct estimates of the changes in the cost of living of many different groups of the population. I have used this material to compile separate index numbers for what are called in the Ministry's report "high income" households and "index" households respectively. The first group—consisting of 460 households, or 3·6 per cent. of the whole sample—includes all households in which the head received a gross income in 1953 of £20 a week or more; and the second group contains all the remaining households except 860 "pensioner" households, these being almost all single or married old-age pensioners living alone. As a rough approximation we may assume that the pattern of expenditure shown by the "high income" group of households is representative of the pattern of expenditure of the first and fifth percentiles in the distribution of income, and that the pattern of expenditure of the "index" households is representative of the other percentiles identified in our tables.

In estimating the weights for these two index numbers it was necessary to make some adjustments to the expenditure figures collected in the survey. As might be expected, people tend to give incomplete reports on their purchases of alcoholic drinks and tobacco. Adjustments can be made with some confidence to the average figures for all households, on the basis of national estimates of sales of these two items, but it is less easy to estimate

the degree of bias in reporting at different income levels. It was assumed that the downward bias on reported tobacco expenditure was proportionately about the same amongst "high income" households and "index" households, but that the downward bias on reported expenditure on alcoholic drinks was slightly less for the former group of households than for the latter.*

Apart from the weights, it was also necessary to have some estimates of price changes for the individual commodities. These were taken from the National Income Blue Book, which contains information from which separate price index numbers for a fairly large number of groups of commodities and services can be computed. Unfortunately, it is not possible to obtain from the Blue Book separate index numbers for subgroups of food in 1938, so that for that year it was necessary to use the average index for all food, weighted by national consumption.† The effect of this may be to reduce differences between the index numbers for different income groups. But this would be more likely to be important if we were estimating changes in the cost of buying the assortments of commodities purchased by different classes before the war. The change in food habits during the war and the substantial rise in the standard of living of the working class will have brought the patterns of food consumption of different classes much closer together than they were in 1938.

The two index numbers which emerge from these calculations (and which are shown in Table 3) exhibit one very surprising feature: namely, the almost identical change in the

TABLE 3

Index Numbers of Consumer Prices

(1949 = 100)

Year	"*High Income*" *Households* (*1953 Weights*)	"*Retail Index*" *Households* (*1953 Weights*)	*Blue Book Index** (*Current Year Weights*)
1938 . . .	52·7	52·5	50·2
1949 . . .	100·0	100·0	100·0
1954 . . .	121·5	123·6	123·2
1957 . . .	134·9	138·0	137·8

* Obtained by dividing total consumers' expenditure in the United Kingdom in each year at current prices by the sum of the same quantities of goods and services when valued at 1948 prices, the result being expressed as an index based on 1949 = 100.

index for each group of households between 1938 and 1949.‡ Since this result appears to run completely contrary to the conclusions of earlier studies, it is desirable to consider the reasons for it in some detail.

It should first be noticed that the dividing line between the upper and lower income groups in this study is not drawn at the same point as in the Seers-Allen studies. The

* The assumed expenditure figures were, for "high income" households 25*s.* a week on tobacco and 30*s.* a week on drink, and for "index" households, 19*s.* a week on tobacco and 17*s.* a week on drink.

† Derived from Tables 26 and 28 of the 1958 Blue Book. A similar procedure was followed for housing, motoring and entertainments, where the subcomponents for 1938 are not available. For recreational goods, chemists' goods and other goods it was necessary to make guesses of the appropriate 1938 figures. These guesses were 67, 60 and 70 respectively (1948 = 100).

‡ An index number for each year (x) was estimated on the formula:

$$I_x = \frac{\Sigma p_x q_{53}}{\Sigma p_{53} q_{53}}$$

These index numbers were then converted to the base 1949 = 100.

latter authors were concerned with the difference between the "working class" and the "middle class". The "working class" was defined as all wage earners and non-manual workers with salaries of less than £250 in 1938, together with their dependants. Seers estimated that the "working class' on this definition accounted for 80–85 per cent. of the population in 1938. The "middle class" were the remaining 15–20 per cent., not all of whom would have incomes above those of the "working class". In this study, on the other hand, the dividing line is drawn at about 5 per cent. from the top of the income distribution.

But this difference of definition is unlikely to account for much of the discrepancy. Differences in weighting may be partly responsible, especially the differences introduced by the choice of 1953 weights in place of 1938 weights. But the main cause of the conflicting results is almost certainly the fact that Seers and Allen were able to use two different sets of price relatives for their two groups of consumers. In this study on the other hand, in the absence of price relatives appropriate to the top 5 per cent. of the population, the same price relatives have been employed in the calculation of both index numbers. Thus the Seers-Allen index numbers reflect differences between the classes both in their expenditure patterns and in the relative price movements of the "same" commodities, while the index numbers given here reflect only the differences in expenditure patterns.

Even so, it is remarkable that the differences of expenditure patterns,[1] which are quite considerable, do not cause any divergence in our two index numbers in the period 1938–49, despite the fact that there was a wide difference in the movements of prices of different commodities in this period. The explanation seems to lie in the fact that between 1938 and 1949 the price of food rose less than average and the price of tobacco more than average. Since both items are of greater significance to "index" households than to "high income" households, the advantage to the former group of the relatively low price of food was largely offset by the disadvantage of the relatively high price of tobacco. Broadly, it can be said that during this period the Government took money away from the male members of working-class households through the tobacco tax and passed it back to the housewives by means of the food subsidies.

From 1949 onwards the two index numbers show a slight tendency to diverge, the index for "retail index" households rising somewhat faster than the index for "high income" households. The principal reason for this is that—as a consequence of derationing of food and the removal of food subsidies—food prices have risen much faster than other prices in recent years. But there was an important offset to this, for poorer families, in that the price of tobacco rose very little. Index numbers of the Seers-Allen type for this period would probably show a wider difference between the rate of increase of the cost of living for "working class" and "middle class" households than is shown by my index numbers for "index" households and "high income" households respectively. Thus the gap between the "class" index numbers, which appeared to have widened so greatly between 1938 and 1949 in the Seers-Allen calculations, has closed—or more than closed—in the past eight years, leaving a relative net change over the whole period very similar to that suggested by my two index numbers. (A recent calculation by Professor Allen (1957) suggests a somewhat different conclusion. But his methods are not the same as they were in the earlier studies.)

With the help of these index numbers it is now possible to deflate the money incomes of the percentiles. For reasons already given, the "high income" index is used for deflating

the first and fifth percentiles and the other index is used for deflating the other percentiles. The results, expressed as index numbers based on 1938, are shown in Tables 4 and 5.

TABLE 4

Index Numbers of Percentiles of Allocated Income Before Tax in Constant Prices

Percentile	1938	1949	1954	1957
First	100	86	84	84
Fifth	100	103	110	117
Tenth	100	111	127	134
Twentieth	100	122	144	163
Fiftieth	100	(124)	(147)	(177)

TABLE 5

Index Numbers of Percentiles of Allocated Income After Tax in Constant Prices

Percentile	1938	1949	1954	1957
First	100	72	74	75
Fifth	100	91	102	105
Tenth	100	104	120	123
Twentieth	100	117	141	153
Fiftieth	100	(119)	(139)	(162)

The broad picture which emerges from these two tables is very clear and definite. In 1949, by comparison with 1938, the real pre-tax allocated income of the first percentile had fallen by 14 per cent. and its real post-tax income had fallen by 28 per cent. In the same period the real pre-tax income of the median had risen by 24 per cent., and its post-tax income by 19 per cent. Since 1949 the real pre-tax income of all the percentiles below the first have risen, the lower percentiles continuing to rise faster than those above them, down at least as far as the median; but the real pre-tax income of the first percentile has not increased at all. In terms of post-tax real income, however, the first percentile has made a very slight recovery, consequent on the reduction in tax rates since 1949. In real terms the allocated disposable incomes of the middle groups are now about 60 per cent. higher than before the war, the allocated disposable incomes of those at or near the tenth percentile are 20–25 per cent. higher, those at or near the fifth percentile are only slightly higher, and those at or near the first percentile are about 25 per cent. lower. These conclusions must not, of course, be assumed to be automatically applicable to personal income in its wider sense.

Shares of Allocated Income Taken by Specified Inter-percentile Groups

Our discussion so far has been in terms of the relative positions of the different percentiles. A closely related method of presenting changes in the distribution of income, which has been widely used by other investigators both in this country and abroad, is to estimate the proportion of total income received by different inter-percentile groups. The relevant estimates have been made by plotting on a double-logarithmic scale the cumulative amounts of income received by units above each published level of income, and interpolating to discover the amounts received by each inter-percentile group. The resulting figures for the distribution of pre-tax income are given in Table 6.

TABLE 6

Percentage of Allocated Income Before Tax Received by Specified Inter-percentile Groups

Inter-percentile Group	1938	1949	1954	1957
Top 1 per cent.	16·2	11·2	9·3	8·0
Second to fifth per cent.	12·8	12·3	11·2	10·2
Sixth to tenth per cent.	9·0	9·5	9·5	9·8
Eleventh to twentieth per cent.	12·0	14·5	16·0	13·5
Top 5 per cent.	29·0	23·5	20·5	18·2
Top 10 per cent.	38·0	33·0	30·0	28·0
Top 20 per cent.	50·0	47·5	46·0	41·5

These figures illustrate the same general trend as we have already noted. The proportion of income before tax received by the top one per cent. of income units fell between 1938 and 1949 from 16·2 per cent. to 11·2 per cent. In the next five years it fell further to 9·3 per cent.; and by 1957 it was down to 8·0 per cent. The change in the fortunes of the next group—those falling between the first and fifth percentiles—was similar in direction but much less catastrophic. In the next band of incomes—that falling between the fifth and tenth percentiles—the proportion of income received was higher in 1954 than in 1938, and increased further in 1957. In the fourth band—running from the tenth to twentieth percentiles—the proportion of income received rose until 1954, but appears to have fallen considerably in 1957.

The movements in the share of post-tax income retained by the different inter-percentile groups are very similar to those exhibited by their pre-tax shares (see Table 7).

TABLE 7

Percentage of Allocated Income After Tax Received by Specified Inter-percentile Groups

*Inter-percentile Group**	1938	1949	1954	1957
Top 1 per cent.	11·7	6·5	5·4	4·9
Second to fifth per cent.	12·4	10·8	10·0	9·1
Sixth to tenth per cent.	9·5	9·6	9·7	9·5
Eleventh to twentieth per cent.	12·8	15·4	16·8	14·5
Top 5 per cent.	24·1	17·3	15·4	14·0
Top 10 per cent.	33·6	26·9	25·1	23·5
Top 20 per cent.	46·4	42·3	41·9	38·0

* These groups are derived from the distribution of income before tax. Thus the table shows what percentage of post-tax income is received by each pre-tax inter-percentile group.

The post-tax share of the top 1 per cent. of income units, for example, fell from 11·7 per cent. in 1938 to 6·5 per cent. in 1949, to 5·4 per cent. in 1954 and 4·9 per cent. in 1957. But the proportionate fall in this group's share of post-tax income has, in recent years, been smaller than the fall in its pre-tax income. This reflects the relaxation of tax rates which, as can be seen in Table 8, have become on the average slightly lower since 1949 for incomes down to the fifth percentile. On the other hand the average tax rate on the next band of income units seems to have risen between 1954 and 1957. Here the upward thrust of money incomes has been more than sufficient to offset the fall in rates, so that the average effective tax burden on this group has increased.

TABLE 8

Income Tax and Surtax as Percentage of Total Income Before Tax in Specified Inter-percentile Groups

Inter-percentile Group	1938	1949	1954	1957
Top 1 per cent.	32	49	48	46
Second to fifth per cent.	10	23	20	20
Sixth to tenth per cent.	2	11	9	14
Eleventh to twentieth per cent.	1	6	6	4
Top 5 per cent.	23	35	33	31
Top 10 per cent.	18	28	25	25
Top 20 per cent.	14	21	18	18
All incomes	7	12	10	11

5. CAUSES OF THE TREND TOWARDS GREATER EQUALITY OF ALLOCATED INCOME

The most obvious cause of the change in the size distribution of income before tax has been the change in the composition of income by factor source. The dominant trends have been an increase in the share of income derived from wages and salaries and a decline in the share of income derived from rent, interest and dividends. Secondly, there seems to have been a tendency towards a greater equalization of income *within* each factor source, for example, by a reduction in the relative differential between wage and salary rates. A third cause of changes in the size of distribution has been the growth in the number of married women at work, which, as was explained earlier, has the effect of shifting income units into higher income ranges. We shall consider each of the causes in turn.

Changes in the Factor Composition of Personal Income

The movements in the components of aggregate personal income before tax (including unallocated personal income) from the different factors of production are shown in Fig. 2. Since we are not concerned with the detailed course of events during the war, figures for the years 1939–45 have been omitted, and the trend over this period is shown by connecting the points for 1938 with those for 1946 by straight lines. The income data are plotted on a logarithmic scale and the slope of each line represents its annual rate of increase.

The most striking feature of Fig. 2 is the extraordinary constancy of the upward trend of employment income, i.e., income from wages and salaries and other employee income.* With the exception of a slight downward kink in 1950, which was a consequence of the Government's urgent appeals for wage restraint, the employment income line has pursued a steady upward path at a rate of about 8 per cent. per annum until 1956. From 1956 to 1957 the rate of increase was slightly reduced to 6·2 per cent. On the average, however, the rate has been the same since the war as it was during the war, and, apart from slight deviations in 1950 and 1957, it has shown no sign of moving differently under different governments or in different economic circumstances. Total employee income is, of course, the product of average employee earnings and the number of employees. Part of the increase in the aggregate figures each year has come from an increase in the employed labour force; but this has not amounted in any peace-time year to more than 1½ per cent.,

* This includes the pay of the Armed Forces and employers' national insurance and superannuation contributions.

and over the whole period 1938–57 the average compound rate of increase has been about 1 per cent. per annum. Hence the average compound rate of increase in employee income per head, in current money values, has been about 7 per cent. per annum.

By contrast with employment income, the other sources of income have followed much less regular paths. The general direction has, of course, been upwards, but there has been considerable variation in the rate of progression, both between the different sources of

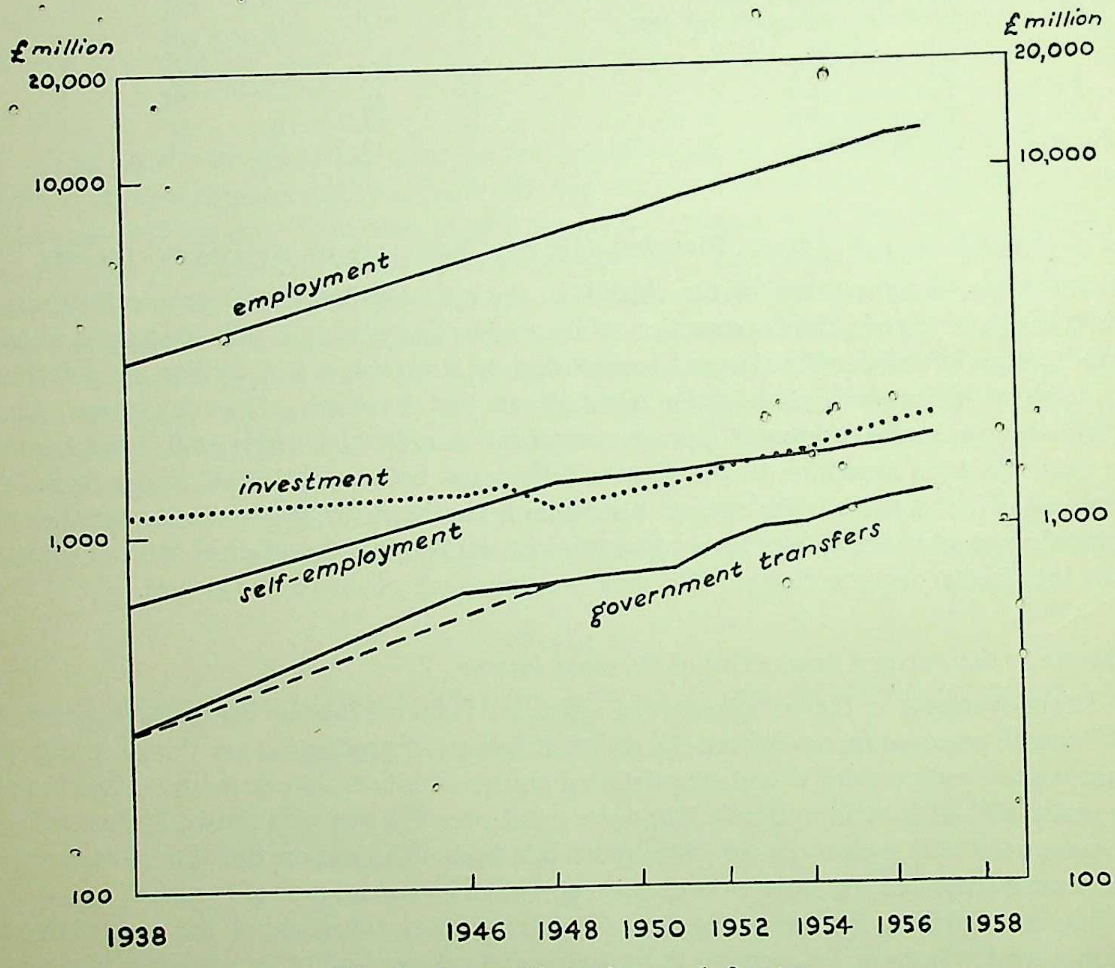

FIG. 2.—Personal factor incomes before tax.

income and between different periods of time. Between 1938 and 1946 income from self-employment increased almost as fast as income from employment, and income from government transfers increased much faster. On the other hand, investment income rose scarcely at all between 1938 and 1946. It is not, in general, surprising that income from self-employment, i.e., the profits of farmers, unincorporated businesses and the professions, should keep broadly in step with employment income; but within the total there was a much larger increase of farm income than of the other types. Farm income before the war had been heavily depressed; and, as a result of the great war-time demand for home-produced food, it rose rapidly, reaching by 1946 a figure more than three times as high as in 1938. Although government transfer income had risen altogether somewhat more than

employment income by 1946, this was chiefly the consequence of a large increase in release leave payments to demobilized members of the Armed Forces. It was not until 1947 that a large jump occurred in retirement pensions, with the coming into force of the National Insurance Act. It would, therefore, be more reasonable to indicate the trend in government transfer payments over this period by connecting the years 1938 and 1948, as is done by the dashed line in the chart.

The most important feature of this period, in its effect on the size distribution of income, was the almost complete stability of personal investment income. One of the main causes of this was the freezing of rents by rent control; but, since government debt interest and the trading profits of companies both increased very substantially, it is at first sight surprising that investment income showed so little improvement. The explanation can be found from a study of the combined appropriation accounts of companies (Blue Book, 1958, Table 3). First, it must be remembered that a very large proportion of extra company profits was taxed away under E.P.T. and ordinary income tax. Between 1938 and 1946 total corporate income before tax increased by 78 per cent. but the increase after tax was only 42 per cent. Secondly, companies had developed a policy of holding back a much larger proportion of their net income in undistributed profits than before the war. Whereas in 1938 dividend payments were three times as great as undistributed profits (after tax, depreciation and stock appreciation), by 1948 net undistributed profits were nearly equal to dividends.

By the end of the war there had been a radical redistribution of personal income before tax, primarily as a result of the decline in the relative importance of investment income. The great change which had taken place can be seen from Table 9. Whereas in 1938

TABLE 9

Percentage of Total Personal Income Before Tax Arising from Different Sources

Source of Income	1938	1949	1954	1957
Wages	37·8	41·9	42·6	42·9
Salaries	17·9	20·5	21·6	22·6
Other employee income	3·8	6·0	6·6	6·5
Total employment income	59·5	68·4	70·8	72·0
Professions	2·3	2·1	1·7	1·5
Farmers	1·4	3·2	2·7	2·4
Other self-employed	9·1	7·8	6·6	6·1
Total self-employment income	12·8	13·1	11·0	10·0
Rent, dividends and interest	22·3	11·4	11·1	11·1
Government transfers	5·4	7·1	7·1	6·9
Total personal income	100·0	100·0	100·0	100·0

employment income represented only 59·5 per cent. of total personal income, by 1949 it had grown to 68·4 per cent. Government transfer income, also, had risen from 5·4 to 7·1 per cent. of the total. The aggregate share of the self-employed was almost unchanged, but farmers' incomes had gained while the others had declined. The investor was the great loser: his share in pre-tax personal income had fallen from 22·3 per cent. to 11·4 per cent. This was indeed a long step towards the "euthanasia of the rentier" which Keynes had

believed to be desirable. But it came out of the exigencies of the war, not from a deliberate policy of changing the distribution of income.*

The course of events since 1946 can readily be studied in Fig. 2. While employment income has held to its steady upward course, the income of the self-employed has dropped behind. Investment income, after a further sharp fall in 1948 following the nationalization acts, has gradually recovered; and since 1951 it has grown at a brisk pace, parallel with employment income. Government transfers, after the big increases in national insurance payments in 1947 and 1948 (which are masked in the chart by the decline in post-war release leave payments) showed a tendency to lag behind, and it was not until 1952 that they rose sharply once more. In recent years they have lagged again somewhat in comparison with employment income; but the substantial increases in rates of payment announced towards the end of 1957 will bring them up sharply in 1958.

The most remarkable characteristic of the post-war period has been the relative stagnation of self-employment income. The upward trend of self-employment income continued parallel with employment income until 1948, but after that year self-employment income rose much less than other forms of personal income. The contrast between the trends in self-employment income and in other types of income can be seen in Table 10.

TABLE 10

Index Numbers of Changes in Personal Factor Incomes Since 1948

	1948 = 100		1951 = 100
Source of Income	1951	1957	1957
Wages	122	185	152
Salaries	129	202	157
Total employment income*	125	191	153
Professions	109	125	114
Farmers	121	145	120
Other self-employed	106	135	127
Total self-employment income	110	135	123
Rent, dividends and interest	110	169	152
Government transfers	111	178	160
Total personal income before tax	120	180	150

* Including other employee income.

Between 1948 and 1951 wages increased by 22 per cent. and total employment income by 25 per cent. but the income of the self-employed rose by only 10 per cent. A similar difference is observable for the years 1951–57, and for the whole period from 1948 to 1957 self-employment income increased by only 35 per cent. as against an increase in employment income of 91 per cent.

What is the explanation of this stagnation of self-employment income? There seem to be several. First, there has been a tendency in recent years—especially since 1949—for

* There was a similar, but not so marked, development in the United States. The share of investment income in that country fell from 17 per cent. in 1938 to 12 per cent. in 1949 and has stayed at that level since.

professional earnings to rise very slowly. This is partly the result of the standstill on doctors' earnings under the National Health Service. (The Danckwerts award, most of which was retrospective to 1948, has been made retrospective to the same extent in the official national income estimates, so that only the further small increase in doctors' rates which took effect from 1951 has influenced the trend.) It is also, perhaps, a reflection of a lag in the standard fees of other professions, such as solicitors and barristers, and it is possible that some of the apparent sluggishness of professional earnings is the result of an increased tendency to reduce taxable income by charging expenses against earnings.

So far as farmers are concerned this last explanation is unlikely to be important, since the national income estimates of farmers' profits are derived from global estimates of the trading account of agriculture. The relative decline in the position of farmers in comparison with other groups is well known and derives from fundamental trends in the world food situation. World food prices have come down a long way since 1952 and the British farmer has had to bear some of the consequences. It is not improbable that in the next few years the share of farm income in total personal income will fall even further in comparison with its post-war peak.

Finally, we are faced by the riddle of the other self-employed. This group—which includes all individuals and partnerships engaged in manufacturing, trade, building, transport, finance and other services—is much more numerous than either of the other two groups. It also receives about two-thirds of the total income from self-employment. In the year 1954–55 the Inland Revenue made Schedule D assessments on nearly $1\frac{1}{2}$ million individuals and 205,000 partnerships. Of this total there were 320,000 farmers, 155,000 engaged in the professions, more than half a million retailers, 47,000 wholesalers, more than a quarter of a million in service trades—including hotels, restaurants, cafés, laundries, hairdressers and undertakers—and 134,000 building firms. The only other large groups exceeding 25,000 in number were road transport and clothing; but there were altogether many thousands of unincorporated firms in different fields of manufacturing.

A study of the changes in Inland Revenue assessments on individuals and firms between 1949–50 and 1954–55 shows that, in general, for industries in which unincorporated businesses are numerous, those which show an increase in the number of firms assessed were also those in which the average profit per firm increased, and vice versa. On the face of it, this relationship might suggest that firms in the former group were prospering, and hence attracting new entrants, while firms in the latter group were suffering a decline. But we cannot necessarily draw this conclusion. The trouble is that the number of unincorporated businesses in any industry can diminish in three ways: first, by the closing down of firms; secondly, by firms being converted into corporate businesses; and thirdly, by individuals joining their businesses together to form a partnership. Similarly, the number of firms can increase by new businesses being set up, by companies reverting to unincorporated status, or by the splitting up of a partnership. Hence if the number of unincorporated firms in an industry declines and the average profit per firm also falls this does not necessarily mean that the average profitability of the firms which have remained in the industry throughout has also declined. It may simply be that some of the more profitable firms have become companies.

It is, therefore, impossible to discover from existing data the precise causes of the relative stagnation of the profits of unincorporated businesses. But a careful scrutiny of the Inland Revenue's assessments for 1949–50 and 1954–55 (which relate broadly to profits

earned in 1948 and 1953 respectively) suggests that in this period both the number of firms and the average profit per firm were declining in manufacturing and that the opposite trend obtained in distribution and other service trades. Some of the decline in manufacturing must certainly be attributable to the conversion of larger firms into companies; but it also seems likely that the profits of the remaining unincorporated manufacturing firms came under pressure from the competition of larger companies. It is probable that in the first few years after the war small manufacturing firms benefited from the high level of demand and the shortage of productive capacity in the economy, while in more recent years the re-establishment of the productive strength of large companies has led to increasing difficulties for small firms. In distribution and other service trades, on the other hand, the small firm can still find a niche for itself. The growth of new towns and the spread of housing estates create new opportunities for the establishment of such small enterprises, with the consequence that there is a steady stream of new entrants. Their average profits are not great—in retailing the average assessment for 1954–55 was only £547—but they are sufficient to attract those who want to be their own masters. But these firms are also doubtless feeling the competition of large companies engaged in distribution. It is noteworthy, for example, that between 1948 and 1955 average income per employee in distribution increased by 47 per cent. while average profit per self-employed owner increased by only 19 per cent. Meantime the profits of companies in the distributive trades rose by 54 per cent. (These figures are deducible from data published in the Blue Book, together with estimates of the average number of employees and self-employed engaged in distribution based on the Ministry of Labour's manpower figures.)

The main conclusions which emerge from this discussion of the changes in the factor shares of personal income are: (1) the share of employment income has grown steadily throughout the period; (2) the share of investment income fell precipitately during the war and up to 1948, but has since maintained itself at the level then reached; (3) income of the self-employed maintained its position—and even increased somewhat—up to 1948, but has since declined. Some indication of the effect of these changes on the size distri-

TABLE 11

Percentage of Total Taxable Income Arising from Different Sources Within Specified Inter-percentile Groups

	Source of Income								
	*Employment**			*Self-employment*			*Investment*		
Inter-percentile Group	1937–38	1949–50	1954–55	1937–38	1949–50	1954–55	1937–38	1949–50	1954–55
Top 1 per cent.	24	28	35	18	33	27	59	38	38
Second to fifth per cent.	53	63	64	19	18	18	28	19	18
Sixth to tenth per cent.	73	86	87	15	8	8	12	8	5
Eleventh to twentieth per cent.	..	91	93	..	5	4	..	5	3
Top 5 per cent.	36	47	51	18	26	22	46	27	27
Top 10 per cent.	44	57	62	17	21	18	38	21	20
Top 20 per cent.	..	68	73	..	16	13	..	16	14

Note: The estimates are interpolations from the results of the Inland Revenue's income surveys for the years mentioned. For the two later years the Inland Revenue have adjusted the original figures for the under-reporting of certain types of income. The above estimates take account, so far as possible, of these adjustments.

* Includes family allowances.

bution of income can be gathered from Table 11, which is based on Inland Revenue statistics collected in the income surveys. Whereas before the war nearly three-fifths of the income of the top 1 per cent. of persons came from rent, interest and dividends the proportion in the post-war years has been less than two-fifths. Between 1937–38 and 1949–50 the share of self-employed income in the total income of this group rose very substantially but has since declined a little. Meantime the share of employment income has risen steadily. To some degree this means that the composition of the top 1 per cent. of persons has been changing: more of them are now salaried and self-employed workers and fewer are rentiers. At the same time it is probable that more of the people whose prime source of income in pre-war days was the ownership of capital are now receiving salaries.

Changes in Distribution Within Factor Incomes

Self-employment income and investment income are both more unequally distributed among their recipients than is employment income. This is primarily a reflection of the fact that the distribution of capital ownership is much more unequal than the distribution of the earnings of employees. These tendencies are illustrated in Table 11, the figures in which show that the higher up the income scale one goes the greater is the proportion of total income which comes from self-employment and investment and the less from wages and salaries. Hence the decline which has taken place since pre-war in the proportion of income derived from the first two sources has created, in itself, a trend towards greater equality of total income. At the same time, there is some evidence that there has been a redistribution of income *within* each of the three major sources.

It is an unfortunate fact that few direct figures are available on this important subject.* But there is one indirect piece of evidence which can be derived from the income surveys. If the figures showing the amount of income from each source received by persons above each level of total income are plotted on a double-logarithmic scale, one can observe a fairly clear tendency for each line to become more steep as the years progress. This seems to imply that the distribution of income within each source has been becoming more equal.

So far as employment income is concerned, there is some additional evidence to support the conclusion that salary scales have tended to rise less rapidly than wage rates (Allen, 1957). For manufacturing industry there are official estimates (in the Blue Book) of the wage and salary bill and of the number of persons covered in each case. From these it can be deduced that from 1949 to 1957 the average annual wage increased by 74 per cent. while the average salary increased by 53 per cent. But these figures are not necessarily indicative of relative changes in wage and salary *rates*, since the composition of the salaried group has probably been changing over the period. The number of salaried workers employed in manufacturing is estimated to have risen by 34 per cent. from 1949 to 1957 while the number of wage earners rose by only 7 per cent. Nevertheless, the difference between the rates of change of average earnings of the two groups is so large as to make it highly probable that average wage rates were rising faster than average salary rates.

The view that the distribution of self-employment income has become more equal

* In recent years the Inland Revenue have begun to publish some data on this subject. Since 1948–49 they have given a distribution of Schedule D assessments on individuals and partnerships; and in their 100*th Report* they started publishing estimates of the distribution of Schedule E income, the first of which relates to income arising in 1954–55.

since pre-war is based solely on the change in the shape of the double-logarithmic curve already referred to. This is not strictly a Pareto-type curve since the quantity of self-employment income received is related to the level of total income. It is, however, of some interest to note that the pre-war line is close to being a straight line, with only a slight but smooth downward curvature, while the post-war curves have a marked downward inflexion at about the £2,000 level of total income. This very obvious change in the shape of the curve strongly suggests that the owners of unincorporated businesses whose total incomes rise above £2,000 are now much more inclined to convert their firms into companies than before the war. The result of such a tendency would be virtually to put a ceiling on self-employment income as such.

As regards investment income, the evidence is again somewhat tenuous. The double-logarithmic line for investment income shows a slight tendency to be more steeply inclined in the post-war years than pre-war; but we should scarcely be justified in drawing very sweeping conclusions on this basis alone. There are, however, some grounds for believing that the distribution of capital is becoming a little less unequal than it was. One important influence here is the incidence of death duties. Each year something like £170 million is paid in estate duty, a very large proportion of which falls on the large estates. Cartter (1953) estimated that in 1947–49 about three-quarters of total estate duty payable was in respect of estates of £50,000 and over. He also estimated that a similar proportion of the total duty was payable by persons in the surtax range of income. Although wealthy people are, on the whole, fairly heavy savers, they would have to save a very high proportion of their net incomes to make up for this steady erosion of their capital. Since people at lower levels of total capital are probably saving more, as a group, than they pay in death duty, the distribution of total capital will by these means gradually become less unequal than it has been in the past. There has also been some redistribution of capital from the old to the young, in order to avoid death duties, which results in a less unequal distribution of investment income.

The Influence of Married Women's Earnings

Although the main forces working towards a redistribution of allocated income before tax are those which have already been discussed, it is necessary to draw attention to the influence on the size distribution of income of the increasing number of married women receiving separate incomes. Since married couples are treated in the tax statistics as having a single income, any increase in the number of married women at work will tend to raise the incomes of married units. Moreover, if two people who are already in receipt of separate incomes get married and continue to receive the same incomes, the effect will be to replace two smaller incomes by one larger one and hence to change the shape of the total distribution.

These effects would not be important in a period in which marriage rates and the propensity of married women to go out to work were fairly constant. But in the period covered by this study there have been considerable changes in these respects. No figures are available about the number of married women, or even of all women, at work in 1938; but according to the Population Census of 1951 the number of women at work in Great Britain in that year was 5·7 million. By 1951—again on Census day—this number had increased to 6·8 million. In the same period the number of *married* women at work rose from 0·8 million to 2·6 million—i.e., by more than the increase in the *total* number of

women at work. The reason for this was that in 1951 more women were married than in 1931. Between these two years the *total* number of women aged 14 and over increased by 2 million while the number of *married* women increased by 2·7 million. It is evident therefore that both the proportion of married couples in the population of income units and the proportion of married couples with two incomes were greater in 1951 than before the war. It seems that this trend is still continuing. The latest estimate of the Ministry of Labour is that in May 1957 there were 3·77 million married women in employment (*Ministry of Labour Gazette*, June 1958).

It is possible to conceive of circumstances in which changes of this sort in the social habits of the population would have no effect on the shape of the income distribution. But the actual changes that have occurred have been of a type which excluded such a possibility. Since a greater proportion of young married women—and especially of working- and middle-class women—go out to work than of those in other age and social groups, the effect of the increase in the proportion of married women at work has been mainly to increase the incomes of the lower and middle income families and to raise their position in the distribution. At the same time the people who are already in the top 1 per cent. of income units cannot usually benefit proportionately very much from their wives' earnings, since the earnings of wives are more equally distributed than incomes in general.

Some evidence in support of these conclusions is provided by the figures of wives' earnings collected in the income surveys. The original data have had to be adjusted upwards in the post-war surveys to offset the tendency not to report all wives' earnings. As a result, the figures are not as precise as they might be; but the trend which is revealed is unmistakable. In 1937 total wives' earnings of all income units down to the tenth percentile amounted to less than 1½ per cent. of total income, and even in the range lying between the tenth and twentieth percentile they were probably no more than 2 per cent. In 1954, on the other hand, the contribution of wives' earnings to total income rose from 2½ per cent. in the top percentile group to about 13 per cent. in the range between the tenth and twentieth percentiles.

In the light of these facts it is clear that the increasing employment of married women in the past twenty years has helped to raise the income level of many working- and middle-class married couples. To that extent it has contributed to the equalization of income. Its principal influence will have been on the distribution of employment income, thereby adding a further force tending towards the equalization of this type of income.

6. The Influence of Other Forms of Income and Benefit

We have found a strong and persistent trend over the past twenty years towards greater equality in the distribution of *allocated* income, both before and after tax. But in recent years some scepticism has grown up about the reliability of the official figures of allocated income as an indicator of the changes in the distribution of real income. It is well known that substantial benefits are distributed by means of goods and services charged as business expenses; it is widely believed that surtax payers can evade a good deal of the surtax chargeable on their investment incomes by converting income into capital gains; and there are a number of other ways in which taxable income can be converted into a non-taxable benefit. Apart from this there are the benefits provided by the social services, the presumed benefit to shareholders of undistributed company profits and the differential effect on the different social classes of the system of indirect taxes.

A strong case can clearly be made for allocating all benefits received, whether in money or kind, in income or in accruals of capital, amongst the various income (or benefit) groups. Only in this way can we make an accurate assessment of the changes in the distribution of income over time. But the difficulties in the way of making a complete allocation over all ranges of income are very great; in some cases, in the present state of our statistical knowledge, they are quite insuperable, unless we resort to the wildest sort of guesswork. I have, therefore, limited myself to trying to answer one question: what difference would be made to the trend in the share of the top 1 per cent. of income units if all the benefits which are not included in allocated income were taken into account? Although the answer to this question will not give us the full picture of the changes in the distribution of benefits over the whole range of persons, it will give us a very important part of that picture. The top 1 per cent. of income units are—broadly speaking—the "rich". They number about one quarter of a million and they occupy the long thin tail of the income distribution on whose shape largely depends all the measures of income inequality commonly in use.

I should emphasize at the outset of this exercise that practically all the figures contained in it are very rough. Only in a few cases has it been possible to employ estimating methods which are even moderately satisfactory. For the rest, the figures are largely guesswork; intelligent guesswork I hope—within the limits of what is known—but guesswork which may easily be influenced by the personal bias of the estimator. In general, my bias has been towards allocating as large a share of any benefit to the top 1 per cent. of income

TABLE 12

Adjustments to Allocated Income

	£ millions							
	All Incomes				*Top 1 Per Cent.*			
	1938	1949	1954	1957	1938	1949	1954	1957
1. Allocated income after tax	4,156	7,890	11,025	13,635	486	513	595	668
2. Unallocated income—								
(*a*) Life assurance and superannuation	119	255	465	649	14	39	64	95
(*b*) Other specified items	−17	54	58	102	3	17	21	31
(*c*) Remainder	313	572	799	920	78	143	200	230
3. Social services	180	756	1,047	1,383	1	8	10	14
4. Net indirect taxes	−544	−1,293	−1,706	−2,088	−54	−65	−85	−104
5. Undistributed company profits	176	457	803	971	135	260	394	476
6. Total of items 1 and 2	4,571	8,771	12,347	15,306	581	712	880	1,024
7. Total of items 1–4	4,207	8,234	11,688	14,601	528	655	805	934
8. Total of items 1–5	4,383	8,691	12,491	15,572	663	915	1,199	1,410

Notes on certain items:

2*a*. Net figure of employers' contributions for life assurance and superannuation, plus employees' contributions for superannuation, plus investment income of life and superannuation funds (after tax), minus pensions included in allocated income.

2*b*. Interest on National Savings Certificates, plus co-operative society dividends, plus excess of imputed rent of owner-occupied houses over their Schedule A value, plus post-war credits, minus part of employees' national insurance contributions included in allocated income.

2*c*. The balance of unallocated income which should be included in a distribution of personal income after tax. Excludes national insurance contributions, depreciation allowances of unincorporated businesses, income of non-profit-making bodies and certain government grants in kind.

3. Central and local government current expenditure on health, education, school meals, milk and welfare foods.

4. Net taxes on consumers' expenditure, after deducting subsidies.

5. Undistributed income of companies operating in the U.K. and of British companies operating abroad, after deduction of tax, stock appreciation and depreciation at replacement cost.

units as is consistent with any plausible hypothesis about the distribution of the benefit amongst the population. There are two reasons for this: first, that this group have much the strongest inducement to find ways of receiving tax-free benefits; and secondly, that I want to see whether, even when the allocations are made on this basis, the apparent trend towards greater equality of income (as measured by the share of income received by the top 1 per cent. of income units) is significantly affected.

In the following paragraphs I shall consider separately each type of adjustment to allocated income. I shall review briefly the reasons advanced for making the adjustment and explain the assumptions underlying the decision on the share of the adjustment to be allocated to the top 1 per cent. of incomes. A summary of the figures which arise from these calculations is given in Table 12, and their effect on the percentage share of income or benefit received by the top 1 per cent. of incomes is shown in Table 13 (p. 31).

Life Assurance and Superannuation

Allocated income includes life assurance premiums paid by persons but it does not include life assurance premiums paid by employers on behalf of their employees. It includes pensions received, but it excludes almost all superannuation contributions—both by employers and employees—and the investment income of life and superannuation funds. All the excluded items are clearly of some benefit to the persons insured or covered by superannuation schemes; and, although there may be some people who would not voluntarily save as much of their income as they are obliged to do under compulsory pension schemes, I think it is reasonable to treat the whole of this flow as forming part of personal income. At the same time, the value of current pensions from previous employers, which is already included in unallocated income, must be subtracted.

The share of the top 1 per cent. of income units in these various adjustments has been determined as follows. In 1938 the number of employees covered by superannuation schemes was probably less than 2½ million, of whom a large proportion were government servants. By 1956 the number in pension schemes was over 8 million (*Occupational Pension Schemes*, 1958, p. 5), and most of the increase seems to have taken place since the end of the war. It is a fair assumption that a larger proportion of employed persons in the top 1 per cent. income group is covered by pension schemes than of other employed persons; and I have hazarded the guess that 50 per cent. of the employment income of the top income group was so covered in 1938, 70 per cent. in 1949, 80 per cent. in 1954 and 90 per cent. in 1957. I have further assumed that in each year the total superannuation contributions paid—whether by employers or employees—amounted to 15 per cent. of the income covered. This is about the maximum contribution possible within the limits of an approved pension scheme. The calculation implies that in 1957, for example, there were about 140,000 persons covered by pension schemes in the top 1 per cent. group (excluding those covered by schemes in which neither the employer nor the employee makes any current contribution, i.e. Civil Servants, Armed Forces and certain local authority employees), that their average employment income was about £3,250 a year, and that the contributions made on their behalf averaged nearly £500. For 1938, the share of the top 1 per cent. income group in total superannuation contributions (excluding direct pension payments) works out at nearly a fifth; and in the post-war years at approximately a seventh. This can be compared with the same income group's share of total employment income

(before tax but excluding all contributions paid), which was about 6 per cent. in 1938 and slightly over 4 per cent. in the post-war years.

The share of investment income of the life and superannuation funds accruing to the top 1 per cent. of income units depends partly on their share in total contributions paid, partly on the length of time for which their contributions have been accumulating and partly on the fluctuations in their incomes over their life time. The number of people with some stake in the life funds is much greater than of those entitled to a pension, so that the share of the top 1 per cent. in this source of income will be less than their share of pension contributions. Further, Civil Servants and members of the Armed Forces, amongst whom there are an appreciable number who are both in the top 1 per cent. and entitled to a pension, derive no income from this source. Thirdly, a fairly large proportion of investment income accrues to the benefit of people who are already retired, not many of whom are still in the top 1 per cent. Against this, of the income accruing to present employees the share of those in the top jobs (who are generally older) will be more than proportionate to their incomes. It is very difficult to assess the numerical importance of all these countervailing considerations; and I have simply made a guess that the share of the investment income of the life and superannuation funds accruing to the top 1 per cent. group has been 10 per cent. in each year.

Of current pensions from previous employers it has been assumed that only 5 per cent. accrues to persons in the top 1 per cent. In general, rather few of those who retire completely will still be in the top 1 per cent.; but there are probably a fair number of pensioners from the Armed Forces, as well as some Civil Servants, who take other employment after retirement from their government posts, thus bringing their combined incomes into the top 1 per cent.

Interest Currently Accruing on National Savings Certificates

There can scarcely be any dispute about the legitimacy of including this source of income in personal income. The difficulty is to know what proportion of it belongs to the top income group. According to the 1953 Savings Survey the top 1 per cent. of income units owned about 13 per cent. of all Savings Certificates (valued at original purchase price). But the survey's estimate of aggregate holdings of Savings Certificates by private households amounted to less than two-thirds of the total stock outstanding. I have assumed that the top 1 per cent. of income units own half the unreported Savings Certificates, so that their share of the total stock is about a quarter.

Co-operative Society Dividends

These are not included in allocated income but they clearly should be in personal income. I have assumed that they all accrue to the bottom 99 per cent. of the population.

The Excess of Imputed Rent of Owner-occupied Houses Over Their Schedule A Valuation

Schedule A valuations have not been revised since pre-war and they now bear little relation to the rental value of property. For the Blue Book estimates of personal income the Central Statistical Office raises the aggregate Schedule A value of owner-occupied property so as to take account of the average rise in rents since the last valuation was made. This extra imputed rent, which is not included in the tax assessment, can most

reasonably be allocated in proportion to the Schedule A values themselves. It appears from the post-war income surveys that the top 1 per cent. of income units are assessed for about 20 per cent. of aggregate personal Schedule A. An unknown proportion of this is in respect of owner-occupied business property, and I have assumed that the figure of 20 per cent. can be applied to residential property taken separately.

Post-war Credits

This is a small item. The size of the individual payments varies quite considerably, and some of this variation will reflect differences in income during the war years, when the credits accrued. But since the credits are not normally paid until the owner reaches the age of 65, the share of the top 1 per cent. of income units in current receipts will be small. I have assumed 5 per cent., which is probably too high.

National Insurance Contributions

Allocated income does not include employers' national insurance contributions but it does contain that part of the contributions of employees and the self-employed which is not deductible for tax purposes. Hence, in order to arrive at a true distribution of disposable income we must subtract this residue. The top 1 per cent. of income units paid no contributions before the war, but in the post-war years they have probably paid slightly more than 1 per cent. of the total contributions of employees and the self-employed. The proportion of occupied persons in this group is about the same as in the population as a whole, but more than a third of them are self-employed, whose contributions per head are somewhat higher than the contributions of employees. I have, therefore, allowed this group $1\frac{1}{4}$ per cent. of the national insurance contributions to be deducted for the post-war years.

The Remainder of Unallocated Income

Unallocated income includes certain items which are not strictly part of personal income after tax. These are employers' and part of employees' national insurance contributions, depreciation allowances of unincorporated businesses, the investment income of non-profit-making bodies and certain government grants in kind (most of which are here included under social services). But after subtracting all these, and the items already discussed above, there still remains a substantial sum of unallocated income. This remainder —which amounted to over £300 million in 1938 and over £900 million in 1957—is a mixed bag of incomes from different sources. About a third of it is employment income, including both income in kind from free or subsidized meals in canteens, cheap coal, meal tickets, etc., and "inconsiderable" incomes (below £50) which are not allocated in the Blue Book. The other two-thirds consists of profit and investment income which has not been assessed for one reason or another. As explained earlier, the method of estimating farmers' profits used in the Blue Book results in a higher figure for this item than that which emerges from the Inland Revenue's assessments of farmers. Something is also added to the assessed profits of other unincorporated businesses to allow for assumed under-assessment. The investment discrepancy is more difficult to understand. It is, unfortunately, a residual of a residual, being the balance of total investment income which cannot be accounted for either by the investment receipts of other sectors or by the assessed investment incomes

of persons (plus the investment items discussed above). It is very probable that it includes some investment income which, though received by persons, is not declared to the tax authorities. Not all of this, however, is necessarily the result of tax evasion, since the bulk of this income is taxed at source. It is also possible that some part of this residual investment income is not strictly the income of individuals but belongs to collective bodies and private trusts.

It is clearly very hazardous to attempt to estimate the share of this remainder of unallocated income which should be attributed to the top 1 per cent. It is not even certain that the aggregate figures are truly representative of the volume of unallocated income still to be dealt with. For the reasons mentioned above they may be too high; but they may also be too low, if tax avoidance and evasion are really greater than the government statisticians have dared to believe. But, for want of an alternative hypothesis, I have assumed that this unallocated remainder is what it claims to be—the balance of true personal income not yet allocated.

I have next assumed that, at the very most, the top 1 per cent. of income units received 25 per cent. of this remainder. This would give each one of them (on the average) a sum of £330 in 1938, £550 in 1949, £770 in 1954 and nearly £900 in 1957. These are very substantial amounts of tax-free income—especially when it is borne in mind that many members of this group are unlikely to be receiving much, if anything at all, in this way.

Social Services

Direct government grants to persons—for national insurance, national assistance, retirement pensions, family allowances and the like—are included in allocated income; but government expenditure on the provision of goods and services in kind is not included in the Blue Book definition of personal income, with a few slight exceptions. When we are considering changes in the shares of income groups in real income we must take these items into account. A suggested criterion for including any item of government expenditure in personal income is that it consists of goods or services which, if they were not provided by the Government, would be purchased privately by consumers. We cannot necessarily assume that consumers would spend exactly the same sum of money on these items as the Government does; but where they are "necessities" it seems not unreasonable to count them as part of personal benefit. On these grounds I have decided to include in personal benefit aggregate current government expenditure on health, education, school meals, milk and welfare foods. These items alone account for about a third of current expenditure by combined public authorities on goods and services (Blue Book, 1958, Table 50). Other government expenditure on current goods and services, which is mostly for defence, administration and local public services, can scarcely be allocated to different economic groups on any rational basis.

For the health services I have assumed that in the post-war years the top 1 per cent. of income units received nearly 1 per cent. of the benefits. Some of the members of this group doubtless employ private doctors and use private nursing homes; but there are no evident grounds for believing that they do not make some use of the services of family doctors, of the drugs provided under the National Health Service, and of certain hospital and dental services. For education, I think it possible that the top 1 per cent. of income units receive more than 1 per cent. of the benefits provided, since, despite the fact that they use the government-supported schools very little, they derive very substantial benefits from

the government subsidies to the universities. For the combined social services—including school meals, milk and welfare foods—I have assumed that the top 1 per cent. have received an average share of 1 per cent. in the post-war years and one-half of 1 per cent. pre-war.

Net Indirect Taxes

The real value of disposable income is affected by the rates of indirect taxes levied, and of subsidies granted, on the various items of consumer expenditure. To distribute the burden of these taxes between income classes we need to know the detailed composition of the expenditure of each class. Since there are no adequate sample studies of the expenditure patterns of the top 1 per cent. of income units, we can only make rough estimates by extrapolating the trend of differences between income classes lower down the scale. Barna's pre-war estimates (1945, p. 184) suggest that the top 1 per cent. of income units paid nearly 10 per cent. of net indirect taxes. In the post-war years the share of the top 1 per cent. in total post-tax income has fallen greatly, and this would tend to bring down their share in indirect taxes. Meanwhile, however, the composition of taxes and subsidies has changed. Tobacco and alcohol taxes have grown fast, and housing taxes slowly; new taxes have been imposed on durable goods, but food, which previously paid some net taxes, now receives substantial net subsidies. The consequence is that the higher income groups probably pay a larger proportion of their allocated incomes in indirect taxes than they did pre-war. I assume that the appropriate figure for their share of net indirect taxes in the post-war years is 5 per cent.

Undistributed Company Profits

It is sometimes argued that, since undistributed company profits belong legally to the shareholders, their value should be added to the personal income of shareholders. Both Barna (1945) and Cartter (1955) made an allocation along these lines. It is, however, questionable whether undistributed profits should be treated as the equivalent of personal income in this way. Shareholders do not, in fact, have much say in the appropriation of company income; and there are no signs that the tendency of companies to plough back a very large proportion of their available profits is weakening. In principle, one might expect that the companies that plough back the largest sums out of current profits would grow most rapidly and that their future profits—and dividends—would reflect this rate of growth. It is, however, difficult to test this hypothesis since we cannot allow for all the other circumstances affecting company profits.

A significant—if not a decisive—test of the value of undistributed profits to shareholders would be to measure the independent influence of undistributed profits on share prices, after dividends and other factors have been taken into account. This has been attempted by Fisher (1957) and Singer (1958), both of whom reached the conclusion that the independent influence of undistributed profits on share prices is very small. Similarly, if we compare the changes in aggregate dividends, aggregate undistributed profits and average dividend yields over the past twenty years, we cannot fail to be struck by the apparent lack of influence of the relative growth of undistributed profits on the dividend yields. For the group of non-nationalized companies taken separately it can be estimated (Blue Book, 1958, Table 35) that, while aggregate pre-tax dividends on ordinary shares increased from £331 million in 1938 to £702 million in 1957, total undistributed profits—after de-

ducting tax, stock appreciation and depreciation at replacement cost—rose from about £150 million in 1938 to over £900 million in 1957. Yet, despite this enormous increase in the relative share of retained earnings, the average dividend yield on ordinary shares (as measured by available series, which are necessarily based on quoted public companies only) did not decline between 1938 and 1957 but may even have risen a little. (See, for example, the series published in the *London and Cambridge Economic Bulletin*.)

It may further be argued that shareholders benefit from undistributed profits through the resulting increase in the capital value of their shares. This is certainly what one would expect. But the aggregate data quoted above suggest that the capital values of shares in fact depend much more closely on the dividends paid out that on any other factor. It is a surprising fact that, although the gross trading profits and other income of all companies increased from 1938 to 1957 more than four times, and dividends plus net undistributed income nearly three times, the aggregate value of shares seems to have just about doubled. Even if we were able to trace the history of the shares of non-nationalized companies alone over this period, it seems unlikely that we would find that their market value (including bonus issues and the bonus element in "rights" issues, but excluding new money subscribed) would have risen by as much as the index of consumer prices. If that is so, the "capital gains" of shareholders over this period have, on the average, not been more than sufficient to keep the real value of their shares (in terms of consumer prices) about constant. If, consequently, we were to attribute part of these capital gains to the ploughing back of profits, we should be implying that, but for this continuous injection of new capital, the shareholders' valuation of companies' capital in this country (after allowing for the replacement of worn-out items) would have fallen substantially in real terms since pre-war. In fact, the value in 1957 prices of the aggregate volume of undistributed profits over the whole period since 1938 is substantially in excess of the rise in the aggregate value of shares since that date; so that, if undistributed profits are assumed to be responsible for capital gains on the stock market, the net capital gain from other causes (the chief of which is the rise in the general price level) would appear to be negative.

For these reasons I am of the opinion that the case for including undistributed profits in personal income—except in the case of closely held family companies—is now a very weak one. Nevertheless, since there may be others who take a different view, I have made some approximate estimates of the "share" of the top 1 per cent. of income units in this source of income.

Barna (1945, p. 269) estimated the distribution of company shares amongst different income groups in 1937. By interpolation from his figures it can be found that the top 1 per cent. of income units held nearly 77 per cent. of the total in that year. I have attributed the same share of undistributed profits to the top income group in 1938. For 1949 and 1954 special new calculations were made, from which it appears that the share of the top income group in total dividends had fallen in 1949 to 57 per cent. and in 1954 to 49 per cent. (Details of the methods used in making these estimates are given in the Appendix.) For the year 1957 I have assumed the same percentage as in 1954, although it is possible that the share of the top 1 per cent. of income units in total dividends has continued to decline. These estimated shares of dividends were then applied to net undistributed profits, with the results shown in Table 12, item 5. Since an important—and increasing—share of total dividends is now flowing to the life and superannuation funds, it could be argued that the top 1 per cent. of income units should be credited with part of that flow also. This

would increase their share of total dividends and hence of undistributed profits. But the difference would not be great, probably no more than £10 million extra in 1957.

We are now in a position to consider the net effects of these adjustments on the share of the top 1 per cent. of income units in total income. In Table 13 I distinguish four types

TABLE 13

Share of Top 1 Per Cent. of Income Units in Various Kinds of Income After Tax (Percentages)

	1938	1949	1954	1957
Allocated income (item 1 in Table 12)	11·7	6·5	5·4	4·9
Personal income (item 6 in Table 12)	12·7	8·1	7·1	6·7
Personal benefit (item 7 in Table 12)	12·6	8·0	6·9	6·4
Personal benefit and undistributed profits (item 8 in Table 12)	15·2	10·5	9·6	9·1

of income: *allocated income* (after tax), which is taken from the Blue Book; *personal income*, which is the sum of allocated and unallocated income, excluding income tax, surtax, most of national insurance contributions and certain other items; *personal benefit*, which is personal income plus the cost of social services minus net indirect taxes; and "personal benefit plus undistributed profits", which, as I have explained, is included only for illustrative purposes.

It will be seen that the downward trend in the share of the top 1 per cent. of income units remains clearly marked for each type of income. Proportionately, the fall in the share of the top income group is greatest in the case of *allocated income*, with a fall of 6·8 percentage points out of 11·7 percentage points over the whole period. In the same period, the share of the top income group in *personal income* fell by 6 points out of 12·7, in *personal benefit* by 6·2 points out of 12·6, and in "personal benefit plus undistributed profits" by 6·1 points out of 15·2. Yet the absolute fall in the share of income is very similar for all types, ranging between 6 and 7 points. It should also be noted that the declining trend in the share of the top 1 per cent. continues through each of the three subperiods and for each type of income. If we focus our attention on *personal benefit*, as being the most meaningful concept of real income, then it seems that the share of the top 1 per cent. in 1957 was approximately half its share in 1938.

In concluding this section I would beg to remind the reader—if he needs reminding—that the estimates contained in it are in many cases only approximate. Nevertheless, I think it has been instructive to discover that, even when the top income group is credited with very large sums of income from superannuation and other tax-free sources, as well as from tax avoidance and evasion on an appreciable scale, its share in total personal income or benefit can be shown to have fallen substantially and persistently since before the war.

7. RETROSPECT AND PROSPECT

Is it likely that the trend of the past twenty years will continue? Clearly our judgment on this issue will depend on whether we regard the period 1938–57 as *exceptional*. Unfortunately, as we have already noted, it is impossible to carry back the type of analysis made in this paper before 1938 (or more strictly before the assessments made in 1937–38). The 1918–19 and 1919–20 estimates relate to an abnormal period and they cover the whole of Ireland. The only other data are the surtax figures and the distribution of 1801, both of

which cover only a small section of incomes at the top of the scale. The best that can be done seems to be to plot the "Pareto" lines from some of these distributions and compare their slopes. These are shown in Fig. 3 for the years 1801, 1913–14, 1919–20, 1937–38 and 1954–55. The 1801 figures are from Stamp (1920); the 1913–14 figures are the super-tax assessments; the others are derived from the Inland Revenue surveys for the years mentioned. The 1954–55 data have been adjusted slightly by the Inland Revenue to take account of the under-reporting of investment income and children's allowances.

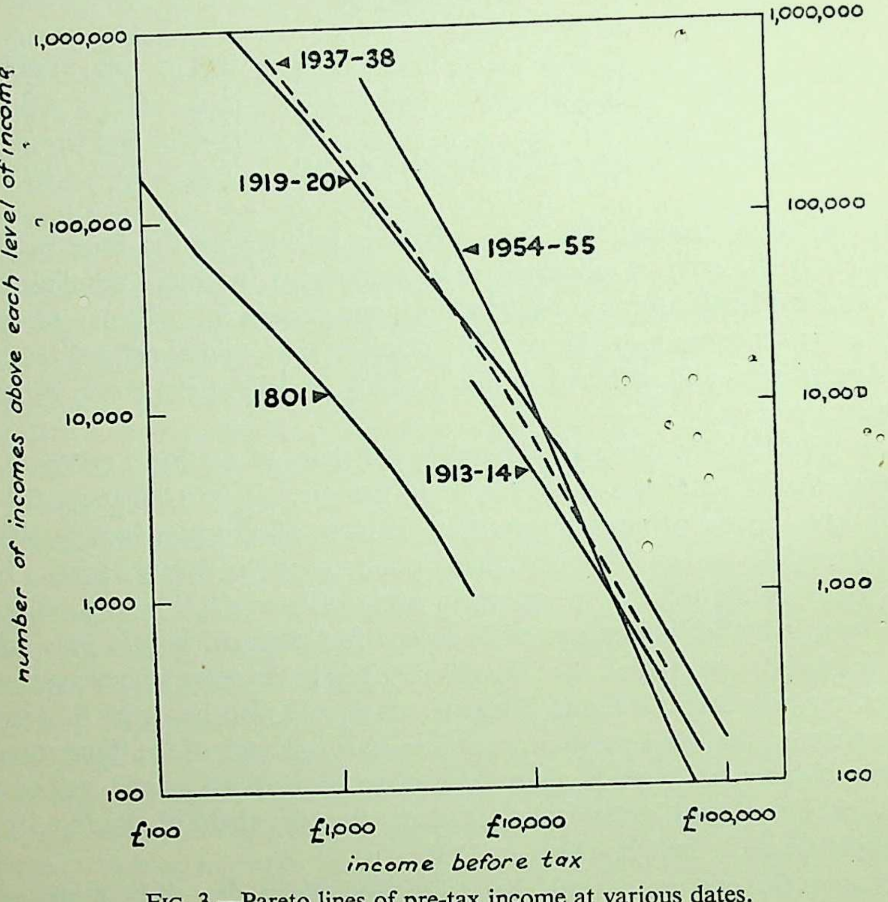

FIG. 3.—Pareto lines of pre-tax income at various dates.

It is obvious from an inspection of Fig. 3 that, whatever changes may have taken place between 1801 and 1937–38, they were negligible in comparison with the great reduction in inequality between 1937–38 and 1954–55. From 1913–14 to 1919–20 there seems to have been little change, at least for the very high incomes; but between the latter year and 1937–38 there appears to have been some movement towards greater equality. If we discard 1919–20 as being an "abnormal" year, we conclude that between 1913–14 and 1937–38 there was a slow process of reduction of inequality at the top of the income scale. The comparison with 1801 is very difficult to make, partly because of changes in the value of money over the period, and partly because of the inherent unreliability of the 1801 figures. The slope of the 1801 line varies quite considerably: for incomes below £1,000 it seems clearly less steep than for the incomes above £5,000 in 1913–14; but for incomes above £1,000 in 1801 the 1801 line is, if anything, a little steeper than the 1913–14 line. There is,

therefore, no conclusive evidence that the distribution of higher incomes changed one way or the other between 1801 and the period before the first world war.

Since the trend in income distribution over the past two decades has been much more strongly egalitarian than in any previous period of our history, it is necessary to consider what may have been the specific causes of this development and how far they are likely to operate in the future. It seems that several different forces have been at work. The most important of these, without any doubt, has been the achievement and maintenance of full employment. Full employment entails the elimination of a large group of very low incomes, not only of the unemployed themselves but also of workers on the margin of unemployment whose wages are particularly depressed. This applies to workers in "sweated" trades, to the unskilled, and to women and young people. All these groups suffer to an exceptional degree from a state of economic depression. It is not surprising, therefore, that one of the main features of the past two decades has been the narrowing of differentials between industries, between skills, between sexes and between age groups. The poor and the weak have been lifted up and brought closer to the level of the more privileged and better organized sections of the community.

Full employment and high demand also bring special benefits to farmers (and with them to farm workers), who are another group especially subject to exploitation in a period of depression. We have already seen how this helped to raise the incomes of the self-employed in the years up to 1949 (and perhaps even up to 1952). Rising prices are also generally assumed to be favourable to all business incomes. But here we need to make a qualification. So long as the characteristic form of business organization in this country was the unincorporated family firm it was probably true to say that rising prices would lead, through expansion of profits, to an elongation of the tail of the income distribution. But with the development of the dominance of companies this conclusion no longer follows. For companies have a high marginal propensity to save and they readily absorb the bulk of increased profits as company savings. The result is that dividends have a "built-in" tendency to rise less rapidly than profits, and perhaps even less rapidly than other money incomes. When we add to this the great increase in the taxation of profits and the pressure from the Government to restrain dividends, both by differential taxation of distributed profits and by periodic exhortation, we can no longer take it for granted that rising prices will redistribute *personal* income in favour of entrepreneurs. They are, indeed, more likely to work in the other direction.

In addition to this, there appears to be a long-term tendency towards a reduction in the economic importance of the unincorporated business. It is partly a question of a decline in the *number* of self-employed business men in relation to the total number of occupied persons. This tendency persists, even though it is offset from time to time by the emergence of new opportunities for small business—such as those in the distributive and service trades. But the major factor is the levelling down of unincorporated business profits, under the combined influence of the competition of large companies and the conversion of successful unincorporated firms into private companies. The consequence is that the proportion of unincorporated business income in total personal income has steadily fallen. Table 9 shows how the proportion of non-farm self-employment income has fallen since 1938, and American figures show a similar trend extending back as far as 1929 (*National Income* supplement to the *Survey of Current Business*, 1954).

But the direct effect of full employment and the other influences mentioned above are

not sufficient to explain the strength of the continuing movement towards greater equality of income in recent years. The elimination of pockets of exploitation is, in principle, a once-for-all operation, and the decline in the importance of the unincorporated business is too small an influence to bear the whole weight of recent changes. We must, therefore, look for a stronger underlying cause. And there is little difficulty in identifying it. From a glance at Fig. 2 it is clear that the leading force in expanding money incomes has been the persistent rise in employment income. The employment line rises every year and keeps an almost constant course, while other types of income have periods of wavering or stagnation. The main reason for the persistent rise in employment income is the annual round of increases in wage rates. The causes of this are complex and a subject of considerable contemporary dispute; but most people would agree that the wage-price spiral can be attributed to a combination of high (or moderately high) demand, strong trade unions and the modern practice of "full-cost" pricing.

The future trend in income distribution seems likely, therefore, to depend largely on the future course of the economy as a whole. If creeping inflation persists, further redistribution of income in favour of the middle and lower incomes is probable. Rents, interest and pensions are forms of income which are absolutely or relatively fixed in money terms; salaries tend to lag somewhat as a consequence of the lack of organized pressure from salary earners; and the incomes of self-employed professional workers suffer from the same kind of disability. All these types of income, therefore, tend to fall behind when there is an upward surge of wages, with the consequence that there is a redistribution of income towards the wage earner.

Even if there were to be no further inflation, and average prices remained stable, there would be likely to be some tendency towards greater equality of personal income in the long run. High surtax rates and the steady erosion of large estates by death duties is causing a slow redistribution of capital; the long-term decline in the position of the unincorporated business is reducing the importance of self-employment income; and the widening of educational opportunities will swell the flow of better qualified people, hence causing a further reduction in wage and salary differentials.

The only circumstances—as I see it—in which the trend towards greater equality of income might be stopped or reversed would be the onset of a major slump. This would, of course, improve the position of the pure rentier, and perhaps also of the "blue chip" shareholder. The workers would suffer through unemployment and the loss of work-opportunities for their wives; the farmers would also suffer as would others amongst the self-employed, including the owners of small private companies.

In face of this awful alternative there can be little doubt that the British electorate, and hence also British politicians, will maintain a permanent bias in favour of expansion. This appears to carry with it; as a corollary, a bias towards greater equality of income. Although the future movement towards equality is unlikely to be as rapid as it has been in the immediate past, I conclude that the long-term trend is set in that direction.

8. Summary

A study of the period 1938–57 reveals a continuous trend towards greater equality in the distribution of allocated personal income. Little of this is attributable to the direct impact of taxation, although the threat of higher tax rates may have worked indirectly by slowing down the rate of increase of pre-tax incomes in the higher ranges.

After allowing for price changes the disposable allocated incomes of the top 1 per cent. of income units appear to have fallen since pre-war by 25 per cent. or more. Their share in total disposable income has fallen from about 12 per cent. in 1938 to about 5 per cent. in 1957.

The principal causes of the changes are: first, a shift in the factor composition of personal income, the percentage share of employment income rising from 60 in 1938 to 72 in 1957, while the percentage share of investment income fell from 22 to 11; secondly, a tendency for the distribution of income within each type of factor income to become more equal; and, thirdly, the growth in the number of married women at work.

When allowance is made for other sources of income and benefit—namely, unallocated income and social services—as well as for indirect taxation, the fall in the share of post-tax income received by the top 1 per cent. of income units is slightly less marked, but still substantial. If, further, allowance is made for imputed income from undistributed profits, which is a procedure of doubtful validity under present conditions, it is found that this adjustment makes less difference to the trend than might have been expected.

Although there are indications that there was a slight tendency towards diminishing inequality before 1938, the rate of change of the past two decades has been exceptionally great. The achievement of full employment has been an important influence; but in recent years the constant upthrust of wages, generated by the wage-price spiral, has been the principal agent changing the shape of the income distribution. For the future, unless there is a catastrophic slump, the trend towards equality is likely to continue, though probably not as fast as in the past twenty years.

Acknowledgments

I am indebted to the Central Statistical Office for assistance in interpreting the data in the Blue Book and for helping me over some particularly rough patches in the construction of Table 12. But they must not be held responsible for the uses to which I have put their figures. Mr. J. L. Nicholson made some helpful criticisms of a previous draft of this paper.

Appendix

The Allocation of Undistributed Profits

Undistributed profits of companies operating in the United Kingdom and of British companies operating abroad are given in the Blue Book (1958), Table 34, item "saving". These are after taxes but before depreciation and stock appreciation. Depreciation at replacement cost of companies is estimated in Table 65 and companies' stock appreciation for the years since 1948 in Table 67. Total stock appreciation in 1938 is given in Table 6 and a suitable proportion (£60 million) of this has been allocated to companies. Thus we obtain net undistributed profits.

An accurate allocation of undistributed profits between income groups would require a detailed knowledge of each shareholder's income and the composition of his portfolio. As an approximation, however, undistributed profits may be allocated to income groups either in proportion to total dividends received or in proportion to the value of shares owned.

For the year 1938 I have used Barna's estimate (1945) of the distribution of shares amongst income groups in 1937. By interpolation in his Table 77 one can deduce that in 1937 the top 1 per cent. of income units owned about 77 per cent. of all shares owned by

persons. I have applied this proportion to the figure of net undistributed profits in 1938.

For the years 1949–50 and 1954–55 data are available in the Inland Revenue's income surveys of the total of "interest and dividends" (a single figure) received by each income group (94*th Report*, Table 90 and 99*th Report*, Table 55). For income groups above £2,000 it is possible to estimate the respective shares of interest and dividends by the following method. Table 55 of the 93*rd Report* and Table 105 of the 98*th Report* show the distribution of various kinds of assets within sizes of estate, as valued for estate duty, in the years 1949–50 and 1954–55. By applying assumed average yields to each asset it is possible to estimate average interest, dividends and total investment income in each range of estate. The proportions of interest and dividends to total investment income at any particular level of total investment income can then be estimated by interpolation.

Next, we refer to Table 129 of the 94*th Report* and Table 98 of the 99*th Report*, which give distributions of investment income by ranges of total income. By applying to total investment income in each range of investment income the appropriate proportions of interest and dividends respectively, we can estimate the amount of each which is received by persons in each range of total income (above £2,000). These results were compared with the data for total interest and dividends in the same income ranges, as given in the income survey tables, and the figures were adjusted slightly (in proportion) so as to ensure that the sum of interest and dividends from the one source was equal to total interest and dividends from the other. In this manner we obtain an estimate of aggregate dividends received by each income group in the surtax range, and so, by interpolation (and for 1949–50 by a slight extrapolation below £2,000), the amount of dividends received by the top 1 per cent. of income units. These figures (£280 million in 1949–50 and £322 million in 1954–55) were compared with the data on total dividends paid by companies in 1949 and 1954 (apart from inter-company payments) given in Blue Book Table 34; and the top income group were assumed to receive the same proportions of net undistributed profits.

It appears that the share of the top 1 per cent. of income units in shares or dividends, and hence in undistributed profits, has fallen from 77 per cent. in 1938 to 57 per cent. in 1949 and 49 per cent. in 1954. This downward trend may be explained by the growth in the importance of life and superannuation funds as investors in equities, by the decline in the position of the rentier compared with those with an earned source of income, and by the subdivision of estates in anticipation of death duties. It is possible, therefore, that the downward trend in the proportion of dividends received by the top income group has continued since 1954.

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DISCUSSION ON MR. LYDALL'S PAPER

DR. BARNA: Mr. Lydall has made a systematic attempt to exploit the official statistics on income distribution which have come to light in the last 20 years. The underlying cause of the greater adequacy of official statistics in this field is the great extension of the area of income tax. Before 1914 only a small fraction of the population was liable to assessment; nowadays practically all recipients of income are assessed. On the other hand, an increasing proportion of the product of industry is distributed in the form of benefits in kind which, as is shown in the paper, is difficult to estimate and even more difficult to allocate to particular income groups. In fact, we are largely restricted to the Inland Revenue's concepts of income and income unit, and these are not the only possible concepts for analysis.

Mr. Lydall comes to three conclusions: first, that there has been an important reduction in the inequality of taxable personal income in the last 20 years; secondly, that this tendency has continued since the war; and, thirdly, that only to a comparatively small extent has the reduction in inequality been due to taxation. These conclusions are reached whichever way we analyse the distribution of income, whether in terms of the relative growth in the income of different classes or in terms of a change in their share in the total. One important fact should be emphasized: the share of the richest 1 per cent. of income earners after taxation has fallen by more than half and this amount has been re-distributed to the working-classes. Broadly speaking, the middle-classes' share has changed little.

However, it is not enough to look at only the taxable income, because the same forces which brought about greater equality in distribution have also contributed to the increase in the amount of income not distributed as taxable personal income. Mr. Lydall has made a brave attempt to deal with this problem, and he could not have done much better in the time and space available to him. I have two quarrels with him, however.

First, there is no explicit discussion in the paper of the problem of tax evasion and tax avoidance and it is not known how much is allowed for this in the final estimates. In this respect pioneers like the late Lord Stamp were ahead of us in their work in so far as they discussed this problem very carefully. However, so far as Table 12 goes, I think Mr. Lydall has taken care not to exaggerate his case and the additions to the income of the richest 1 per cent. which he made are probably sufficient.

The second point where we disagree concerns undistributed profits, as regards both the argument and the calculations. I think it is accepted that undistributed company profits are income: they are savings made by the company on behalf of the shareholders which ought to result in dividends in later years. Mr. Lydall's argument that undistributed profits should not be regarded as income can be accepted as valid only if one assumes that company directors act in an irresponsible fashion, which does not appear to be the case. The two statistical works quoted by Mr. Lydall are not relevant to the issue. Dr. Singer himself reached a different conclusion from Mr. Lydall's, namely, that the stock-exchange valuation of different shares is not necessarily rational. It also seems to me that it is surtax payers rather than institutional investors who are interested in companies with a high proportion of undistributed profit, but it is not necessarily the same people who are most influential in determining market values.

With regard to the calculations on undistributed profits, the figure for 1937 quoted from my book and Mr. Lydall's own estimates are not comparable. The two methods of calculation are widely different for the reason that new statistics are now being published and my own figure relates to shares in personal ownership only.

The great statistical puzzle is that while Mr. Lydall has shown that a significant improvement in the distribution of income has occurred, the share of the domestic national product going to employed and self-employed persons has changed very little. The percentage according to the Blue Books was 74½ in 1938 and 76 per cent. in 1957. It seems therefore that the fundamental distribution of the national product between capital and labour has not changed much over this period. Mr. Lydall refers to indirect evidence on the distribution of capital, but there is direct evidence contained in Mrs. Langley's estimate which indicates that the highly unequal distribution of property between persons has not changed much. It seems therefore that taxation must have played a much greater part in bringing about a diminution in inequality of income than is suggested by Mr. Lydall.

I have re-calculated the figures in Table 13 to take incomes before instead of after tax, and have arbitrarily assumed that two-thirds of undistributed profits are allocated to the richest 1 per cent. in each year. I find that, instead of the figures contained in the last line of that Table, the share of the top 1 per cent. in 1938 was 19½ per cent. and in 1957 15 per cent. It follows from this that Mr. Lydall's final conclusions hold no matter what kind of calculation one does, although on a different basis the improvement in the distribution of income may appear different from that shown by him.

One very important conclusion emerges from the paper as a whole and that is that both the top and the bottom of the income distribution have been cut off. I do not think that this is different from what is observed in the United States and a number of other countries. Fifty years ago the richest one or two per cent. of the population accounted for a high proportion of total income, total expenditure and property, and at the same time it was shown by social investigators that a fairly large proportion of the population lived below the so-called poverty level, especially families with many children. To-day, this situation has fundamentally changed; extreme riches have disappeared or ceased to be effective, while at the other end of the scale few families live below the poverty line. There is a great deal of luxury spending which may give the appearance that the official statistics are somehow wrong, but a more careful consideration would suggest that even if allowances are made for tax evasion and unallocated income, the share of the richest element, as Mr. Lydall has shown, has fallen. My observation would be that the apparent luxury spending of the rich (on e.g. motor cars and meals out) represents a change in the pattern of expenditure due to the lack of domestic servants.

Mr. Lydall attributes the main responsibility for this state of affairs to full employment and inflation, but I feel that the causes of the long-term trend towards equality are of deeper origin. There has been a change in social philosophy and a long-term development of social services. I would emphasize the permanent effect on distribution of the health services and even more of the extension of education.

It is 22 years since the Society last had a paper on the distribution of income before it; this was by the late Lord Stamp and covered only a small part of the field. The subject was

more fashionable in the nineteenth century, but I think it fair to say that the Society never had a paper on this subject as comprehensive as Mr. Lydall's. I have great pleasure in proposing a vote of thanks.

Mr. PAINE (in seconding the vote of thanks): I have no criticism to make of Mr. Lydall's conclusions, and do not think that any of the small criticisms which I am going to offer affect his results in the least bit. Firstly, I wonder whether the definition of income or benefit used by Mr. Lydall is the best or most useful one for his purpose. Mr. Lydall has been limited very largely by the information available to him in general and by the scope of the Inland Revenue statistics and the other official statistics in particular. He has also been limited by the definition used in national income accounting and by the definitions of income used for tax purposes. Now, there are several other ways in which income or accretions of wealth could be defined. For instance, I find it curious on the one hand to include as income the accretions of wealth in the pension funds, and on the other hand to exclude the capital gains of any sort. It is very difficult for a person to spend the increase in his pension fund before he starts drawing his pension, but it is quite easy for a person to realize a capital gain and to spend it there and then. If, therefore, we are looking at spending power it is quite possible to imagine a more suitable definition of income or accretion to wealth than the one used by the Inland Revenue, the one used in the national income accounts or the one taken by Mr. Lydall. I suggest that what is really wanted in such an exercise as this is a definition of income that includes such things as capital gains which can be realized, but excludes the accruing claims upon superannuation pension and life assurance funds. The problem is a very difficult one—indeed the Inland Revenue themselves make no attempt at a statutory definition of income.

May I now turn to Table 12. For the year 1957 Mr. Lydall starts with £600 million of allocated income going to the top 1 per cent. and works back from there to a total of £1,400 million. I shall consider only the item 2(*c*) where he adds £200 million to the £600 million he started with, thus increasing the share going to the top 1 per cent. by a third.

He has been very fair throughout his paper in deliberately taking an extreme view by allocating as much as 25 per cent. of the unallocated income to the top 1 per cent. This allocation has been described by Dr. Barna as reasonable. I want to suggest some reasons for thinking that it might be over-generous.

A large proportion of this remainder is described by Mr. Lydall as income that evades or avoids tax—income which, therefore, escapes the Inland Revenue net and consequently is left out of the distribution of allocated income given in the national accounts.

Much of the income that evades tax which is one of the components of this item belongs to people with small incomes or at any rate to people outside the top 1 per cent. In 1948 legislation was brought in requiring expenses payments and benefits in kind to directors of companies and employees with incomes over £2,000 to be treated as income subject to deduction only of proper expenses. This special treatment of expenses claims and benefits and the consequent adjustment of taxable income (to adopt Mr. Lydall's term) must be an important factor for the top 1 per cent. No such legislation has been brought in for incomes less than £2,000. On the face of it, therefore, smaller incomes may have had a greater opportunity since 1948 to avoid or even to evade tax. Another indication of this can be found in the statistics of the amounts of Back Duty, published by the Inland Revenue in their Annual Reports. These figures give both the numbers of cases and the amounts of tax. After excluding the penalties the average amount of tax collected is only £800 per case. This sum may sound a lot, but a Back Duty case is normally brought into the figures only once, and may cover five, six or even ten years. When you remember that the standard rate of tax is nearly half the income, the average amount of income per year represented by this tax of £800 is quite likely to be nearer £100 than £1,000. It seems probable, therefore, that quite a large amount of evasion does belong to incomes outside the top 1 per cent. Furthermore, it is common knowledge that evasion can be practised by small income earners, e.g. by not returning income from small jobs at the week-end or in spare time or by omitting items of receipts from business accounts.

Consider also what happens to the distribution of incomes when you alter your definition of income by adding a new component. The re-definition alters the whole distribution of incomes. Some members go up in rank and others down. The composition of the top 1 per cent. will contain a different set of individuals. Consider one such change. An individual who was outside the top 1 per cent. has an addition to his income which brings him into it thus displacing an existing member. But only a part of this additional income is a net addition to the aggregate income of the top 1 per cent. as the remainder has been taken up in displacing the sitting member. So that the amount of this new element owned by the new top 1 per cent. is more than the net addition of 25 per cent.; it may be 30 per cent. or more. For these reasons the 25 per cent. of this item which Mr. Lydall has allocated to the top 1 per cent. seems very generous, and indeed may be unnecessarily so even for his purposes.

There is one detail I should like to call Mr. Lydall's attention to. When on p. 34 he is considering the future prospects of investment income and dividends he seems to have overlooked the fact that profits tax is now at a flat rate and no longer charged at a higher rate on distributed profits.

I have very great pleasure in seconding the vote of thanks so ably moved by Dr. Barna.

The vote of thanks was put to the meeting and carried unanimously.

Professor Paish: It is always a very great pleasure for an amateur to listen to a professional and I feel that in Mr. Lydall's paper we have a thoroughly professional performance. I am grateful to him for closing a number of loopholes which people had discerned in the approach made by myself and others even though it is good to be assured that there was nothing there.

I should like to support Mr. Lydall against Dr. Barna about the question of company profits. As far as I can see a shareholder can benefit from his investment in only two ways. One is in dividends and the other is capital appreciation, and unless and until he gets one of these he does not get any benefit, however much the Company ploughed back. It would be reasonable to add in capital appreciation but, here again, even if a man had held his capital from 1938 entirely in a typical holding of ordinary shares, he would have had capital appreciation appreciably less than the rise in the general level of prices. I do not think it is fair to assume that the top 1 per cent. would have their investments entirely in ordinary shares, and in their fixed interest securities they would have had a substantial money loss, let alone a real loss. The individuals who have had their capital in physical assets, especially if they bought them on borrowed money, have been the only people to profit. If you are going to take in capital appreciation of the ordinary shares of the top 1 per cent., you must also take in the capital appreciation on the house of the owner-occupier, particularly if he bought the house on borrowed money, not only because of the appreciation of the house but the depreciation of the liability.

If I have any criticism to make of Mr. Lydall's paper, it is that it omits two or three things I should have liked to see included. One is some estimate of what was happening to the bottom half. If we can go down halfway we must be able to get some idea of the bottom half. My own impression was that it was the middle which was doing best, that there was a redistribution of income towards the middle both from the top and the bottom, and the bottom half was probably not much more than keeping level with the average. It was people with incomes round about the full weekly wage of the male wage-earner who were doing the best, while people above and below were not doing so well.

A second question on which I should have liked to have information is the effect of redistribution of income on taxable capacity. With a highly progressive income tax and surtax system, the greater the equality of incomes, the lower the yield of a given rate of tax. My own rough guess is that if the present total of personal incomes had still been distributed in the same proportions as before the war we should have got about the present yield from personal income tax at about pre-war rates of tax, or about two-thirds of the present rates. In other words, the extremely high rates of income tax are necessitated not

only by increased Government expenditure, but by the redistribution of personal incomes, which has made any tax rate give a lower yield. If personal incomes had not been redistributed, we should have been able to afford 5s. or 6s. income tax. This, if true, is quite an interesting reflection.

Mr. K. J. BURTON: Previous speakers have dealt with some of the points which occurred to me in reading this interesting paper which Mr. Lydall has presented to us. It is always interesting to have one's "hunches" confirmed by factual analysis of the type Mr. Lydall has provided. I suspect there are no two people in this discussion who would define personal income in precisely the same way. The Inland Revenue authorities are bound to have regard to the forms of income on which they can levy tax, and the obvious practical purpose for which they want some idea of the distribution of income is in order to determine the effect of changes in the rates and form of assessment to tax.

I am not an economist but I am not at all clear that even economists are agreed as to what personal income is. Although the book produced by the Central Statistical Office to describe the national income statistics bravely starts out with a simple definition, as one finishes the first chapter one discovers all sorts of questions and qualifications attached to this simple definition.

The first thing to do is to decide the purpose for which one is examining the distribution of income. The author has provided us with three or four different measures, the last of which I think he does not regard as measuring anything which is immediately of significance to his purpose in demonstrating what changes have occurred over the past 20 years. He rejects the idea—as I should—that one should take into account the undistributed profits of companies. The question of how one deals with capital gains is a difficult one; as far as I am aware there is no means by which they could be computed for the purpose of an analysis such as we have in the paper.

There are two detailed points to which I would like to make more specific reference. I cannot understand why pensions under private pensions schemes should be treated differently from those of civil servants merely because the taxpayer pays all the cost of such pensions as it emerges without the establishment of any fund. Pensions so far as they are paid from employers' contributions are generally inalienable, at any rate up to the time they are receivable, and it is wrong to regard a contribution made towards a pension as if the employer is providing an immediate source of income to the employee. It is far more realistic to take account of the pensions when they are payable, and to look immediately at the actual income which a man can dispose of and use for his own purposes; that is, if we are talking about personal spending income.

I find some difficulty over the question of owner-occupied houses. I am sure the economists also find difficulty over this question, and I cannot see the justification for treating the current enjoyment of one particular form of capital goods as a source of income when no money passes, the more so as one does not deal similarly with the enjoyment of all the goods which one puts into the house. It seems to me that there is no valid reason for the present form of tax treatment. Schedule A assessments are themselves pretty arbitrary. If, however, one were to try to find out what a house is actually worth to its owner-occupier by imputing an element of "excess rents" one should pay some regard to the depreciation of the asset. People overlook the fact that houses are a wasting asset. On the particular point raised by the last speaker regarding capital appreciation of houses, I think that near the top of the upper income bracket there has often been capital depreciation, judging from what has been happening to the houses one sees round the country which were occupied by families at no great distance of time.

These two questions of superannuation and owner-occupied houses are both becoming increasingly important factors in the national economy. It seems to me, however, that we must be careful about imputing income to people if it is not clear that it is income which they can immediately dispose of, at any rate if we are talking about personal spendable income.

Mr. UTTING: Professor Paish has already asked the question which is the main point of my contribution. Mr. Lydall's paper deals wholly with the top 50 per cent. of the income distribution—and the main part of it is concerned with the top 20 per cent. I am equally interested to see what has happened to the bottom 50 per cent., who are of major importance from the point of view of social policy, although their impact upon many aspects of the national economy is very much less than that of the people at the top. I have made a few, rather rough, calculations based on the Blue Book figures and would like to give you some of them now. Mr. Lydall may well have more detailed and more carefully calculated figures and, if so, I hope that he will put them into his written reply so that they can appear in the report of this meeting.

I have tried, first of all, to see what has been happening to those below the median, between 1949 and 1957—you cannot go back to 1938 on the basis of the Blue Book. It seemed to me that from 1949 to 1957 the before-tax share of the bottom half of the income distribution increased only by about 2 percentage points, from something like $23\frac{1}{2}$ to $25\frac{1}{2}$ per cent., while the share of most of the deciles above the middle was increasing relatively more than that. I tried also to look down the scale to the 70th percentile. I chose that because it can just be done with the Blue Book figures for 1954 and 1957, and Mr. Lydall himself has commented that the 70th percentile seems to have lagged behind those above it from 1949 to 1954. Between 1954 and 1957 it seems to have caught up a little (but only a little) again. So far as I can see, the before-tax share of the bottom 30 per cent. increased from $9\frac{1}{2}$ to 11 per cent., but its after-tax share increased by rather less than that, by about 1 percentage point, from 11 to 12 per cent.

Added in writing:

Percentage of Allocated Income before Tax Received by Specified Inter-percentile Groups

(Continues Lydall Table 6)

Inter-percentile group	1949	1954	1957
21st to 30th	11·5	12·0	12·5
31st ,, 40th	9·0	10·0	11·5
41st ,, 50th	8·5	9·0	9·0
51st ,, 60th	—	7·0	8·0
61st ,, 70th	—	6·5	6·5
Top 30 per cent. . . .	59·0	58·0	54·0
,, 40 ,, . . .	68·0	68·0	65·5
,, 50 ,, . . .	76·5	77·0	74·5
,, 60 ,, . . .	—	84·0	82·5
,, 70 ,, . . .	—	90·5	89·0

Percentage of Allocated Income after Tax Received by Specified Inter-percentile Groups

(Continues Lydall Table 7)

Inter-percentile group	1949	1954	1957
21st to 30th	11·6	12·1	13·0
31st ,, 40th	10·5	11·5	12·0
41st ,, 50th	9·5	9·0	9·5
51st ,, 60th	—	8·0	8·5
61st ,, 70th	—	6·5	7·0
Top 30 per cent . . .	53·5	54·0	51·0
,, 40 ,, . . .	64·0	65·5	63·0
,, 50 ,, . . .	73·5	74·5	72·5
,, 60 ,, . . .	—	82·5	81·0
,, 70 ,, . . .	—	89·0	88·0

N.B.—All my estimates are to the nearest 0·5.

Mr. E. INNES: May I turn briefly to superannuation. Firstly, a point to which Mr. Burton referred, that employers' superannuation contributions are in a sense only contingently allocable as entitlement to benefit does not usually arise until retirement takes place—a person leaving employment before retirement age would not necessarily be entitled to any benefit except in respect of his own contributions.

Secondly, and more important, Mr. Lydall's assumption is that the total superannuation contributions, whether by employers or by employees, of the top 1 per cent. amounts to 15 per cent. of the income covered. This assumption would seem to have some quasi-statutory backing in that the maximum annual contribution payable by a self-employed person under the new arrangements introduced by the 1956 Finance Act is 15 per cent. of earnings, and also in that the maximum annual contribution which an employee may pay to an approved Superannuation Fund is 15 per cent. of remuneration. So far as employed persons are concerned, therefore, the total contribution, including that of the employer, could well be above 15 per cent.

As inflation moves slowly on the pensions of the top 1 per cent. will move in step as it may be supposed that they will be linked to retiring salary. However, the increase in cost is accelerated since the increase in the provision for depreciation, as it were, has to be written off over a shorter period. This concentration of cost can and does give rise to total superannuation contributions greater than 15 per cent. of the remuneration and in individual cases they may be substantially greater. In a scheme providing pensions of two-thirds of retiring salary it would be by no means unusual to find that the total contribution in respect of the senior employees amounted to 25 per cent. or more of their total remuneration. The indication would seem to be therefore that the figure of 15 per cent. is on the low side.

Mr. J. W. S. WALTON: I should like to add something to what has already been said about the definition of income or benefit and the treatment of superannuation. One should be clear whether the benefit is the current benefit or includes claims on a future benefit. Mr. Lydall has said that the case for regarding undistributed profits as a personal benefit is weak. I think that a similar kind of argument applies to an individual's current share in a future benefit from funded private superannuation or life assurance. In the present this is an unsubstantial thing. If one were looking at the distribution of wealth one would, of course, be fully entitled to include the accrued value of this claim on a future benefit or, at any rate, that part of it which is immediately accessible. In looking at the distribution of income, however, the kind of thing one has in mind is how changes in the distribution affect purchasing power, expenditure patterns and the social structure of the community. For this purpose it seems to me that the appropriate measure of income or benefit is that currently received.

I think the difficulties inherent in Mr. Lydall's approach to this question can be seen quite clearly from his treatment of pensions from private superannuation and life assurance funds, on the one hand, and of the national insurance retirement pension on the other. Aggregate personal income in the national income figures includes all the revenue of private superannuation and life assurance funds—their investment income, the employers' contributions and the employees' contributions, which are part of employment income. The Inland Revenue data and the distribution of income in the Blue Book exclude most of the contributions and all the investment income of the funds, but include the pensions received by individuals from them. Mr. Lydall has tried to allocate the contributions, etc., of the top one per cent. on the ground that they are part of personal saving. In order to avoid double counting he has deducted the pensions received by them. In the case of national insurance, however, he has allocated the retirement pension along with the other social service benefits and has deducted the employers' and employees' contributions. The argument here is presumably that the contributions are a form of forced saving and are best regarded as a tax. Contributions to private superannuation, on the other hand, while they may be a contractual obligation are in essence voluntary and it would in the last resort be open to an individual to refuse to save to this extent, although he might

have to change his job to do so. I think that the operative distinction is not between what is voluntary and what is compulsory saving but between what is a current benefit and what is merely a claim on a future benefit. One should note that Mr. Lydall's treatment means that people receiving the national insurance retirement pension would be counted in the distribution of income but those getting a pension from private superannuation alone would be left out. This makes little sense to me. For one thing, the estimate of the number of incomes would have to be reduced by the numbers who have private pensions and no other income over £50, and the value of all the percentiles would change on this account.

One should remember when considering the two measures of income—contributions or pensions—that there might be a substantial difference in the trends of the distribution of income according to the measure used. A distribution taking the contributions, etc., throughout would always appear less equal than one in which all pensions count as income and all contributions, etc., are excluded. Shifts over time in the balance of the active and inactive population would be reflected in quite different ways in the two distributions.

It seems to me that from the national income point of view it is quite simple to concentrate on the current benefit when drawing up the distribution of income. Life assurance and superannuation funds could be set on one side as channels of contractual saving. The income or benefit of individuals would be defined as what they currently receive and it would, therefore, include the pensions received from private superannuation and life assurance funds, as current transfers from the institutions for contractual saving. Although the annual increase in the funds of these institutions is ultimately the property of individuals, it would not be allocated to individuals because it does not form part of their current benefit.

I now turn to the question of wives' earnings. I welcome the prominence which Mr. Lydall has given to this subject. The period from 1938 to 1957 has certainly been an unusual one in relation to wives' earnings. Many married women went out to work during the war for the first time, and this trend has perhaps been encouraged since the war by the favourable taxation treatment of their earnings.

As Mr. Lydall points out, the way in which this is reflected in the income distribution depends on the choice of the income unit. The husband-and-wife unit is quite a real one, but it does mean that the shape of the income distribution will depend on the proportion of the population which is married and on the number of married women who go out to work. One should remember that occupied married couples are at their peak earning capacity and may come fairly high up the income scale. The average male industrial worker has been some two-thirds of the way up the income scale in the post-war years and his wife has only to go out to work for the couple to get close to the top 10 per cent.

The effect of the increase in the number of working wives can be seen by looking at the percentiles of distributions of income including and excluding wives' earnings. There are various difficulties in the way of using the data of wives' earnings in the Inland Revenue Survey for 1937–38, but there are indications that since 1949 the various percentiles, down to the 70th, of the distribution of income including wives' earnings have increased consistently more than in the case of the distribution excluding wives' earnings. In the upper half of the distribution of income including wives' earnings for 1949, 1954 and 1957 there was less dispersion (as measured by the ratio of the 1st percentile to the median) than in the upper half of the distribution excluding wives' earnings. But within this range, there was much less dispersion between the 1st percentile and the 10th percentile of the distribution including wives' earnings, than between those of the distribution excluding wives' earnings, and rather more between the 10th percentile and the median. In other words, the upper half of the distribution including wives' earnings is more equal than that of the distribution excluding wives' earnings and this applies particularly to the range between the 1st and the 10th percentiles. This reflects the concentration of the income of couples with wives' earnings between the 10th percentile and the median. There appears to have been little difference, as between the two distributions, in the dispersion between the median and the 70th percentile.

Few wives were working in 1938 and so there would have been relatively less difference between the two distributions in that year. It follows that the increase of wives' earnings since 1938 has led to a reduction in dispersion between the median and the upper percentiles. But I am not sure that this movement can be said without qualification to have been one towards greater equality of income. The Inland Revenue data tell us nothing about movements since 1938 in the dispersion between the median and the bottom end of the scale. There are relatively few married couples among those with the lowest incomes. One would therefore expect the increase of wives' earnings since 1938 to have been a factor contributing to an increase of dispersion at the bottom end of the scale, as well as to the known decrease in dispersion at the top end of the scale.

Mr. LYDALL subsequently replied in writing as follows :

I am grateful to all those who took part in the discussion for their interesting and helpful comments. They have raised at least three important issues.

The first is how to deal with undistributed profits. Dr. Barna considers that undistributed profits form part of personal income ; Professor Paish takes the opposite view. Mr. Paine, who is in favour of including capital gains in income, would, presumably, include undistributed profits to the extent to which they lead to capital gains. I would not disagree with anyone who says that undistributed profits *ought to* lead to a rise in capital values (which is what Dr. Singer suggests at the end of his article (1958)), but what troubles me is the lack of evidence that they do *in fact* lead to capital gains. Moreover, even if undistributed profits did produce appreciable capital gains in current money values, we should still be faced with the difficulty of deciding whether an increase in the money value of capital, which falls short of the rise in consumer prices, is really a capital gain or not. There is clearly room for some further thought, as well as for more empirical work, on this whole problem.

The second major issue is whether pension fund contributions should be included in the current income of employees on whose behalf the contributions are made. Mr. Paine, Mr. Burton and Mr. Walton all felt that they should not be included since they are not part of spendable income. I agree that this is a matter of legitimate dispute ; but I think we should recognise the logical consequences of taking the line suggested by these speakers. If pension contributions are excluded from income, then why not exclude life assurance contributions ? Mr. Walton actually suggests this. But if life assurance contributions are excluded, then why not exclude mortgage repayments, which are equally binding ? And why not exclude all regular contributions to saving schemes of any kind ? In the end we may be brought by this kind of thinking to the point of defining spendable income as income which is spent, which would be absurd.

To me it seems obvious that pension fund accumulations are a form of personal saving and that, since all saving comes out of income, they are part of personal income. No one seriously objects to belonging to a pension fund and most people regard this as a valuable addition to their current income. Good pension arrangements are widely advertised as an inducement to join a particular firm or institution. The difference between private pension contributions and national insurance contributions is that the former create a personal right to a specified pension or other benefit while the latter create no binding rights. National insurance contributions are really part of the taxation system, and this is how they are regarded by the public. I admit that my own treatment of Civil Service pensions was not consistent with the treatment of other pensions and I now think that I should have made it so.

The third important point, made by Professor Paish and Mr. Utting, is that I devoted little or no attention to the bottom half of the income distribution. I accept this criticism in principle and agree that much more thought is needed about this matter. But the real difficulty is the lack of data on the lower incomes, especially for pre-war. Mr. Utting's figures are a useful addition to my own ; but they are necessarily rather rough, and they add little to our understanding of the processes at work. My own guess is that the apparent lag in the lowest incomes is mainly a reflection of the growth in the number of old age

The initial size of each "model year" generation is, however, not directly given. The figures in Table A2 for this item are:

1930–34.—R. L. Polk annual registration figures as reproduced by Smith (1942).

1935–42.—R. L. Polk figures of current model registrations on July 1st of the year concerned *plus* next twelve months' registrations *minus* the number of current model registrations at the later date.

The latter registration figures have also been taken from *Automobile Facts and Figures*.

The fractions in Table A3 are the quotient of the number of surviving cars of a given model year and the initial size of that model year generation. For the treatment of the period 1941–46, see main text.

Discussion on Mr. Cramer's Paper

Professor Stone: I am very pleased to be given the opportunity of proposing a vote of thanks to Mr. Cramer for his most interesting paper. His subject is an important one and it is good to see it studied as a statistical phenomenon amenable to empirical examination. Accountants and economists alike are concerned with the problem of how the gradual wearing out of durable goods should be reflected in private and national accounts and will welcome attempts, such as Mr. Cramer has made, to show by detailed study the means by which an appropriate formulation of the phenomenon of depreciation can be made.

I find Mr. Cramer's paper particularly interesting because I have been working for some time on the demand for durable goods and find, in doing so, that it is necessary to introduce a concept of consumption, which I think of as depreciation, as well as the more straightforward concept of purchases. In order to have a definite model for demand it is necessary to define the depreciation process and estimate its parameters. Partly because of its favour among accountants in this country and partly because it fits comfortably into the formulation of my demand model I have adopted the reducing balance method of estimating depreciation which in a sense is the deterministic analogue of Mr. Cramer's stochastic formulation.

Mr. Cramer does not stop at a theoretical formulation but tries by various means to estimate the parameters in the model of depreciation. This is also important for the demand analyst. For different periods Mr. Cramer arrives at estimates which, if true, are useful, and somewhat different from what might have been expected. They reveal however a complication, namely, that there has been a large apparent fall in the depreciation rate for motor-cars since before the war. This means that the assumption of a constant depreciation rate over time may be seriously misleading and not merely a minor source of inaccuracy. I hope we shall hear this evening from those with practical knowledge of the motor trade what they think of Mr. Cramer's estimates and of the large change which they suggest has taken place. I wonder, for example, how far the apparently large depreciation rates relating to the pre-war period can be taken to reflect a high degree of obsolescence in a rapidly developing product and how far the low rates of the post-war period reflect a much higher degree of technical stability in which obsolescence is relatively unimportant.

I hope that this paper will stimulate further the intensive study of depreciation. The author and the Society may well feel proud of it and I have great pleasure in proposing a hearty vote of thanks to Mr. Cramer.

Sir Robert Shone (seconding the vote of thanks): I am delighted to have the opportunity of speaking on Mr. Cramer's paper, as depreciation is one of the matters on the border line of economics and accounting which, in spite of its great importance, has not been effectively analysed. A more scientific assessment of the magnitude of depreciation

REPORT OF THE COMMITTEE ON THE SUPPLY OF AND DEMAND FOR STATISTICIANS

[Introduced by PROFESSOR E. S. PEARSON, C.B.E., and discussed before the ROYAL STATISICAL SOCIETY, December 3rd, 1958, the PRESIDENT, SIR HARRY CAMPION, C.B., C.B.E., in the Chair]

1. ORIGIN AND APPOINTMENT OF COMMITTEE

IN February 1956 Mr. A. W. Swan, acting on behalf of the Operational Research Society, wrote to the Honorary Secretary, referring to the difficulties experienced in the recruitment of suitable candidates for industrial Operational Research and asking whether Council would consider taking steps to bring the existence of useful careers in this field to the notice of young statisticians.

This matter was considered by Council at its meeting on March 8th, 1956, when it was agreed that the difficulty of attracting statisticians into Operational Research was part of a larger problem—the general shortage in supply of trained statisticians. Council, therefore, decided to appoint a committee with very wide terms of reference to investigate and report on the matter. The following persons agreed to serve on the committee:

Professor E. S. Pearson (Chairman,) Professor G. A. Barnard, Mr. H. Campion, Professor D. G. Champernowne, Dr. H. E. Daniels, Professor M. G. Kendall, Mr. G. W. Sears, Sir Robert Shone, Mr. L. H. C. Tippett.

The committee held four meetings during 1956–58.

It is well to make clear at the outset of this Report that the Committee dealt only with part—though what it regarded as a very important part—of the possible field of enquiry. We were concerned mainly with the supply of and demand for university graduates and we decided that the problem of shortage of statisticians in this category was of sufficient importance and magnitude to justify us in reporting back our findings to Council. At the same time, in the memorandum for schools on *The Career of Statistician*, prepared by the Committee and now in draft, it has been made clear that there is valuable work to be done in the statistical field by boys and girls who do not go to a university.

2. PRELIMINARY DECISIONS ON COURSE OF ACTION

At the first meeting of the committee it was decided:

(*a*) That more concrete information was needed on the extent and nature of the current demand for statisticians.

(*b*) That some enquiry should be made of the universities as to the recent output of statistically qualified graduates.

(*c*) That from a long-term point of view improvement in the position must depend to a large extent on getting across to the schools information that interesting and varied careers for mathematically minded boys and girls were to be found in Statistics.

(*d*) That the possibility of obtaining exemption from national service for statisticians should be explored.*

* Owing to the later decision to discontinue national service, further action was dropped.

In connection with (*a*), it was decided that advertisements for vacant statistical posts announced during the first six months of 1956 should be followed up with a Questionnaire asking the employers concerned for confidential information. Under (*b*), it was decided to confine attention in the first place to the recent output of mathematical statisticians. It was considered that one of the most effective methods of reaching the schools, (*c*), would be by the circulation of an appropriate brochure on careers in statistics and steps were taken for its preparation.

3. Enquiry Regarding Advertised Posts

3.1. A record was made of all posts which appeared to require statistical qualifications advertised in *Nature* and *The Economist* between January 1st and June 30th, 1956. In addition, a certain number of statistical vacancies which had been notified directly to university departments during the same period were added. From all this information a list of about 150 different vacant posts was obtained*. The questionnaire shown in the Appendix with an appropriate covering letter was then sent out in December 1956 in respect of all these posts and ultimately replies were obtained in 135 cases. Table 1 shows a broad classification of the answers.

3.2. *Classifications in Table* 1

Under the heading "Government, Nationalized Industries etc." are included posts in the Statistician Class, in the Scientific Civil Service (e.g. both in Government Departments and in Research Establishments under the D.S.I.R., or Agricultural Research Council, etc.); also posts under the National Coal Board, Central Electricity Authority, B.O.A.C. etc.

It was thought desirable to group separately 15 "lesser posts" with salaries of under £550, because it seemed likely that at the bottom of the scale there would have been a good many more posts for statistical clerks, computers, etc. which would have escaped the enquiry.

While the categories of "medical statistician" and "operational research worker" were based without difficulty on the wording of the advertisement, it was much harder to decide whether to classify a post as for a "mathematical" or "economic" statistician. Clearly no precise definition exists, but it was thought useful to make a rough division into the more mathematical and the more economic. This division could frequently be based on the wording of the advertisement and in other cases on information supplied on the questionnaire, including that on the background training and experience of the successful applicant.†

It will be seen that Table 1 contains a column "qualifications inadequate". The entry involved a subjective judgment on the part of the compiler of the table; it was based on such information as was provided on the form as to the degree courses and experience of the successful applicant. It will be seen that out of the 24 cases entered in this column, 13 relate to mathematical statistical posts. In the great majority of these, the successful applicant had a mathematics degree but there was no evidence suggesting that he could

* There was some uncertainty as to the exact number as it was suspected that in a few cases the same post was advertised twice during the period in rather different terms. Further, during the course of the enquiry information regarding a few additional vacancies came to light.

† Admittedly, by using the qualifications of a candidate in deciding on the type of post, there is a risk of classing him as suitably qualified when he is not.

have had more than a rather superficial knowledge of the theory and practice of mathematical statistics.

TABLE 1

Summary of Results of Enquiry Regarding About 155 *Statistical Posts Advertised in* Nature *and* The Economist *or Heard of Privately by Members of the Committee During the First Six Months of* 1956

		Number of Posts: Advertised Total	Advertised New Posts	Filled Total	Filled Qualifications Inadequate	Not Filled
1. MAIN POSTS						
University	Math. statistics	9	7	7	1	2
	Economic statistics	10	8	9	—	1
	Medical statistics	2	2	1	—	1
	Total	21	17	17	1	4
Government (central or local) and *Nationalised Industries* etc.	Math. statistics	19	11	11	4	8
	Economic statistics	24	11	15	1	9
	Medical statistics	3	—	3	1	—
	Operational research	5	3	3	—	2
	Total	51	25	32	6	19
Industry and *Commerce*	Math. statistics	19	14	15	8	4
	Economic statistics	23	15	14	4	9
	Operational research	6	4	5	—	1
	Total	48	33	34	12	14
2. LESSER POSTS (salaries below £550)		15	5	9	5	6

SUMMARY		Advertised Total	Advertised New Posts	Filled Total	Filled Qualifications Inadequate	Not Filled
MAIN POSTS	Total for Mathematical statistics	47	32	33	13	14
	Economic statistics	57	34	38	5	19
	Medical statistics	5	2	4	1	1
	Operational research	11	7	8	—	3
LESSER POSTS		15	5	9	5	6
	Grand Total	135	80	92	24	43

No replies received—20.

3.3. *Conclusions from Table* 1

It will be seen that during the period in question the university vacancies were filled rather more successfully than those in Government service or in industry. A number of the unfilled Government vacancies were for appointments in the Statistician Class at home or in the Colonial service. Taking the totals, it is seen that 43 out of 135 posts, or 32 per cent., had not been filled at the time of the enquiry i.e. 6 months or more after the advertisement. In addition, it seems possible that a further 18 per cent. or more of the successful

candidates had scarcely adequate qualifications, and may only have been appointed for want of better qualified applicants.

It will be noted that 80 posts (59 per cent.) were described as "new posts". The unfilled posts were equally distributed among the new posts (26 out of 80 unfilled) and the replacements (17 out of 55 unfilled).

If it is asked whether it is possible from the information obtained from this limited enquiry to draw any numerical conclusions as to the extent of un-balance between the flow of demand for statisticians and the flow of supply, we think the answer must be no. In the first place there may well have been a number of advertised posts which were missed by limiting our search to *Nature* and *The Economist*. Again, certain posts are filled without advertisement, by direct application to the heads of university departments. At a time when the stock of trained statisticians is seriously insufficient for the potential demand, large numbers of unsatisfied advertisements for new and existing posts are to be expected during a period of six months, but it is difficult to draw any conclusions from these numbers which are directly relevant to estimating the extent of inadequacy of the net flow of trained statisticians to meet the potential demand for them. Nevertheless some further interesting comparisons are obtained by summarizing other items entered on the forms.

3.4. *Training and Experience of Successful Applicants* (*Table* 2)

Some details regarding the persons who filled the 92 posts are shown in Table 2, using the same broad classification for the nature of post adopted in Table 1. Clearly, without a detailed enquiry into the syllabuses of different university courses a classification by degree subject provides only a first approximation to the kind of statistical training a student has received. The following points should however be made:

Statistics or Mathematics plus Diploma or M.Sc.—Under this heading were included: graduates who had taken an extensive course in mathematical statistics at University College, London (8); mathematical graduates of Oxford (1) and Cambridge (3) who had gained Diplomas in Statistics at these universities; mathematical graduates who had taken an M.Sc. in Statistics (2).

Mathematics.—A number of these 28 graduates had undoubtedly received some statistical teaching with their mathematics but there was no evidence that any of them could be regarded as adequately trained in mathematical statistics on leaving their universities.

Economics.—The amount of statistical teaching involved in these degree courses will have varied greatly, but a high proportion of the 30 persons graduating in economics appear to have gained some useful statistical experience after leaving the university.

Other subjects.—The degrees obtained were as follows : Physics (1), Engineering (2), Geography (4), Medicine (2), Modern Languages (3), History (2).

It is clear that the type of post covered by the advertisements considered was intended primarily for the university graduate. Only six of the 92 appointees had no degree; one of these had qualifications from the Institute of Electrical Engineers and three others had passed some or all of the examinations of the Association of Incorporated Statisticians.

As might be expected, the universities picked persons with previous university research and teaching experience. While industry claimed half of those who were entering their first jobs, the Government and nationalized industries chose persons who had had experience outside the universities, generally of an economic statistical character.

TABLE 2

Training and Experience of Persons Filling the 92 Posts

Nature of Employment		Number of Posts Filled	Subject of 1st Degree: Statistics or Maths. + Dip. or M.Sc.	Subject of 1st Degree: Maths.	Subject of 1st Degree: Econ.	Subject of 1st Degree: Other	No Degree	University Statistical or Economic Experience: Research	University Statistical or Economic Experience: Teaching	Appropriate Non-University Employment	1st Job
University posts	Math. stat.	7	1	6	—	—	—	4	3	1	2
	Econ. stat.	9	—	1	7	1	—	8	4	2	1
	Medical stat.	1	—	1	—	—	—	—	—	1	—
Government, Nationalized Industries, etc.	Math. stat.	11	6	4	—	1	—	—	—	4	4
	Econ. stat.	15	—	1	11	2	1	4	—	13	2
	Medical stat.	3	—	—	—	2	1	—	1	1	—
	Op. res.	3	—	2	1	—	—	1	—	2	—
Industry	Math. stat.	15	5	8	1	1	—	2	—	2	9
	Econ. stat.	14	—	1	7	4	2	1	—	6	5
	Op. res.	5	2	—	2	1	—	—	—	4	1
Lower grade posts		9	—	4	1	2	2	—	—	3	4
Total		92	14	28	30	14	6	20	8	39	28

3.5. *Salaries Associated with Posts*

Adequate information about salaries was available in the case of 77 posts filled in Great Britain; leaving aside 7 posts of a junior character, there are left 59 posts filled by men, and 11 filled by women. Analysis of the data showed that little information would be lost by classing together those with (*a*) no previous experience and (*b*) experience of less than four years. Combining men and women and using the three main types of employment of Table 1, the figures for median and quartile salaries shown in Table 3 were obtained.

TABLE 3

Opening Salaries of 70 *Successful Applicants*

		University	*Government, etc.*	*Industrial*
Little or no experience	Number	(7)	(15)	(24)
	Q_1	£550	£615	£625
	Median	650	720	659
	Q_3	700	795	735
Considerable experience	Number	(8)	(8)	(8)
	Q_1	£850	£865	£950
	Median	975	1,050	1,150
	Q_3	1,025	1,095	1,500

For the women (who are included in the above), 9 had little or no experience and for these the median salary was £650 with lower and upper quartiles at £600 and £700, figures which do not differ appreciably from those for the combined sexes. The two women with considerable experience were given salaries of £728 and £1,145.

Finally, Table 4 compares salaries of posts of a mathematical character with those of an economic character.

TABLE 4

Salaries of "Mathematical" and "Economic" Posts

		Mathematical Statistics	*Economic Statistics*
Little or no experience	Number	(26)	(19)
	Q_1	£650	£600
	Median	700	650
	Q_3	800	710
Considerable experience	Number	(8)	(12)
	Q_1	£925	£900
	Median	1,000	1,000
	Q_3	1,320	1,050

This table excludes five posts which were neither mathematical nor economic in character, e.g. the medical posts. The slightly better salaries of the mathematical-type posts may be due in part to a tendency to classify posts requiring especial skill as being mathematical.

4. UNIVERSITY OUTPUT OF MATHEMATICAL STATISTICIANS

4.1. The enquiry was not a complete one, but it covered the main centres in Great Britain at which there had been substantial teaching in mathematical statistics during the years 1950–56. A summary of the information collected is shown in Table 5 where the first posts secured by outgoing students are classified under a variety of headings.

4.2. *Cambridge, Oxford and London*

A number of points are brought out by the first part of Table 5.

(a) *National Service.*—For a detailed analysis of the effect of national service, it would be necessary to distinguish between men and women graduates. However, we can see evidence of the different results following from what has been the usual Oxford and Cambridge practice of asking students to do their national service before coming up to the university and the London practice of generally taking them direct from school. There have, however, been further concealed consequences in the choice of employment. A number of the London graduates who have taken posts in Government service or industry have entered the Scientific Civil Service either with one of the Service departments or the Ministry of Supply or have taken up industrial work in connection with Defence, in order to obtain indefinite deferment of national service. In the same way, entry to a Life Office, to the Government Actuary's Department or to a firm of consulting actuaries has had an attraction, because it involved deferment while studying for actuarial examinations.

(b) *University teaching posts.*—The high proportion of these which have been filled by Oxford and Cambridge graduates is not altogether surprising, remembering how many of the best school mathematicians are drawn off to one or other of these universities. But here, again, the call-up has had some influence. The London student who has followed his first degree with a two years' Ph.D. course, has generally not been eligible for an academic

TABLE 5

First Posts Secured by Students with Statistical Training at Certain English Universities and Colleges

Students Completing in:	*Nat. Service*	*University Posts*	*Gov't., Nat. Industries, etc.*	*Industry*	*Actuarial*	*School Teaching*	*Abroad**	*Unknown or Died*	*Total*
1. (a) *Cambridge.*—Diploma in Statistics—									
1951 . .	—	1	1	1	—	—	2	—	5
1952 . .	1	3	2	—	—	—	—	—	6
1953 . .	—	2	—	6	—	—	3	—	11
1954 . .	1	3	1	3	—	—	—	—	8
1955 . .	—	3	1	2	—	—	2	—	8
1956 . .	—	2	—	2	—	—	4	—	8
Total . .	2	14	5	14	—	—	11	—	46
1. (b) *Cambridge.*—Research Students (1951–56)—									
	—	4	—	1	—	—	5	1	11
2. *Oxford.*—Diploma in Statistics—									
1949–56 . .	1	6	11	6	—	1	6	7	38
3. *University College, London.*—Graduates (Special and General) and Postgraduates in Statistics—									
1950 . .	2	1	3	4	—	—	1	—	11
1951 . .	6	1	2	2	—	1	3	—	15
1952 . .	1	1	4	2	3	—	1	—	12
1953 . .	5	1	—	2	1	—	3	1	13
1954 . .	4	—	1	4	1	—	—	—	10
1955 . .	2	—	4	4	4	1	4	—	19
1956 . .	2	—	2	4	—	—	1	—	9
Total . .	22	4	16	22	9	2	13	1	89
Allocation of those completing Nat. Service		1	5	7	—	—	—	7	20

contd.]

TABLE 5 (*contd.*)

Students completing in:	*Nat. Service*	*University Posts*	*Gov't., Nat. Industries, etc.*	*Industry*	*Actuarial*	*School Teaching*	*Abroad**	*Unkown or Died*	*Total*
4. *Imperial College, London.*—Maths. Department—Students taking first degree who subsequently took statistical posts—†									
1952 . .	—	1	1	4	—	—	—	—	6
1953 . .	—	—	—	4	—	1	—	—	5
1954 . .	—	1	1	3	—	1	—	—	6
1955 . .	2	1	4	1	1	—	—	—	9
1956 . .	—	1	1	1	1	—	—	—	4
Total . .	2	4	7	13	2	2	—	—	30
5. *London School of Economics.*—B.Sc. (Econ.) Graduates specializing in Statistics—									
1950 . .	4	—	—	3	—	—	—	6	13
1951 . .	2	1	1	3	—	—	1	7	15
1952 . .	2	—	—	2	1	—	—	5	10
1953 . .	3	1	—	2	—	—	—	2	8
1954 . .	3	1	—	2	—	—	3	—	9
1955 . .	2	2	1	6	1	—	—	1	13
1956 . .	5	—	—	3	1	—	1	1	11
Total . .	21	5	2	21	3	—	5	22	79
6. *Leeds University.*—Maths. Department—Students whose qualifications included the passing of some examination in Statistics—									
1951–56									
Maths. Special	2	—	—	13§	1	8	—	5	39
General .	7	2	1	11§	—	5	—	1	27
7. *Liverpool University.*—Students with appreciable statistical qualifications—									
1952–56 . .	—	—	3	3	1	—	3	—	10
8. *Manchester University.*—Maths. Department—Diploma, M.Sc. or Ph.D. students in Statistics—									
1952–56 . .	3	1	2	3	—	—	—	—	9

Notes—

* The category "Abroad" refers in general to overseas students who returned to their countries after graduating, but there were a few cases of "home" students taking posts overseas.

† All mathematics students at Imperial College (approximately 30 per annum) take some statistics.

§ Leeds. All 13 Special and 5 of the 11 General students took posts in the aircraft or armaments industries.

post until after a further two years of national service. By that time financial or other considerations may have deflected his interest elsewhere.

(c) *Economic statistics.*—The special third year course at the London School of Economics represents, at a high level, the statistical training that can be included in an Economics first degree course. Largely owing to the inadequate mathematical qualifications of the school entrants, this course has not been able to play as useful a part as it might in the supply of economic statisticians. It will be seen that in so far as information is available, a high proportion of those graduating in this group have taken industrial posts.

(d) *Students from overseas.*—The table shows the distribution of some of the considerable number of overseas students who have received a statistical training in this country, generally at the postgraduate level.

4.3. *Leeds, Liverpool and Manchester*

These figures are not comparable with one another. The Leeds students were either: (i) Mathematics students taking at least a one-year statistics course (certain of them a

statistics option extending over three years), or (ii) General students taking a statistics course over two years. It is likely that many of the 24 who entered the aircraft or armaments industries were influenced in their choice because of the deferment of national service involved. Six of the 10 Liverpool students took higher degrees and three of the others had taken national service before coming to the university.

The information from Manchester concerns only home postgraduate students. Information was not available concerning undergraduates, but Professor Bartlett wrote: "Those taking the Statistics Subsidiary course in their second year and then opting to take Statistics in their third year Honours final degree examination are reasonably well qualified and many would in fact obtain either a statistics post or perhaps more likely a mathematics post in which statistical qualifications are considered an advantage."

5. The Supply of Economic Statisticians

The evidence given earlier in Tables 1 and 4 accords well with the general impression that both the degree of shortage and the levels of salaries of economic statisticians are about the same as those of mathematical statisticians. The nature of the problem involved in training more economic statisticians, however, differs from that concerning mathematical statisticians in the following respects.

(*a*) Many of the jobs available to economic statisticians require in addition to academic training considerable outside experience, for example in industry, of a kind which it is not easy to make available during a period of training at a university.

(*b*) A knowledge of economic statistics would lose most of its value unless it was based on a wide understanding of the economic background. For this reason economic statistics is normally taught as one part only of an economics degree and it is difficult to present a convincing case for establishing an independent degree course in economic statistics with economics as a subsidiary subject.

(*c*) In most universities economic statistics appears as one *optional* subject within an economics degree course: the danger is that either it is made so difficult an option that only a few candidates offer it, or else that it is made so elementary as to be of only moderate value as a qualification.

(*d*) Although the position of the postgraduate economic statistician is not very different from that of the corresponding mathematical statistician if they are working for a written examination, he may suffer some disadvantage if he is writing a thesis for a doctorate. This is because the nature of the statistical material available is such that it is far harder to find a thesis subject in economic statistics that is within the competence of a single research worker to cover in two or three years, than to find one either in strictly mathematical statistics or in applications to the sciences.

In economic statistics as in mathematical statistics the best graduates are often tempted away by attractive offers from industry or government so soon as they have graduated, and find little incentive to spend a further year or two at the university acquiring a specialist training in economic statistics.

Under these conditions it is almost inevitable that the universities, whilst turning out an adequate number of economists with a sketchy understanding of statistics, only produce a trickle of fully qualified economic statisticians.

6. THE SUPPLY OF MATHEMATICIANS

6.1. For a number of reasons the question of the supply of statisticians at graduate level cannot be separated from that of the supply of mathematicians. Apart from the question of training in mathematical statistics, the university departments which teach statistics as a special or optional subject in an economics degree course are at present faced with difficulties because of the lack of mathematical qualifications among their students. What these departments would like to do, is to draw off into economics a greater number of school leavers who have passed in Pure Mathematics at the Advanced Level of the General Certificate of Education.

We must recognize the fact that in statistics as in other scientific disciplines, some basic knowledge of mathematics is becoming increasingly important if full use is to be made of some even of the simpler tools which theoretical research has provided. The Committee therefore was of the opinion that the inclusion of the information assembled below in sections 6 and 7 was very relevant to their enquiry.

6.2. While the figures discussed in sections 3 and 4 cannot form the basis of a precise estimate of the lack of balance between the demand for and supply of trained statisticians, we think that they do help to clarify the essential features of the situation existing to-day. When we turn to ask how the position can be improved, there are a number of questions to be answered:

(*a*) Are the existing facilities for teaching mathematical and economic statistics being used to the full?

(*b*) If not, what are the reasons?

(*c*) How far is the real difficulty the general inadequacy of the supply (in quantity and/or quality) of mathematically qualified boys and girls coming up from the schools?

(*d*) Can it be said that mathematical statistics is getting its fair share of this limited supply?

(*e*) If not, what steps should be taken to draw more mathematicians into our field?

The answers to these questions vary with the conditions holding at different universities. Before summarizing (in section 7) the views we have obtained from a number of university schools of statistics it will, however, be useful to record certain figures regarding the supply and employment of mathematicians, relevant to questions (*c*) and (*d*) above.

6.3. *Entry to Mathematical Courses in the Universities*

We know of two recent enquiries into the numbers of entrants to university mathematical courses.

(a) *That undertaken by the University of Birmingham* for entry for 1951–52 session (see R. E. Peierls, *Math. Gazette*, 1952, No. 315, pp. 33–35). This concerned the entry to universities and university colleges of England and Wales, excluding Oxford and Cambridge. It was found that for this year the total number of applicants* was just over 700. The maximum number of places which could have been filled by the institutions was 480–490. The actual number of places finally filled was 385. While it is likely that some of the 300 applicants who failed to get entry might have been acceptable at some institution had there been an effective pooling

* This figure is for applicants, not applications, since multiple applications were identified and allowed for. Successful applicants for Oxford and Cambridge were not included.

of information, Professor Peierls was of the opinion that over 200 of the applicants were not really of a suitable standard for entry to a university mathematics course.

(b) *That undertaken on behalf of the Committee of Vice Chancellors and Principals of the Universities of the United Kingdom*, entry for session 1955–56. (See Report on *Applications for Admission to Universities*, (1957) by R. K. Kelsall published by the Association of Universities of the British Commonwealth, 36 Gordon Square, London, W.C.1.)

We have taken from this Report information with regard to Subject Group No. 15: "Astronomy, Mathematics and Statistics". In addition to the universities and university colleges of England and Wales including Oxford and Cambridge, the information covers Belfast, Glasgow, Edinburgh, and St. Andrews. The figures for Home as distinct from Overseas applicants were as follows:

	Males	Females	Total
Applicants admitted—			
Oxford and Cambridge	189	40	229
Other universities	282	163	445
Total	471	203	674
Applicants not admitted—			
Other universities only	184	95	279

It was estimated that of those not admitted, very possibly over 200 did not qualify for a place. The number of places left unfilled for this mathematics group in "other universities" was given as 85.

Having regard to the differences in date and in the universities covered the results of these two enquiries are consistent. Between 600 and 700 applicants are being entered annually for "Special" mathematical courses at the universities of the United Kingdom. This figure is considerably greater than the figure for those graduating in mathematics.

In its *Return from Universities and University Colleges in receipt of a Treasury Grant* Academic Year 1955–56, the University Grants Committee gives a table (Table 8) showing the number of students awarded honours degrees in 1956, according to subject, at the universities and university colleges of Great Britain (i.e. England, Wales and Scotland). In mathematics by itself there were only 391 degrees (284 men and 107 women); if we include degrees in Mathematics and Physics, Mathematical Physics, Astronomy, Statistics etc. the total is raised to 441 (326 men and 115 women).*

There are, of course, many reasons for the difference between the initial entry and final output of mathematicians. After a first year of mathematics a number of transfers to other courses occur, e.g. at Cambridge about 150 students have recently been taking mathematics in their first year, but some 50 of these have changed to another course, e.g. to the National Sciences or the Mechanical Sciences Tripos, after completing Part I Mathematics. Again, the weaker students are often transferred to a "general" course.

There are also a large number of students admitted initially to the universities for a General or Combined Honours Course in Science,† including Mathematics and several universities are providing within such a syllabus increasing facilities for giving those who

* In addition there were that year 14 External Students of the University of London at non-university institutions in Great Britain who obtained an honours degree in Mathematics.

† The University Grants Committee (loc. cit) reports 436 students (362 men and 74 women) graduating in General Science in 1956, but does not mention how many of these had mathematical qualifications.

wish some elementary training in statistics. In general, however, the potential specialist in mathematical statistics must be drawn from among the abler members of the group of 600–700 school mathematicians who qualify annually to start on "Special" mathematical studies, and of whom only some 400 are carrying through as mathematical specialists to the degree stage.

6.4. *Employment of Mathematicians*

The competing demands for these qualified mathematicians are very many. A Report on *Scientific and Engineering Manpower in Great Britain* issued jointly in 1956 by the Ministry of Labour and National Service and the Advisory Council on Scientific Policy gave figures for (i) the number of qualified scientists and engineers employed in 1956 and (ii) the estimated requirements for 1959. Below, in Table 6 are reproduced the figures for qualified mathematicians* classed according to profession. It will be seen that just over 800 additional mathematicians a year were thought to be needed; two-thirds of these were required for teaching. No figure was given for the number required annually to make good wastage and retirements, but if this is put at 400, it would appear that a new entry of over 1,200 mathematicians a year would be required to satisfy the anticipated

TABLE 6

Estimates of Numbers of Qualified Mathematicians Employed in Great Britain in 1956 *and of the Increase Required by* 1959

Character of Employment	*Employed in* 1956	*Required in* 1959	*Increase*
Manufacturing industry	1,152	1,766	614
Industrial Research Associations	60	80	20
Nationalized industries	175	227	52
Central Government: Defence Depts.	544	599	55
Central Government: Civil Depts.	76	104	28
Central Government: Research Depts.	145	164	19
Local Authorities	28	28	0
Total	2,180	2,968	788
Universities	431	520	89
Schools and Tech. Colleges	8,871	10,426	1,555
Total	9,302	10,946	1,644
Grand Total	11,482	13,914	2,432

demand. This is a figure far in excess of the university output of specialist mathematicians. Clearly no great accuracy can be claimed for figures such as these, but we think they show that in order to make any substantial increase in the supply of mathematical statisticians it will be necessary to attract mathematically qualified undergraduates or graduates from a limited pool in face of strong competition.

6.5. *Occupations of Outgoing Cambridge Mathematicians*, 1955–56

An indication of what is at present happening to mathematical graduates is given in the following tables based on information kindly supplied to the committee by the Cam-

* The "qualifications" were defined as a university degree or an associateship of certain bodies such as the Manchester College of Technology.

bridge University Appointments Board. Table 7 shows what happened to a large proportion of the mathematicians who took Parts II or III of the Mathematical Tripos or completed their postgraduate courses in 1955 and 1956. Those under the category "further study" either went on to Part III (in the case of Part II students), to research training at Cambridge or elsewhere, or to a Diploma course. In 1955 six students who had taken Parts II or III of the Tripos, proceeded to the Diploma in Statistics, but none in 1956.

Table 8 gives a rough classification of the type of work which the 71 graduates entering Government service, the nationalized industries and general industry or commerce were required to undertake. The very heavy emphasis on work in connection with defence is clear. Even if this should fall off in the future there is a growing demand for mathematicians in connection with the various aspects of electronics, automation and the production of atomic energy, while from a long run point of view it is essential that the supply of teachers of mathematics should be maintained and if possible increased.

TABLE 7

Outgoing Cambridge Mathematicians, 1955–56

Year	*Course Completed*	*Further Study*	*National Service*	*University Posts*	*Gov't., Nat. Industries, etc.*	*Industry*	*Actuarial*	*School Teaching*	*Abroad*	*Total*
1955	Part II .	15	13	—	—	22	4	6	—	60*
	,, III .	10	1	—	—	6	—	2	—	19
	Research .	—	1	2	4	2	—	—	—	9
	Totals .	25	15	2	4	30	4	8	—	88
1956	Part II .	8	5	—	5	16	7	9	—	50*
	,, III .	14	1	—	4	3	—	3	1	26
	Research .	—	—	4	5	4	—	—	—	13
	Totals .	22	6	4	14	23	7	12	1	89

* These figures are for male students taking Part II in their third year. Thus in 1956 in addition to the 50 male Part II students shown there were 4 women and also 44 students who took Part II at the end of their second year, making 98 Part II candidates in all.

TABLE 8

Classification of Posts Taken by Cambridge Mathematicians with Government, Nationalized Industry and Industry

Year	*Aircraft and Guided Weapons*	*Other Defence*	*A.W.R.E. and A.E.R.E.*	*Electronics and Computing*	*Miscellaneous*	*Total*
1955 .	10	6	—	10	8	34
1956 .	11	3	6	9	8	37

7. THE SUPPLY OF MATHEMATICAL RECRUITS TO UNIVERSITY STATISTICAL COURSES

Our enquiries have led us to certain broad conclusions as to the present position at universities in regard to the supply of statistically trained graduates and postgraduates. At any rate as far as the more advanced courses are concerned, the existing or potentially existing facilities are on the whole not being used to the full because suitable students are not coming forward in adequate numbers. We have used the word "potential" because

of the very real difficulty in filling all the established lecturer or assistant lecturer posts in statistics; in the earlier part of this year there were certainly some half dozen unfilled posts in the main statistical teaching schools.

While the type of the courses varies from one university or college to another, we have no doubt that provided the existing establishment of teaching staff is kept filled, an appreciable increase would be possible in the numbers of undergraduate students taking the more specialized second or third year courses. This is true in London, at the Imperial College, the London School of Economics and University College, in Aberdeen, in Leeds and in Manchester. All those universities which have postgraduate diplomas in statistics, with the possible exception of Cambridge, have recently found their entry disappointing, at any rate as far as home students are concerned. Again, apart from overseas entry, the recent entry for research training in statistics has been falling off and from this has undoubtedly followed the difficulty in filling university teaching posts.

There is a good deal of current criticism among university mathematicians of the standard of the mathematical applicants from the schools and all available places in "special" mathematics courses are not being filled because applicants do not reach the acceptable minimum standard. Among those accepted, a high proportion (and we suspect that this is a symptom in most fields of study) seem to show little real interest in the subjects studied and to regard a university career primarily as a means of passing the examinations necessary to obtain a job. This attitude, which stimulating teaching should to some extent be able to remove, works against the drawing of good material into the statistical field in two ways.

The good mathematician with "applied" leanings will have less trouble if he continues with subjects whose ground work is already built into his standard mathematical training, and he can be sure of a choice of perfectly satisfactory mathematical posts immediately on graduating. Thus he sees no point in taking a statistics option in the second or third year of his mathematics degree course, nor in staying on for a fourth year diploma course.

On the other hand a statistical option in mathematics, or even a full statistics degree course as is possible in London, may appeal to the more mediocre student who believes that it will provide an easier route to a degree. In fact, such a student is likely to find statistics harder than the more conventional mathematics and statistical departments are continually having to protect themselves against acceptance of an undue proportion of the second rate.

There is no doubt that both at school and at the university there is an impression present, which is stimulated consciously or unconsciously by mathematical teachers, that mathematical statistics is not a mathematically interesting subject. It is possible that this prejudice might be overcome at the universities by making the advanced statistical courses mathematically more attractive. The difficulty is that this can only be done at the cost of reducing the genuinely statistical content of the course which teachers in statistics are naturally reluctant to do.

It would be possible to develop a mathematically attractive diploma course in "applied probability theory", including in it besides some "statistics", work on Markov chains, renewal theory, modern computing methods, games and information theory and linear programming, coupled with the relevant mathematical techniques such as measure theory, linear algebra, functional analysis and a modern approach to differential and integral equations. Such a course, however, would seem likely to turn out industrial mathemati-

cians rather than statisticians, and while its introduction would be an interesting experiment, it would hardly provide a universally appropriate solution.

Indeed it is clear that it is for each university to work out its own scheme of attracting a larger proportion of competent mathematicians into its statistical courses and, beyond putting on record this summary of the views which we have received from many quarters, we have not felt it appropriate as a Committee to make any recommendations in this respect. There are, however, two points at which definite action seems needed, where we have some concrete suggestions to make, namely with regard to (*a*) the supply of information to the schools, (*b*) the provision of grants for potsgraduate training and research.

8. Approach to the Schools

8.1. As stated in paragraph (2*c*) of this Report, the Committee agreed at its first meeting that the most hopeful long-term solution of the problem lay in an approach to the schools. Means must be found of getting across to headmasters, careers and mathematics masters—and through them to the boys and girls—that through a training in statistics mathematical talent can find a varied, interesting and nationally important outlet.

There are a number of points on which emphasis should be laid. In the first place it should be made clear to mathematics masters that the more advanced branches of the subject *are* mathematically interesting, and that the abler school mathematician who might be expected to look for a career in the academic world will find full scope for his research talents in mathematical statistics. In developing the theory of probability with the aid of modern mathematical techniques he will be following on the lines of some of the great European mathematicians of the past.

While the post-war concentration of attention on the nuclear and engineering fields has strongly attracted the young mathematician towards mathematical physics there are bound to be some whose inclination is to keep away from these fields. The scope for mathematics, through statistics, in the biological and medical sciences and in economics and sociology, would have an undoubted appeal to this latter class of school mathematician if sufficient information were available. For example, the potential training facilities in statistics of an institution like the London School of Economics are not being used to the full because there is so little awareness in the schools that the scientific treatment of economic and social problems is making an increasing call on mathematics and mathematical statistics.

8.2. *Memorandum on Careers in Statistics*

Our first step has been to prepare a memorandum on *The Career of Statistician* primarily aimed at informing the school-master who is concerned with giving advice on careers. We recommend that a copy of this memorandum be sent to all appropriate schools in Great Britain. An official list* shows 2,388 Secondary Grammar Schools, Independent and Direct Grant Schools in England and Wales. In addition, there are 201 Secondary Technical Schools, some of which might provide boys or girls interested in junior statistical posts. The total list of corresponding Senior Secondary and Independent Schools in Scotland is given as 265.

* Published in *The Education Authorities Directory and Annual*. 1958.

8.3. *Other Means of Maintaining Contact with the Schools*

In addition to the issue of the memorandum, we are agreed that other steps should be taken to maintain an active contact with the schools. Recently a number of efforts, independently organized, have been made to bring the subject of statistics, in its relation to mathematical education, to the attention of school-masters. Thus in 1957 a summer school was held at Cambridge for sixth form mathematics masters to give them some idea of the type of work their pupils would meet in coming up to Cambridge. Talks were included on the part played by statistics in the Tripos examinations. There was also an Easter conference at Oxford in 1957 and we understand that a similar conference is planned at Liverpool next year.

Earlier this year the Association of Incorporated Statisticians held a meeting for mathematics masters at which the chair was taken by a member of our Committee. In July 1957 another member contributed an article on Careers in Statistics to the Bulletin (No. 74) of the Public Schools Appointments Bureau.

We think that the Society in co-operation with other interested bodies should encourage the extension and co-ordination of activities of this kind. In this connection it should be recalled that in 1951 Council published the Report of a representative committee of Fellows on "The Teaching of Statistics in Schools" (*J.R.S.S.* Series A, Vol. 115, Part 1, 1952). This dealt both with the teaching of statistics as a more specialized study and with its place as part of a general education.

There is undoubtedly a growing interest in the subject in the schools, which the Society should do all in its power to encourage. In the recently published book, *The Teaching of Mathematics*, issued by the Incorporated Association of Assistant Masters in Secondary Schools (Cambridge University Press, 1957) there is a chapter on the teaching of statistics. Further, it is understood that the Mathematical Association is planning a report on "The Teaching of Statistics", for use in schools in conformity with its series of similar reports on the teaching of algebra, calculus, mechanics etc.

9. Postgraduate Grants

9.1. Apart from the importance of having funds available for research, a subject which was discussed at a recent meeting of the Society,* the training of statisticians whether carried to an advanced level or not, depends on the supply of teachers. At present, not only in the universities but in certain polytechnics, it has proved impossible to fill all the established posts. Unless the universities are able to find means of retaining some of their best students for postgraduate training, it will become increasingly difficult to fill the existing complement, let alone add to the number of adequately qualified teachers in statistics.

9.2. *Awards Made by the Department of Scientific and Industrial Research*

In broad terms, two types of grant are needed:

(*a*) For a Diploma course, in which the graduate in mathematics or economics—or indeed in some other branch of science or technology provided that his mathematical qualifications are adequate—receives a training in the theory and application of statistical method.

* See *J.R. Statist. Soc.* A, Vol. **121**, pp. 196–215.

(*b*) For research training leading to an M.Sc. or Ph.D. degree.

Starting from 1957, the financing from Government sources of postgraduate awards in science and technology has become the responsibility of the D.S.I.R.† This Department now offers Advanced Course Studentships to deal with (*a*) above and Research Studentships for (*b*). It is still too early to judge how far these grants will meet university needs. For example it is not quite clear how far sufficient grants will be available for research in economic statistics, nor can we yet be sure that the new scheme will be sufficiently flexible to cope fairly with the chance yearly fluctuations to be expected when dealing with small numbers. Thus in a subject like statistics, a particular college or university may have one or no suitable candidate for a research award in one year and then, in the next year, perhaps four or five. A scheme whose supply is based on average demand over the years in many subjects must be able to deal with such variations.

9.3. *Research Grants from Industry*

Because they were unable to recruit enough trained statisticians from the universities the Imperial Chemical Industries have recently initiated a scheme under which they intend to choose freshly graduated mathematicians, give them a year or two of industrial experience and then send them back to a university to obtain a Diploma in statistics. This is undoubtedly one method of increasing the number of qualified mathematical statisticians in industry and it is to be hoped that other firms—and possibly Government Departments—will develop similar schemes.

It is likely, however, that smaller firms having appointed a mathematician, given him industrial experience and put him on to statistical work would not readily spare him for a further university course. Thus the need for uncommitted postgraduate training must continue. The fact that many of the advertisements for statisticians refer to the desirability of the candidate having research experience shows that for certain posts this is regarded in industry as adding to the graduate's ability to handle new problems and to apply in practice newly developed techniques.

In the physical and chemical sciences and in engineering there already exist a considerable number of research studentships financed by industry, where the recipient is in no way tied to take a post later with a particular firm. We should very much welcome the foundation of a few similar research studentships specially assigned to mathematical statistics, which could be held at any university in Great Britain where active research in statistics is in progress.

10. Other Methods of Solution

10.1. *Intensive Post-university Courses*

The proposals for an approach to the schools which we have put forward can hardly provide an immediate solution to the problem of supply. Their aim is to make it easier, through spread of information, to draw away from other channels of employment some of the present limited supply of mathematical talent. The ending of national service may speed the process and, if the ground is prepared now, the statistical profession may

† The Research Studentships of the Agricultural Research Council can also be granted for postgraduate work in Statistics.

secure a larger share of the increased number of mathematically qualified school leavers who it is hoped will be looking for university places in the early 1960's.

If industry and government wish for an immediate improvement it seems necessary for them to select persons trained in other disciplines, either just leaving the university or already in their service, and send them to special intensive courses in statistics. The Committee were of opinion that the universities would respond to a request to provide such courses, but that the initiative in asking for assistance and in finding the persons to be taught must come from the would-be employers.

10.2. *Statistics as an Ancillary Subject in Science and Engineering Degree Courses*

The present investigation has been mainly concerned with specialist graduates in statistics or in mathematics with statistics. We are aware that there is also a considerable industrial demand for graduates in physics, chemistry or engineering who additionally have a good working knowledge of statistical method and experimental design. The importance of introducing some knowledge of statistics as an ancillary study in the curriculum of students specializing in other scientific fields, both at universities and technical colleges, was given considerable emphasis in the Council's 1947 Report on "The Teaching of Statistics in Universities and University Colleges" (*J.R.S.S.*, Vol. 110, Part 1, 1947).

In the eleven years which have intervened since this Report was published we know that some progress has been made in this direction, but the obstacles to be overcome are considerable. There is firstly the difficulty that, with the advance in knowledge, the main subject time tables become over full, and unless a Professor of Physics, Chemistry or Engineering is himself convinced that it is important for his students to know some statistics, the difficulty in finding time for a subsidiary statistics course may be insuperable. Further, unless the statistics course can be seen to be dealing with techniques which make an essential contribution to the main subject, the students in physics, chemistry or engineering, with an already crowded programme, will tend to regard this as an unnecessary extra which may be neglected as giving little credit in his examinations.

The problem in front of the teachers of statistics would be easier if those large industrial firms who want a supply of physical scientists and technologists having some knowledge of statistics, would make their wishes known to the university departments from which they draw their recruits: namely that they would like the curriculum to include a little statistics at the expense of some physics or chemistry or engineering. The alternative solution seems to be for industry to send certain of its scientists and technologists to appropriate short courses in statistics after appointment.

10.3. *The Filling of Junior Posts*

We are aware of the need of government and industry to recruit non-graduates with some statistical qualifications to junior posts. The existence of many openings of this kind has been referred to in our memorandum on careers in statistics, but we have not made any study or recommendations in the matter. We consider that it is properly a subject requiring joint discussion with the Association of Incorporated Statisticians.

11. Statistics and the Actuarial Profession

There are many close links between the Royal Statistical Society and the Institute of Actuaries. A number of Actuaries have filled senior statistical posts both in Government

Departments and in industry and commerce. Under the revised examination syllabus a Fellow of the Institute will have reached that standing after taking mathematical statistics and probability theory in at least three and possibly five of the nineteen papers set at different levels of his examinations. There is also some integration with university examinations in that certain classes of graduates may obtain exemption from some of the Institute's examinations. Again, both the statistical and actuarial professions seek to draw into their ranks school leavers with very similar interests and qualifications.

The Committee, however, took as the basis of its enquiry the filling of statistical posts of a character typified by the 150 vacancies discussed in section 3. We found no evidence that there existed a pool of younger actuaries available and anxious to satisfy a demand of this character. While recognizing, therefore, that actuaries form a particular, and well qualified class of statisticians, we did not consider it would be appropriate for us to consider the question of supply and demand within this class.

12. Summary of Information Collected

(*a*) In approaching its task the Committee decided that it was essential to collect and put on record some more precise data than had hitherto been available (*a*) on the extent and nature of the current demand for statisticians; (*b*) on the recent university output.

(*b*) Section 3 of this Report describes an enquiry made regarding statistical posts, mainly for university graduates, advertised during the first six months of 1956. These vacancies are analysed in Tables 1–4. It was found that 43 out of 135 posts (32 per cent.) had not been filled six months or more after advertisement and that, in addition, some 18 per cent. or more of the successful applicants had scarcely adequate qualifications.

(*c*) It was not possible to make a complete survey of the university output of statisticians, largely because statistics is generally taught as part of a mathematics or economics degree course. The thoroughness of the statistical training varies from one university to another and records were generally not available showing whether the mathematician/statistician or economist/statistician did in fact take a post where his statistical knowledge was used.

(*d*) We obtained, however, a certain amount of information from most of the main university teaching schools in statistics for the years 1950–56 and this is set out, showing the types of first posts taken up, in Table 5 and is discussed in section 4.

(*e*) The information most readily come by concerns the output of mathematical statisticians and the possible means of increasing this. The position with regard to the supply of economic statisticians is discussed briefly in section 5.

(*f*) Because the techniques of present-day statistical methods are becoming increasingly mathematical, the source of supply upon which the university teachers of statistics must draw consists of the output from the schools of mathematically minded boys and girls. The Committee therefore considered it relevant to collect information from various published sources concerning the present supply of school and university mathematicians, as well as official estimates of the annual national demand for qualified mathematicians. These matters are discussed in section 6.

(*g*) The salient facts appear to be as follows. Between 600 and 700 mathematically qualified school leavers are now being accepted annually for a 1st year honours mathematics course. A considerable number of these students change after one year either to the natural or mechanical sciences or are transferred to a "general" degree course. Only about 400 students graduate annually in Great Britain as specialist mathematicians.

(*h*) In 1956 a joint committee of the Ministry of Labour and the Advisory Council on Scientific Policy estimated that during the three years 1956–59 the total number of qualified mathematicians employed in Great Britain needed to be increased by about 2,400 of whom some two-thirds were required for school and university teaching. If we allow an addition of 400 a year to make good wastage, it appears that a supply of some 1,200 a year for three years was needed to satisfy the anticipated demand. This figure is far in excess of the current university output of specialist mathematicians and there would seem no chance of satisfying the demand within the period specified.

(*i*) It seems clear therefore that to increase the supply of mathematical statisticians it will be necessary to attract mathematically qualified undergraduates or graduates from a limited pool in face of strong competition.

(*j*) To illustrate the activities into which mathematicians are being drawn we have set out in Tables 7 and 8 of section 6.5 the nature of the employment taken up by Cambridge mathematicians leaving the university in the years 1955 and 1956.

(*k*) In section 7 are summarized views obtained from various university teaching schools regarding the supply of mathematical recruits to university statistical courses. The broad conclusion is that as far as the more advanced courses are concerned, the existing or potentially existing teaching facilities are not being used to the full because suitable students are not coming forward in adequate numbers.

13. Summary of Conclusions and Recommendations

(*a*) In our opinion the most hopeful long-term solution lies in an approach to the schools; as a first step towards this we recommend the publication and circulation of the memorandum on *The Career of Statistician* which we have prepared (section 8.2, p. 61).

(*b*) We think that consideration might also be given to the appointment of a small committee representing the points of view of the Royal Statistical Society, the Association of Incorporated Statisticians, the universities and the schools whose task would be to explore steps which could be taken to increase the contact between the schools and the statistical profession (section 8.3, p. 62).

(*c*) The continued supply of university teachers in statistics depends on it being possible for the universities to retain some of their best students for a two or three years postgraduate research training. From the wording of advertisements, it is also clear that industry has need for a certain number of statisticians with research experience. It follows that it is of considerable importance that a sufficient number of postgraduate grants should be available for students in statistics.

(*d*) At present the Advanced Course and Research Studentship schemes of the Department of Industrial and Scientific Research provide the main source for such awards. A new form of these schemes was introduced in 1957 and it is still too early to judge how far they will meet the university needs in regard to postgraduate training and research in statistics (section 9.2, p. 62).

(*e*) The scheme instituted by a large industrial firm of choosing freshly graduated mathematicians, giving them a year or two of industrial experience and then sending them back to a university for a "diploma" course in statistics is to be welcomed. It is to be hoped that other large firms and possibly some Government Departments will develop similar schemes. Apart from such "tied" arrangements, it would be of great value if industry were to found a few research studentships in statistics, as it has done in other

scientific fields which could be held without commitment as to future occupation at any university in great Britain where active research in statistics is in progress (section 9.3, p. 63).

(*f*) If industry and government wish for an immediate improvement in the supply position, it is considered that they must take the initiative, asking the universities to help in arranging special intensive courses in statistics and themselves finding the persons to be taught (section 10.1, p. 64).

(*g*) It is realized that there exists in industry some demand for graduates in physics, chemistry and engineering who additionally have a good working knowledge in statistics. The difficulty is to find any place for ancillary statistics teaching in an already overcrowded main-subject curriculum. Progress might be eased if the large industrial firms who recognize the value of this additional knowledge, would make their views known to the university departments of science and technology from which they draw their recruits. The alternative is to arrange for special courses for graduates already in employment (section 10.2, p. 64).

(*h*) This Report has been primarily concerned with the supply of university graduates although the memorandum for schools indicates clearly that there are many junior statistical posts to be filled. We consider that any action taken with regard to increasing the supply of statisticians for junior posts should be taken in co-operation with the Association of Incorporated Statisticians (section 10.3, p. 64).

Appendix

CONFIDENTIAL

Royal Statistical Society Enquiry regarding statistical and allied posts advertised as vacant, January–June 1956.

Post ....

Source of Committee's information..

(1) Was it a new post or a replacement ?

(2) Has post been filled ?

(3) If filled, sex of successful candidate

(4) If filled what were the qualifications of the successful candidate?:

(*a*) University and degree or degrees: Subject of degree

(*b*) Previous post and other experience

(5) Opening salary (if no objection to giving this)

(6) Any further comments, particularly on the adequacy of the condidates applying for the post, in numbers and in qualifications and experience.

Discussion on the Report

Miss Keen: I would make a plea not only for the second-class mathematician but for arithmeticians and those who for one reason or another are not likely to get university degrees. My own mathematics stopped short at the Intermediate stage; and on the question of qualifications for non-graduates, I have a special interest as Chairman of the Education Committee of the Association of Incorporated Statisticians.

On the mathematics question, I think we should be careful not to proceed on the assumption that only the really first-class mathematician has an opportunity in this field. In industry—at least on the technological side in which I am particularly interested—there is scope for statisticians who are not first-class mathematicians; in fact, really good mathematicians may find much of the average statistical work lacking in opportunities for the use of higher mathematics. Even in my own industry (electrical engineering) which should provide plenty of opportunities, our mathematicians complain that they are forgetting even their elementary mathematics through lack of practice.

At the junior end, I think we might stress that boys and girls who are only just mathematically adequate but very good at arithmetic are likely to make good statistical clerks. The type of person who likes figures has quite a career in front of him in this field.

At the other end, I think that if we took out of the Society all those people who did not start on the mathematical side, we might find the financial position of the Society a little more difficult than it is now. At the moment, there is a shortage of statisticians and many people who initially came out of other disciplines have picked up their statistics since graduating. Quite a number of graduates take the Incorporated Statisticians' Registered Statistical Assistant examination, which is a clerical level qualification. On the other hand, during last year the A.I.S. exempted some 40 graduates from the Intermediate examination of the Association so that they might proceed straight to the Final and, of these, 17 at least did not have a B.Sc. degree. It is difficult to tell what the 17 were—they may have been mathematicians from a university which prefers to think that mathematics is an art—but these figures prove that there are some people who, even after graduating in another discipline, still feel the need for a statistical qualification.

In the following table I have tried to collect together from Table 5 of the Report, those people, ranging from economic degree holders to the Cambridge diploma holder, who might be said to have had at least some statistical training.

Year	*Total No. of Graduates*	*Final examination of A.I.S.*
1951	50	7
1952	53	7
1953	57	9
1954	53	14
1955	69	20
1956	54	16
1957	—	17
1958	—	21

Column 3 of the table shows the corresponding number of people passing the Final examination of the Association of Incorporated Statisticians.

It appears from these figures that quite a substantial and probably increasing proportion (10 to 20 per cent.) of those who may be said to be statistically qualified are not coming directly from the university with a statistics degree but from quite a number of other fields of interest, some already graduates in another field. I think this reinforces Professor Pearson's plea for more postgraduate grants for training those people who, if they had known about statistics a little earlier, might have chosen it as their career.

The A.I.S. examination probably also caters for those to whom mathematics is not a prime interest; by the time these candidates come to the Final examination they will have been examined in mathematics, English, applied statistics, statistical theory, scientific method, organization of a department, and an applied paper in any field of their choice which is acceptable. Fields chosen so far are economic statistics, market research, industrial

research and control techniques, agriculture, medicine, and this year one candidate is taking biometry; the odd mathematician who slips through the university net can, if he likes, take mathematical statistics as his special subject.

At present then, a substantial minority of those qualifying with an adequate statistical background do so through the A.I.S. Whether the need for examining such people will gradually disappear I do not know, but I suspect that there will always be a number of people who cannot graduate via the university or who have graduated in another discipline and want a statistical qualification as well.

Dr. F. R. HIMSWORTH (I.C.I): First of all I should explain that I am not putting forward any official Company views. I am giving my own views, which I know to be shared by my colleagues but they are not in any sense official.

The Report covers the ground so adequately that there are no new matters to raise.

In Section 7 there is a suggestion that people think of statistics as a not very interesting branch of mathematics, which is quite wrong. It can be one of the most interesting branches of applied mathematics, and one duty of the Society should be to remove this belief that it is a dull and dusty subject; it is anything but that.

It is suggested in Section 7 that University Statistics Departments are having to "protect themselves against acceptance of an undue proportion of second grade material". I hope this will not be misconstrued. I agree with what has been said about openings for other than first-class mathematicians. There is plenty of room for the second class, by which I mean those who have taken a second or third class honours degree. In accepting students in statistics, the course should be regarded as part of a degree course rather than a post-graduate course, and should therefore produce some first, second and third class honours graduates.

There is a general demand for more and more graduates in all branches of science and technology, which cannot be met by producing more people with first-class brains, all of whom now have the opportunity to graduate. Statistics, being still in its first stage of expansion, can expect to divert to itself some first-class mathematicians, but it must also be prepared to produce some of the expansion by training other than first-class material.

The suggestion in Section 7 that it would be possible to develop a mathematically attractive diploma course is a very important one, and is, I think, dismissed too lightly. It ought to be possible for a graduate to pick up the more elementary statistical theory, and also to study some of these more advanced and mathematically more interesting subjects. The industrial statistician is being more and more called upon to handle these techniques. The suggestion is worth serious thought.

Section 8 is extremely important and much can be done in the way of interesting boys and girls while still at school. Undoubtedly it is easier for them to follow a statistical course if they realise the potentialities at the time of leaving school. I should be interested to know if there are any concrete results from the Oxford Conference held in 1957.*

There is another conference to be held in Liverpool. These conferences will not achieve any result unless there is some sort of follow-up.

In Section 9.3 the words "uncommitted postgraduate training" suggest that the I.C.I. arrangement does commit the student. This is not so. He is employed by the Company, and then goes back to a university to take a Diploma. He is free to resign if he wishes, though we naturally hope that he will not do so. I can only assume from what is said about advertisements from industry calling for research experience that this is so, but I think it is quite wrong. It may be a company's custom to expect research experience in, say, chemists or physicists, and therefore to ask for it automatically. But it is not necessary for industrial statisticians.

In Section 10.1 there is a suggestion that industry should send some of its employees for an intensive course of statistics. I very much doubt the wisdom of doing this. I am quite convinced that an industrial statistician should first be a mathematician. Industrial statistics very seldom turns out to be as simple as it looks in the textbooks; industrial

* Oxford Mathematical Conference, Trinity College, Oxford, April 8–18, 1957: London Technology (Times Publishing Co.), 1957.

experiments never quite turn out as they are meant to, and a statistician without a good mathematical background will often find himself out of his depth with only simple statistical techniques.

It is quite true that industry would like its graduates in other branches of science to have a working knowledge of statistics. If the universities put on such a course of statistics they would have the full support of most industrial firms, but it is quite irrelevant to the supply of statisticians; and all it can do is to increase the demand. The more the chemists know of statistics, the more work they expect from the Statistical Department.

What we have done as an experiment, instead of sending people to a university for a course, is to give a course of seven or eight lectures to entire sections of research or works departments to give them a statistical outlook. We have found this to be successful; it does not convert them overnight, but it gets perhaps 10 or 20 per cent. interested.

I entirely agree with Miss Keen that junior posts should not be overlooked; unless you have enough of the junior type of statistician the senior statistician is loaded with a lot of dull and unnecessary work. I would support anything which will increase the supply of junior statisticians or computers.

Turning to the Committee's recommendations. I entirely support the first, but any committee which is appointed should be a small active one, and I see no reason why it should not, throughout the country, form contacts with schools, arrange for people to give lectures to schools, draw up syllabuses for the teaching of statistics. It might suggest ways in which different masters could collaborate, for example, by arranging laboratory experiments for statistical analysis. Statistics can be made interesting by bringing in specific examples referring to subjects which the children know about.

The other recommendations I would also support, with the exception of (*f*). I do not think the sending of graduates in other sciences for an intensive course of statistics is a really sound way of providing statisticians.

Mrs. Barritt (R.A.E. Farnborough): We have had a fair amount of follow-up from various schools—mainly girls' schools—as a result of the Oxford Conference. A proportion are definitely interested in opportunities for girls who are not likely to get honours degrees. In the past the lack of such opportunities for girls who cannot go far beyond A level, but who would like to go in for this type of work has been a real difficulty. Several schools and technical colleges have written, and the latter are prepared to try any reasonable course which would teach statistics together with mathematics.

Miss P. Deane: I should like to say a word or two about the shortage of economic statisticians from the point of view of a university research worker. It looks from the table as if the universities have been relatively easily suited. Whereas one-third of the economic statisticians' posts in government or industry have remained unfilled only one of the university posts was unfilled six months after the vacancy was announced; this may be because universities do not readily advertise posts unless they are optimistic about finding people to fill them. At the Department of Applied Economics in Cambridge the important limiting factor in starting new research projects has usually been staff rather than money. It is not a shortage of the reasonably well-qualified rank and file of economic statisticians; it is the senior worker who can both design and direct an inquiry, and without whom the projects fall to the ground, who is in short supply.

In so far as the shortage of "top men" is a consequence of the fact that adequately qualified university entrants are attracted to such ends as mathematical physics rather than to statistical economics there is not much one can do about it except to make the economics degree course more intellectually interesting. But in so far as it is a problem of the economic statisticians or the potential economic statisticians being attracted away from economics after they graduate and before they have time to specialize because they receive attractive offers from Government and industry, then I think the alternative is to create competing attractions. In this connection it seems to me that the I.C.I. scheme of taking people into the firm for a year or two to give them industrial experience and then sending them back

to the university to specialize may prove very suitable for economic statisticians. I do not know whether it would be as useful for people to go into United Kingdom Government departments for practical experience before specializing; but I imagine that it would be a very good thing for intending colonial statisticians, for instance, to go to their colony to acquire some local experience before specializing. In any case I think that both Government and industry might solve part of their problem by choosing their staff from fresh economics graduates and providing grants for their statistical training.

Mr. Hutton: The supply of mathematicians is something which we know about. The figure of six or seven hundred admitted to the mathematical faculties of universities comes out of the 12,000 boys and girls passing G.C.E. at advanced level each year. If you can talk in terms of the full-time equivalent of a mathematician (that is, a boy who passes G.C.E. at "A" level in mathematics and in one other subject as half a mathematician, and one who passes in mathematics and two other subjects as one-third of a mathematician), admissions to the mathematics faculties at universities form only a small proportion of the 6,500 "A" level mathematicians produced each year. Can we dismiss the other 5,800-5,900 as having inadequate qualifications to go on with mathematical-statistical work ? We must try to make mathematics a little more attractive to the school leavers, and perhaps lower our standards a little.

As the father of four children I think we could do more to arouse interest in statistics, in the same way that the Royal Institution stresses the romance of science to school leavers. The only statistics which the average school leaver thinks of is in relation to smoking and lung cancer. Christmas lectures given by statisticians might begin to arouse their interest in statistics as a possible career.

The number of school leavers with "A" level passes in mathematics knocking at the doors of the universities, or seeking admission to courses of comparable level, is likely to double over the next 10 years. Will the mathematics faculties be able to absorb these students, or will they become even more exclusive and not try to persuade these students to come into their ranks.

Finally, there is the question of quality. The demand for teachers of mathematics predominates amongst the total demand for mathematicians. Only a minority of them will need to be senior wranglers. I can see the importance of having good mathematicians for the sixth forms in grammar and other secondary schools, but sixth form teaching amounts to less than a quarter of the teaching in these schools, and most of the other teachers require considerably less than a first or second-class honours degree as a minimum. Recognition of this fact provides our best hope of bringing more people into mathematics, and thence into statistics.

Dr. Easterfield: Many operational research workers are statisticians but many are not. Professor Pearson has said that during the war biologists did extremely well in operational research, not because they rapidly learned statisticial methods but because they had learned to handle complicated collections of data. (This *Journal*, 1948, p. 218.) A war-time colleague said that many statisticians employed as operational researchers were so interested in the "significance" of their results that they could not look at them to see what they meant.

There are a great many approaches to statistics. I sometimes feel that there is a danger of mixing up research which uses statistics as a tool and research which is developing statistics. When there was mention of research experience I wondered whether the firms required advanced mathematical statistics or whether the statistician should be able to investigate a problem by the use of statistics and not have to do any tool building. These are very different things and the second obviously requires a great deal less mathematical ability.

Most people can apply mathematics up to some level, but this varies enormously. It has been suggested that more should be done in the schools to interest children in mathematics and particularly in statistics. At a very early stage one can introduce some of the

ideas set out simply by Hogben, or by Tippett. This will give them an idea both of the basic science of classifying material into categories, the basis of descriptive statistics, and also of the distribution of probabilities which for many of them will lay the foundation of mathematical statistics. Many will end as scientists or go into other disciplines, but they will be the better for being able to appreciate statistical argument and may be able to do some mathematics themselves.

I was a mathematician myself, and such statistics as I have learned I have picked up by asking other people, by reading papers, and by trying to dig things out of textbooks. I have picked up enough to be useful for the things I do, and if people would do that and would learn enough to be able to go and ask the expert, it would increase the number of people who could use statistics when they needed it and reduce the demand for statisticians by relieving them of many of the more trivial jobs for which higher mathematics are not necessary.

Mr. Gorman: Unwary readers of this very useful report may come away with the impression that economic statisticians are in relatively good supply, at least compared with their mathematical colleagues. I think that this is probably incorrect as applied to economic statisticians as a whole; as applied to econometricians, contrasted with political arithmeticians, it is quite wrong. They are very scarce indeed—as one might expect, since they have decided to read economics rather than science in the university despite being good mathematicians, and must be interested in applied research.

It has always been difficult to recruit them from the schools. The crusade for technical education is likely to make it more difficult and, I hope that there is nothing in the Society's new pamphlet which might make it more difficult still.

This brings me to my second point. Table 5 reads like a list of the main statistical centres in the country. Yet it does not mention Nottingham, North Staffordshire, Belfast or Birmingham, all of which, I believe, offer joint honours courses in Mathematics, Economics and Statistics.

Added in writing. Professor Pearson has asked me to give some details about Birmingham. Between 1949 and 1958 20 econometricians graduated here, of whom six later took university posts either here or abroad. This does not include political arithmeticians or those who read statistics in the Departments of Mathematics, Genetics, Production Engineering or Medical Statistics. This year we expect to turn out about a dozen economic and social statisticians, of whom half will be "econometricians". The latter will have read two fairly advanced mathematical courses, and, in the case of the undergraduates, two courses in "political arithmetic". The others will have read a "methods" course in addition to the course in elementary statistics which is compulsory for everyone.

According to Table 1 a smaller proportion of the posts in economic statistics was filled than of those in mathematical statistics. The definition of an adequately trained mathematical statistician is fairly clear. My impression is that the Committee thought of an economic statistician as being adequately trained if he had had some practical research experience. This may well be true for many of the jobs but, equally well, people with training in statistical theory are frequently employed as economic statisticians in cases where statistical training would really be a great advantage.

There are four special courses in all: two in economic statistics and two in social. May I say here that sociologists with a full statistical training are also very scarce. I have counted them with econometricians in the figures quoted above.

Dr. C. Eisenhart: I believe firmly in statistics as a useful tool in the work of non-statisticians. I feel that there is a real danger that the traditional development and teaching of mathematical statistics may tend to discourage people who would otherwise come forward and become effective users and advisers on statistical methods in other fields.

In the first place, I think that we need to recognize a distinction between a person who has a true appreciation of the meaning of a mathematical result, and a person who is capable of a rigorous mathematical derivation of it. For example, there are many people

who are not capable of a rigorous mathematical derivation of Student's t-distribution, yet they fully comprehend the general theorem that if X is normally distributed with mean 0 and standard deviation σ and s is an estimate of σ, *independent* of X and such that $\nu\, s^2/\sigma^2$ has a χ^2-distribution for ν degrees of freedom, then the ratio X/s has the t-distribution for ν degrees of freedom; and some of these become quite skilled at twisting expressions round until they have this form.

People are encouraged to go on in a particular field, be it music, mathematics, or statistics, by recognized achievement, approbation, and acclaim. A shortcoming of our traditional training procedure in statistics is lack of in-training rewards for those potential users and advocates of statistical tools who are not fortunate enough to be skilled in mathematical derivation. Such individuals need to be given opportunities to whet their appetites with some not-too-difficult real-life practical problems, under appropriate supervision, to gain incentive and to offset the great advantage that the mathematician has. The potential statistical practitioner needs data to sharpen his tools on—data obtained from some one else, whose needs and reputation he must respect. The mathematician, on the other hand, is able to find his own problem, to work on it in isolation, and, if he cannot come up with a tidy solution, then he simply changes the problem. Furthermore, in mathematical contributions it is easier to distinguish right from wrong and good from bad, so that mathematical papers tend to get published faster, and one can build up a reputation more quickly. In consequence, the Committee on (Honorary) Fellows of the American Statistical Association, of which I have been a member for several years, has had to be continually on guard against electing to Fellowship a disproportionate number of mathematical statisticians simply because mathematical statisticians as a group seem to be better able to agree among themselves who really is top-notch, and who is not. Dr. Abraham Flexner is said to have had the same experience when he established the Institute for Advanced Study in Princeton—that is why the Department of Mathematics got off to a start several years before the others.

To get back closer to our subject, what we need to do in our university statistics programme is to attract and cultivate the people who are going to be good statistical tool users, and to see to it that those who succeed in obtaining approximate answers to real problems are given fully as much encouragement as those who obtain exact answers to artificial problems. In the latter connection, for example, much time, effort, and paper has been devoted to the development of criteria for the rejection of outlying observations such that the probability of rejecting a good observation can be controlled exactly when the conditions are precisely as postulated. Comparatively little attention has been given to the performance of such criteria when the measurement-taking is really sour. This latter is a much more difficult problem—to begin with one has to define "sour"; and for each definition there is a different answer to the problem. Worthwhile progress in this direction has been made recently by empirical sampling and mathematical *tours de force*. The answers obtained are less elegant and not as clean-cut as those to the simpler problem, but they are *the* answers that count in practice.

Finally, I should like to suggest a way to get young people interested in statistics. If you are a true believer in the value of statistical tools in experimental work, then it behoves you to take steps to ensure that a few well-chosen examples of the effective use of statistical principles and techniques are incorporated in the lecture materials and laboratory exercises of elementary courses in other applied fields. This takes a bit of time and tact; you may have to sit through the lectures and attend the laboratory sessions; and you will certainly have to get on the "right side" of the instructor; but, if you have your wits about you, you will find some places where unobtrusive use of a few simple statistical tools (e.g., sign test, Wilcoxon's rank-sum, Kendall's τ, distribution-free confidence limits) would add elegance to the lectures, or increase the value of the laboratory work (e.g., through revealing the presence and magnitude of observer differences)—there's your chance!

If this does not bring students to your door, then there is a final way: have the statistics course forbidden!

Mr. Edwards (Department of Scientific and Industrial Research): There are three things I would like to say. First of all, if more suitable candidates had been forthcoming this year, D.S.I.R. could have awarded more studentships.

Next, the flexibility of the D.S.I.R. scheme has been mentioned. Heads of departments who have applied earlier in the year to D.S.I.R. for studentships are notified early in June of their initial quotas; in addition, most universities have "unearmarked" awards which are virtually in their own gift; and there is the "appeals" procedure under which further awards can be obtained; no candidate need be named until the summer. Thus the D.S.I.R. scheme is quite flexible.

Thirdly, I should like to bring to the notice of this meeting the D.S.I.R. schemes designed to encourage the movement of research workers to and from this country. If there is any head of department who is interested then we should be grateful if he would come and see us and we will do what we can to help.

Dr. Finney: Mr. Edwards's remarks encourage me to comment on the question of "wastage" of statisticians. I am doubtful about the extent to which an advertising campaign to schools will reduce the present shortage. I entirely agree that we need to make sure that schools are well informed about the possibility of statistics as a career. It is evident from the Report, however, that even though the demand for statisticians is increasing greatly they will continue to form only a small part of the annual output of mathematicians. In any advertising campaign, we shall be competing with the bigger users of mathematicians; if we seek to persuade school boys and girls to take up statistics in preference to other lines of mathematical employment, then we merely invite the other employers to do likewise and nobody will be any better off.

My own experience in the last few months may be of some interest. I have a quota of only one D.S.I.R. grant per annum, yet I have been unable to fill it! I thought I had one good man lined up for it; he graduated, and then decided that he would prefer to be an actuary. Is anything gained if he is persuaded against being an actuary? I am sure the actuaries would say "No", and I did not try. There were one or two others who graduated at the same time whom I also considered. The best was a first-class mathematician, who was taken on to the staff of the Department of Mathematics without delay. You can imagine the reactions of my colleague the Professor of Mathematics had I tried to persuade this man to take up statistics. There were two others, good second-class, who would have made good competent statisticians, and who, like the first, had included mathematical statistics as a subject for final honours in mathematics. One was determined to be a school teacher and nothing was to be gained by persuading her otherwise; it is surely excellent that some school teachers should have a small knowledge of mathematical statistics. The other young lady wanted to be a policewoman; that is perhaps "wastage", I do not know.

Professor M. G. Kendall: In view of what has been said the meeting might be interested to know how we are tackling the problem at the London School of Economics. We went to industry, collected some money and have set up scholarships, which will come into operation in a year's time, for persons prepared to come to L.S.E. to read for the B.Sc.(Econ.) with special subjects statistics or numerical computation. Each scholarship is worth £450 per annum, and in giving it no attention whatever is paid to any other income of the child or of its parents. I think there are quite a few people in the middle-class income group who are debarred by means tests from sending their children to a university. This is one field of recruitment which we hope to use by making our scholarships means-test free; and I recommend this method to my colleagues in other universities.

We also have availed ourselves of the D.S.I.R. grants, and I am glad to hear that there are plenty going. We are in the same difficulty as Professor Pearson and Dr. Finney, in that we have not been able to fill all vacancies we would like, but we have three this year and hope to have three next year.

There is a mortality among these students. These scholarships are at the most worth £400 and sometimes rather less. The median salary at which a qualified statistician can

start to earn money is £650. Thus, if a student wants to get married he has to get a job outside; and marriage is a quite serious risk for people who are designated for postgraduate work.

There is one other point which the universities may have to consider. We often say that the proper thing to do is to second men from the Civil Service or industry back to the university. This raises the serious difficulty that very often a man cannot be seconded from business for a year or two without serious risk to his career. If we are going to take people back from industry and train them we may have to give them their courses in two or three months or at summer schools, and although I hesitate to suggest anything which would encroach on the universities' long vacation, this may be the answer, to take people back for a short intensive course lasting two or three months.

May I also remark that at the London School of Economics we do turn out mathematical statisticians. It is true that they understand economics as well, but we have not found this a disadvantage.

Dr. D. R. Cox: One way of increasing the supply of mathematical statisticians, not discussed in the Report, is by continued part-time university work. For example, there are at present at Birkbeck College, London, over 20 people working part-time for M.Sc. or Ph.D. degrees in statistics or statistical mathematics. Most of these people combine a full-time post in statistics, usually in industry, with evening academic work. Such a method of study makes great demands on the student, but has a number of advantages, notably in its balancing of the theoretical and applied aspects of the subject. I hope that the possibilities of part-time study will be taken into account in any final recommendations that are made.

Mrs. Barritt (R.A.E. Farnborough): In the latest scheme to be sponsored by the Ministry of Supply in conjunction with the Atomic Energy Research Establishment and Atomic Weapons Research Establishment resources have been pooled and both lecturers and students sent to an interested college of advanced technology for one day a week. About 30 students have started a three-year course and 20 a two-year course. It is expected that many people will wish to express their views on the usefulness or otherwise at the end of this first academic year. Resistance to granting one day's leave has been low, especially in comparison with other suggestions in the nature of a six-months' session.

Mr. H. R. Fisher: There is only one qualification by virtue of which I can presume to offer comment and that is the undeniable qualification of being a parent. With two boys in or nearing the sixth form I looked at section 8.1 where it says: "Means must be found of getting across to headmasters, careers and mathematics masters—and through them to the boys and girls—that through a training in statistics mathematical talent can find a varied, interesting and nationally important outlet". The message would perhaps be more forceful if one could put in "and well paid". My experience is that this consideration will be very much in a boy's mind. I doubt whether anything I as parent could say would remove it. How else may the boy judge? And a master may well think he ought himself to have it in mind as adviser.

Whether this addition can be legitimately made I would not venture to judge from the figures in Tables 3 and 4, because I do not know what to compare them with; but on hearing of the incident just mentioned where the actuarial profession was preferred it occurred to me to wonder. I do not know the facts, but if, in fact, it is possible to make what I would call that interesting addition to section 8.1, then I suggest it should be done.

Professor Pearson subsequently added the following remarks in writing:

I am sure that the very full discussion when printed after the Report, will add much to its value. On behalf of the Committee I should like to thank the speakers for treating our difficult task with much sympathy, for bringing forward useful points which we had

overlooked and for finding time to come to this special meeting which was inserted at rather short notice into the Society's normal programme of meetings.

There is very little, I think, arising from the discussion which it is necessary for me to answer or explain here, but perhaps I should say a word on a point raised by Dr. Himsworth. In the fifth paragraph of section 7 of the Report we were certainly not equating the phrase "the second rate" with students capable of obtaining degrees in the Class II as distinct from Class I. We had in mind, I think, the university applicant who was hardly suited to profit by an honours degree course, but who hoped it might be easier to get through either if he registered in statistics rather than in mathematics, or if he took a statistical option rather than a methematical one in the later stages of his mathematics course.

Speakers have asked why industrial firms have mentioned the desirability of some research experience. I imagine that it may have been because they recognized the value of a postgraduate period in which the student has time to assimilate the considerable range of knowledge which he has acquired in his degree course, to learn how to make a critical study of the literature and to gain some training in tackling the difficulties of a new problem, under guidance of an experienced supervisor.

It is to be hoped that there will be some follow-up of our recommendations. In connection with the schools, it was unfortunately not possible to secure a representative speaker in the discussion as it seemed that the day of our meeting was unluckily chosen. The Memorandum on *The Career of Statistician* is now with the Printer and it is hoped that it will be available for wide circulation among schools during the early summer. Personally, I hope that our recommendation, (*b*), of section 13 can be implemented, but for this to be possible there must be forthcoming individuals who are willing to form a small, active committee, ready to formulate a plan of campaign and see it through. The great majority of the members of the Committee which prepared the Report, have probably too many other commitments to be able to undertake such further work.

I would still think that the initiative in the matter of special university courses (paras. (*f*) and (*g*) on p. 67) must come from industry or government, rather than from the Society, although it can be argued that nothing will happen unless some central body starts the ball rolling.

Size of Company and Other Factors in Dividend Policy

By P. Sargant Florence*

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1. The Importance and the Data of Dividend Policy

Recent investigation into changes in share prices and capital gains over a middle-range period of fifteen years has shown the importance of the dividends paid, in accounting for these changes, rather than merely of the profits earned. The gains or losses of the ordinary shareholder I found (Florence, 1958) to depend on the policy of its directors when declaring dividend quite as much as on the total of the earnings or profits. It is generally held that share prices depend upon the expectation of future profits, but it would be more accurate to read "dividends" for "profits". The two are very far from being one and the same thing or from even having a very direct relationship.

The importance of prospective dividend policy in addition to prospective earnings has been unduly neglected by economists. Keynes (1936, pp. 153–8) "digresses" to a lower "level of abstraction", in discussing the considerations which enter into the market valuation of the yield of an investment. He emphasizes the precariousness of this valuation when based on the convention that the existing state of affairs will continue indefinitely, and lists several important factors which accentuate this precariousness; but the uncertainties of directors' dividend policies do not appear among the factors.

At a meeting of the Royal Statistical Society Florence (1946) analyzed the degree of concentration of voting shares in twenty of the largest English Companies with a view to discovering the seat of control and of policy making. Over three hundred large English industrial and commercial companies have now been similarly analyzed. But before the analysis is published, it will be useful to study in some detail that important type of policy on which growth and investment hangs, namely, the allocation of profit to dividend.

In the course of the last war and its aftermath there was a fairly general drop in the ratio of dividends to profits, not only in Britain where it is often attributed to the official urging of dividend limitation but also in America, and the considerably lower ratios touched have been largely maintained in both countries. The high ploughing back of profits away from dividends about the year 1947 appears not to have been a mere flash in the pan (whether due to Government prompting or not), but to have become the fairly settled policy of at least the larger British Joint Stock Companies and American corporations. Considering industrial and commercial companies in aggregate they seem to have changed policy, from a fifty-five to forty-five relation of ordinary dividends to reserves ploughed back before the war to a reverse relation of about forty to sixty—that is, only about 40 per cent. of total earnings for ordinary dividend is now in fact distributed in dividend.

A deviation more intriguing than that of aggregate dividends from profits appears, however, when comparing the deviation of individual companies at any one time. The

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TABLE 1

Ratios of Dividend to Net Earnings—Individual Companies 1948–51 *Classified by Size and Industry*

Very Large Companies (Capital £3m. and over in 1951)	
Distributive trades	12·8, 22·6, 25·5, 31·7, 34·2, 37·3, 39·0, 42·9, 43·1, 44·7, 56·1, 65·3
Engineering	12·5, 14·0, 17·2, 18·8, 19·2, 19·4, 20·9, 21·4, 24·4, 26·5, 27·4, 30·2, 33·5, 34·2, 35·8, 36·6, 37·9, 40·7, 42·6, 55·0, 62·7
Food	5·7, 9·8, 30·8, 32·2, 36·8, 39·0, 45·9, 48·9, 56·1, 69·8
Motor vehicles	17·3, 18·7, 27·5, 46·9, 48·0
Textiles	11·5, 12·2, 15·2, 16·9, 17·6, 19·8, 23·5, 32·8, 35·7, 39·8, 43·3, 49·0
Other industrial and commercial*	*11·0*, *11·7*, *12·5*, *17·5*, 19·4, *19·7*, *20·6*, *23·9*, 28·9, 29·4, *29·9*, 30·9, 31·0, 32·8, *33·5*, *40·0*, 41·4, *43·2*, *46·4*, *50·9*, 53·5, 60·2, *61·3*, *62·1*, *65·8*, 67·8, 69·2, 80·5, 86·7, 87·1, *87·6*
Breweries	41·8, 42·1, 43·5, 46·7, 52·8, 57·5, 62·0, 62·5, 64·6, 67·9, 72·6, 82·5
Medium Large Companies (Capital £1m. up to £3m. in 1951)	
Distributive trades	9·7, 15·6, 21·5, 25·4, 27·5, 28·8, 32·7, 34·0, 37·4, 37·4, 38·6, 44·3, 58·8, 67·0, 69·3, 93·6, 98·6, 100·0, 138·4
Engineering	0·0, 6·2, 12·2, 12·8, 16·9, 17·6, 17·9, 18·7, 19·5, 20·8, 21·0, 29·6, 30·2, 30·3, 31·5, 32·1, 32·3, 35·9, 37·2, 39·2, 39·6, 39·9, 40·5, 41·6, 49·6, 52·8, 63·7, 101·3
Food	16·1, 16·7, 25·4, 29·8, 35·8, 36·1, 49·5, 56·9, 68·4, 72·9
Motor vehicles	19·1, 24·6, 26·1, 29·5, 32·1, 34·5, 35·0, 41·0, 41·7, 46·7, 46·9, 47·2, 49·6, 70·8
Textiles	11·3, 13·5, 14·9, 16·0, 16·5, 20·6, 25·5, 25·5, 25·5, 31·1, 31·7, 34·4, 36·5, 39·5, 49·8, 56·6, 71·3
Other industrial and commercial*	13·6, 14·9, *15·7*, 17·4, 21·2, *21·6*, 22·0, *22·6*, 23·2, 23·5, 23·6, *24·1*, 24·9, 25·7, *26·0*, 27·7, *28·7*, 29·7, 30·2, 33·9, *35·1*, *38·5*, 40·3, *41·2*, *41·5*, *44·3*, 47·9, 50·3, 58·8, 65·1, 65·2, *68·3*, *68·7*, 68·8, 69·1, 69·8, 75·0, *79·4*, *87·6*
Breweries	35·6, 37·0, 42·5, 43·5, 44·7, 45·2, 49·8, 53·6, 54·3, 80·0, 90·3
Smaller Large Companies (Capital £0·2m. up to £1m. in 1951)	
Distributive trades	21·8, 23·5, 27·3, 30·2, 31·8, 32·6, 33·2, 33·9, 34·9, 43·8, 53·3, 53·7, 54·0, 56·3, 57·2, 61·0, 61·9, 69·2, 69·8, 70·5, 77·9, 79·9, 93·9, 103·1, 112·4, 115·5
Engineering	6·8, 11·9, 15·5, 22·3, 24·4, 25·2, 28·9, 31·0, 31·4, 33·9, 35·0, 35·2, 35·4, 35·6, 39·8, 47·5, 53·8, 64·4, 71·9, 77·9, 78·9, 115·5
Food	26·0, 57·4, 60·0, 62·3, 76·8
Motor vehicles	7·2, 30·1, 38·6, 50·0, 68·2, 69·2, 70·3, 98·6
Textiles	14·9, 19·2, 22·9, 23·6, 24·2, 26·0, 27·0, 28·8, 30·8, 34·1, 34·3, 56·9, 59·7, 85·7, 89·3
Other industrial and commercial†	*10·8*, *16·9*, *19·5*, *19·7*, *32·4*, *34·3*, *46·0*, *48·6*, *61·0*
Breweries	18·5, 33·7, 37·3, 40·1, 42·2, 43·0, 44·2, 46·9, 51·2, 51·9, 64·9, 69·2

* Includes industries with less than twenty-five companies altogether and less than five in any one size class, i.e. Paper, Chemicals, Tobacco, Building Materials, Clothing, Miscellaneous Manufactures and Services. Paper and Chemicals are in italics.

† Paper and Chemicals only.

wide variety of company policies is, in fact, the burden of this article. Since the new relation of dividends to profits seems to have set in about 1947 we may take the years 1948–51 for comparing the policy of allocation to dividend of the different companies, particularly since most companies had by 1948 consolidated their accounts under the Act of that year. There is nothing sacrosanct, however, about the dates 1948–51 and it is to be hoped that subsequent writers will investigate dividend policy in other years.*

We are familiar with the Stock Market's expression "times covered" to denote the ratio of net profits (or, more specifically net earnings for ordinary dividends) to dividend, and in any column where the "times covered" are quoted the range of variation will be found striking, even within a fairly homogeneous industry group. Opening the *Investors' Chronicle* at random I find for the week ending 11th October, 1957, for instance, that among "Aircraft and Motor" companies the times that profits covered dividends varied from company to company from 0·2 to 12·3.

To the layman in matters of company finance "times covered" will perhaps be better understood as its reciprocal, the percentage ratio of net earnings available for ordinary dividend that is actually distributed in ordinary dividend. Table 1 presents a comprehensive tabulation of these "dividend ratios" averaged for the four-year period 1948–51. The ratios were obtained from the "historical record" given in Moody's Services "memoranda", as follows. For each of the four years the percentage on ordinary stock paid in dividends was divided by the percentage earned; the quotients for the four years were then averaged. In the few cases where consolidation was not actually effected until 1949, or where in one year of the four no profit was made and there was nothing currently to divide,† the three remaining years only were averaged. Public companies not finally constituted until 1949 were not included.

The dividend ratios are grouped in Table 1 by different industries for *all* the very large (or "giant") and all the medium large companies for which the required data were available,‡ and for a sample of the smaller large English companies listed in the 1951 edition of the *Stock Exchange Year Book*'s category of "industrial and commercial". Sizes were in the first instance measured by issued capital as of 1951. Very large (or giant) refers to all companies with over £3 million, medium large to all companies with £1 million up to, but short of £3 million, smaller large to a sample of companies with a fifth of £1 million up to but short of £1 million of issued capital. It must be noted that companies with even a fifth of £1 million issued capital are still among the larger half of companies quoted in the Year-Book, hence the seemingly silly title of "smaller large companies". These sample smaller large companies were planned to number about a hundred—a sample of about 1 in 15—and were obtained on a stratified basis allowing proportional representation of various sub-sizes and main industries. The industries represented, however, were confined to the major groups (i.e. distributive trades, engineering, textiles, motors, food, paper and chemicals) and thus within these groups formed a more generous sample.

* This hope would hardly need expression except that past experience shows critics of this type of enquiry to fasten on to the particular dates chosen. Particular dates have to be chosen by any one enquirer, and 1948 to 1951 has not proved peculiar in its dividend policies. This particular choice of dates in no way excludes other enquirers choosing other dates.

† Apart from three companies (one of each size) not earning for at least two of the years and not paying dividends in *any* of the four years, only 15 cases of no "earnings for ordinary" occurred or 1·3 per cent. of the approximately 1,200 possible company-years.

‡ This enquiry into dividend ratios was part of a larger enquiry into financial success 1936–51. Many medium-sized companies had no official price quotations for 1936 and had to be excluded.

For purposes of comparison all very large and medium large English brewery companies were similarly analysed and also a sample of the smaller large breweries. The criteria of size and the method of sampling were the same as for the industrial and commercial companies.

The division by industry is difficult where a company "integrates" (as many of larger size do) over several branches of industry. Following the census practice, however, companies were assigned to the industry they mainly pursue. The standard industrial classification and the company assignment of the National Institute of Economic and Social Research* were used as guides, but not invariably since some consideration had to be given to Stock Exchange groupings.

2. Influence of Industry and Size of Companies upon their Dividend Ratios

Three features stand out from Table 1.

(1) A wide variation in dividend ratios. This wide variation is remarkable when it is borne in mind that for each company the ratios are averages, normally of the same four years—1948, 1949, 1950 and 1951—with the same general business conjuncture; and borne in mind also that companies suffering a loss in more than one year are excluded and that ratios in any case cannot for very long exceed 100 without bankruptcy.

For all the 303 large industrial and commercial companies in our list, excluding breweries, the average proportion of net earnings for ordinary distributed was 40·5 per cent. The total range extended from 0 to 138·4 per cent.

The degree of variation within the larger size-classes and industrial subdivisions as measured by standard deviations given in Table 3 will be discussed later.

(2) The wide variation between individual companies can, however, be reduced to a certain order. The averages for different sizes and for different industries, given in Table 2, suggest that industry and size both have some influence over the dividend ratios. Distributive trades averaged highest, with food and motor vehicles ratios definitely higher than textiles, and engineering somewhere between. And if, for the moment, we turn attention to brewery shares, their dividend ratios are found to have been, on average, as liberal to their shareholders as the distributive trades at 50·4 per cent. as against the textiles' stingy 31·0 per cent. at the other end of the scale. The difference between industries will be more precisely considered in paragraph (4).

(3) In this scale of liberality and stinginess the size of company also appears to have considerable influence. Since the type of industry makes a difference, it is important to isolate the industry factor as far as possible. The five industry groups distinguished in Table 2 are homogeneous enough and are represented by enough companies of each size to make comparison between the averages for each size worth considering. In all these five industry groups, given separately and together in Table 2, the average dividend ratio rises as we pass from very large to medium large, and from medium large to smaller large companies. The unweighted average for these five homogeneous and sufficiently represented industries rises from 32·7 per cent. for the very large to 38·8 per cent. for the medium large to 49·5 per cent. for the smaller large companies. When weighted according to the

* *A Classified List of Large Companies Engaged in British Industry*. Since this list was published a few of the companies in our original listing have been reclassified, entailing some slight changes in averages for specific industries when compared with earlier articles.

TABLE 2

Average Ratios of Dividend to Earnings 1948–51 by Size and Industry of Company

(Number of Companies in brackets)

Industry group / *Size of Company:*		*Very large (Capital £3m. and over)*		*Medium large (Capital £1m. up to £3m.)*		*Smaller large (Capital £0.2m. up to £1m.)*	*All sizes of large company*
(1) Distributive trades	(12)	37·9	(19)	51·5	(26)	57·8	51·6
(2) Engineering	(21)	30·0	(28)	31·8	(22)	41·9	34·4
(3) Textiles	(12)	26·4	(17)	30·5	(15)	38·5	32·2
(4) Food	(10)	37·5	(10)	40·8	(5)	56·5	42·6
(5) Motor vehicles	(5)	31·7	(14)	39·1	(8)	54·0	42·1
Average of homogeneous five industries							
(6) (Unweighted)		32·7		38·8		49·7	40·3*
(7) (Weighted)	(60)	32·3	(88)	38·0	(76)	49·0	40·4
Weighted average for all industrial and commercial companies							
(8) Seven specified industries; including paper and chemical but excluding other miscellaneous	(77)	33·4	(103)	38·6	(85)	47·1	39·8
(9) Including other miscellaneous	(91)	36·2	(127)	38·9		†	40·5
(10) Breweries	(12)	58·0	(11)	52·4	(12)	45·3	51·9

* Average of averages for 3 sizes of company.
† No miscellaneous smaller companies sampled other than in paper and chemicals, average as in line (8).

number of companies in each industry, the average rises in much the same way from 32·3 per cent. to 38·2 per cent. to 49·4 per cent.

No other industrial and commercial groups, except the five given in Table 2, contain a total of as much as 25 companies altogether and of at least five companies in every one of the size-classes. The two industries, paper and chemicals, which are nearest to providing such a "quorum" are moreover far from homogeneous. Paper includes many newspaper publishing companies as well as pulp and paper manufacturers. Chemicals includes heavy chemicals and paint manufacturers as well as pharmaceutical companies, with a retailing business attached. These two industries we shall usually include with building materials, clothing, tobacco, miscellaneous manufactures and services in "other industries". Occasionally, however, for stricter comparison with the smaller companies, which have no "other" miscellaneous industries except these two, paper and chemicals will be formed with the homogeneous five industry group into a seven specified industry "ensemble".

Three industry groups within the industrial and commercial section (distributive trades, engineering and textiles) contain enough companies (not less than twelve in each size group) to justify working out a measure of the variation of dividend ratios and, based on that, of the reliability of the average dividend ratios, and the differences between them for the very large (giant) and the smaller companies. The standard measures are given in Table 3. Taking the same industry groups and comparing the two sizes of companies, a difference of 19·9 appeared in the distributive trades between the average dividend ratio of very large and smaller companies in 1948–51, more than three times the standard error of 6·58. Similar calculations show rather less significance in the differences between the average dividend ratios for the two sizes of company in the textile and engineering industries,

TABLE 3

Measures of Dispersion in Dividend Ratios of Giant and Smaller Large Companies in Several Industries and of the Significance of Differences between their Average Ratios 1948–51

	(*Number of companies*)	*Average (A. mean) ratio*	*Standard deviation*	*Test of unreliability (Standard error)*
Distributive trades				
Very large (giant) companies	(12)	37·9	13·70	3·97
Smaller large companies	(26)	*57·8*	26·64	5·22
Difference in averages		19·9		6·58
Engineering				
Very large (giant) companies	(21)	30·0	12·77	2·78
Smaller large companies	(22)	*41·9*	25·31	5·38
Differences in averages		11·9		6·06
Textiles				
Very large (giant) companies	(12)	26·4	12·51	3·61
Smaller large companies	(15)	*38·5*	22·61	5·82
Differences in averages		12·1		6·85
Breweries				
Very large (giant) companies	(12)	58·0	12·53	3·62

where the differences of 12·1 and 11·9 were respectively 1·77 and 1·97 of the standard error.

(4) Table 3 also shows that taking similar sizes of companies there was in 1948–51 a significant difference in their average dividend ratios between at least some industrial groups, for instance textiles and distributive trades. Among the very large companies, textile companies showed an average dividend ratio of 26·4, distributive trades of 37·9. The standard error of this difference of 11·5 was 5·4 and the difference is thus more than double the error. Among the smaller companies, textile companies showed an average dividend ratio of 38·5, distributive trades of 57·8. The standard error of this difference of 19·3 was 6·8 and the difference was, again, more than double the error. Similar calculations for the other sub-divisions of industrial and commercial companies show less significance in the difference between their average dividend ratios; but if we compare the very large companies in these subdivisions with very large brewery companies, all of them, distributive trades and engineering as well as textile companies, show a significant difference between their average dividend ratios and the particularly high average for very large breweries of 58·0.

Brewery companies behave conversely to the industrial and commercial groups of industries, in that the large companies have on average the higher dividend ratio. So much so that, while among the very large companies breweries have an average dividend ratio 20 points above any other industry in Table 2, among the smaller large companies, breweries have a rather lower average dividend ratio (45·3 per cent.) than that of the seven specified industries (47·1 per cent.). It is for this reason that Table 3 does not include measures for the smaller brewing companies. The difference between them and other smaller companies is clearly not significant.

Brewing, measured by net output or assets per worker and other tests, is a capital intensive industry and it made early use of joint stock finance. The industry is organized

in large companies but, comparing 1930 and 1951, is on the whole a static industry. It is not a developing industry in the sense, (e.g. engineering) of a generally growing industry, or yet in the sense (e.g. the distributive trades) of an industry growing in size of firm, or even in the sense (e.g. the textile industry) of an industry requiring to keep up to date in machinery and equipment to hold its own. The brewery industry is fairly stationary and logically—to be precise, *technologically*—does not require much ploughing back of profit. We shall revert later to this distinction in industrial requirements, particularly in its effect upon the policy of the very large joint stock companies.

3. The Size Differential Further Considered

The criterion for dividing companies into very large (giant), medium large and smaller large was, as already stated, their issued capital in 1951. Other criteria are available and have been used: for instance the market value of capital and debentures and the net tangible assets. The disadvantages of the market value of capital as an index of size is that the value depends so much on the dividend being paid or expected to be paid at any given moment and thus, paradoxically, the more paid out in dividend and the less ploughed back as assets the larger the company appears. Again, compared with the assets criterion it seems more relevant, when dealing as we are with share prices and dividends or shares, to use the number of issued and marketable shares as the criterion of size. Issued capital, besides, is not affected year by year by the ploughing back of earnings to form additional assets in the shape of reserves; its value during any fairly short period is thus more stable and "neutral".

In any case, the different indices do not appear to yield very different size classifications. Hart and Prais (1956, p. 16) found the correlation coefficient between the market valuation and capital (including debentures) of a great number of companies in 1950 to be 0·87.

My own enquiry noted the net tangible assets as well as the nominal capital of each large company. The following differences appeared. If, instead of the threefold division into sizes by nominal capital in 1951, a threefold division by net tangible assets at the beginning of the dividend study (i.e. 1948) had been adopted, bounded by round figures, (very large companies £5m. of assets or more; medium large £2m. up to but not including £5m.; smaller large, less than £2 m.), there would have been no overlap at all between the very large and the smaller large companies as measured by the two indices. Companies in the medium-large *asset* class, as rounded out above, were twenty fewer than in the corresponding *capital* class.* Fourteen (or 11 per cent.) of medium capital go into the "very large" asset class; 19 (or 15 per cent.) into the smaller asset class. Thirteen companies are added—three from the smaller capital companies and ten from the very large capital companies. The transfers between size-classes are largely marginal and, with a few exceptions, the order of size is not very different whether measured by issued capital in 1951 or net tangible assets in 1948.

Issued capital may, then, be accepted as the main measure of size of company, but net tangible assets used as a subsidiary measure to check upon such far-reaching conclusions as that the larger the size of the company, the less liberal the dividend allocation. Eliminating differences due to type of industry, Table 4 gives the (weighted) average dividend ratios for the very large, medium large and smaller large companies in the five homogeneous

* 83 as against 103 in Table 4.

groups of industries, size being measured both by issued capital and by net tangible assets. The average ratios for companies in the seven specified industries, including the less homogeneous paper and chemicals are also given for the three sizes measured by both tests. Companies in other miscellaneous industries are not included in the comparison since they were not included in the small company sample. The three asset-size classes show, similarly to the capital-size classes, lower average dividend ratios for the larger companies.

TABLE 4

Comparison of Average Ratios of Dividends to Earnings 1948–51 *by Size of Company as Measured in* 1951 *Issued Capital*† *and in* 1948 *Net Tangible Assets*

	Very large companies‡		*Medium large companies*‡		*Smaller large companies*‡		*All sizes*	
	Asset measure £5m. and over		*Asset Measure £2m. up to £5m.*		*Asset measure under £2m.*			
	Capital measure £3m. and over† (*from Table* 2)		*Capital measure £1m. up to £3m.*† (*from Table* 2)		*Capital measure under £1m.*† (*from Table* 2)			
Weighted dividend averages of ratios (%)								
Five homogeneous industries*	(63)	32·5	(72)	37·0	(89)	45·8	(224)	40·4
	(60)	*32·3*	*(88)*	*38·0*	*(76)*	*49·0*	*(224)*	*40·4*
Seven specified industries** .	(83)	33·8	(83)	38·8	(99)	44·4	(265)	39·8
	(77)	*33·4*	*(103)*	*38·9*	*(85)*	*47·1*	*(265)*	*39·8*

* Distributive trades, engineering, textiles, food and motor vehicle industry.
** Paper and chemicals added to five industries.
† In italics.
‡ Numbers in brackets.

4. EFFECT OF DIVIDEND POLICY ON CAPITAL GAINS AND THE *Financial Times* INDEX NUMBER

The lower allocation of profits to dividend among the larger industrial and commercial companies is probably one of the reasons why not only their shareholders' gains by dividend over the whole period 1936–51 were found considerably lower than those of the smaller companies but also their shareholders' capital gains. The findings which I have already published (Florence, 1957*a*, pp. 81–111) for large Companies, wherever data were available both for 1936 and 1951, may be summarized as follows:

	Shareholders gain 1936–51		
	81 *Very large companies* (%)	134 *Medium large companies* (%)	89 (*Sample*) *smaller large companies* (%)
Arithmetic Mean			
Dividend gain	+79	+103	+108
Capital gain	+50	+85	+93

The market value of shares, and hence capital gains, we find associated not so much as supposed with the profitability of companies but more than supposed with liberal

allocations to dividend out of profit. The implications of this association are not always realized. It has been noted that if market valuations of capital are used as a measure of the relative growth of larger and smaller companies, smaller companies will appear to grow though they are, by the very fact of their liberal allocation to dividends, adding less to their assets in the form of reserves. A further implication is that, with the wide variance in dividend distribution, already shown, any index of share prices based on the shares of a few, and particularly a few larger companies, will not be representative. The index of industrial ordinary shares that is most frequently quoted is that of the *Financial Times*, formed by averaging the prices of the ordinary shares of thirty companies. According to Parkinson (1951) five of these companies were in 1951 comprised in the iron, coal and steel section, two were Scottish, one a brewery and one in shipping—totalling nine companies outside our immediate field of enquiry into English industrial and commercial companies. The price of two further "index" companies were not available as far back as 1936, but we are still left with nineteen out of the thirty index companies whose shareholders' capital gain for 1936–51 can be compared with the average of all our companies.

The *Financial Times* index for the thirty ordinary shares showed a rise from 100 to 114 between September, 1936 and September, 1951; the average capital value of all our companies a rise from 100 to 180. This is a large discrepancy which cannot be fully explained by the fact that the *Financial Times* index included some iron and steel, Scottish, brewery and shipping shares, and uses a geometrical mean, while our average is the more usual arithmetic mean; nor by any neglect of bonus shares, of which the index takes account, as we do. It can mainly be explained by the particular selection of shares—at least within the category "industrial and commercial"—for the index.

If the nineteen companies common to our list and the *Financial Times* index are separated from the total of our companies, their average capital gain from September, 1936 to September, 1951 is found to be as from 100 to 111 (instead of the 100 to 180 of *all* our companies)—a rise very near the 100 to 114 rise in the *Financial Times* index. Clearly these nineteen companies are *not* representative of the larger industrial and commercial companies whose shares are traded on the Stock Exchange. Considering the smallness of the sample the *industrial* composition of the index seems fair enough. But its *size* composition is quite unrepresentative. As many as sixteen out of the nineteen companies had issued capital of over £3m. in 1951 and fall into our very large group.* But even within this group the index seems to have hit on the companies paying low dividends between 1936 and 1951. Table 5 gives the relevant details for the nineteen individual companies, selected for the index in 1951. Only three had dividend gains (i.e. the sum of dividend yields in 1936–51) higher than the average of 79 per cent. for all our own very large companies and, probably as a consequence, few (in fact only two, Dunlop and Spiller) showed a capital gain, i.e. a price appreciation, above the average for all the very large companies.

Since companies amalgamate or change in their importance on the Stock Exchange in the course of years, the composition of an index number must itself be changed from time to time. Earlier in the period under review (1936–51) the *Financial Times* index made use of some industrial and commercial companies other than those cited by Parkinson

* One further company, Murex, ranked as very large by the asset criterion of over £5m. Among the eleven companies used in the *Financial Times* index, not among those industrial and commercial companies whose fortunes we surveyed, ten ranked also as very large. Parkinson (1951, Appendix) gives the amount of the issued capital in 1951.

TABLE 5

Characteristics of Nineteen English Industrial and Commercial Companies, the Price of whose Shares are Included in the Financial Times *Index*, 1951, *Compared with Similar Companies as a Whole*

	1.	2.	3.	4.	5.	6.
			Shareholders' gain in 1936–51		*% of voting shares concentrated on largest* 20 *holders*	
	Nominal capital 1951	*Net tangible assets* 1951	*capital appreciation* (%)	*dividend total* (%)	1936	1951
Very large companies (capital £3m.)						
Imperial Chemical	85	257	29	72	10·1	9·3
Imperial Tobacco	50	123	−39	65	19·0	15·8
Courtaulds	32	69	−11	43	21·0	7·1
Woolworths	20	27	30	70	87·5	66·3
Dunlops	17·3	51	91	106	1·7	9·6
General Electric	9·8	37	4	63	14·9	13·3
Patons & Baldwins	7·0	7·7	9	75	36·5	14·4
Turner & Newall	6·7	25	3	54	20·4	21·2
Assoc. Portland Cement	6·5	19	2	52	16·5	9·3
Tate & Lyle	6·4	10·7	26	67	64·9	27·2
Morris Motors	5·7	10·2	23	97	99·9	29·7
Harrods	5·3	10·2	22	55	4·6	6·2
Spillers	4·1	12·5	63	102	21·0	11·9
Electrical & Musical	3·4	7·5	−6	50	25·3	82·1
Pinchin Johnson	3·4	7·4	11	57	11·4	15·8
London Brick	3·0	4·8	−16	59	14·9	32·2
Medium large companies						
Murex	2·4	7·1	4	67	—	—
Leyland Motors	2·0	1·4	2	54	*	21·2
Rolls Royce	1·5	4·7	0	63	*	30·3
Average for nineteen index companies			11	67	29·4†	22·6†
Average for all very large companies*			50	79	46·0‡	34·5‡
Average for all large companies*			80	100	n.a.	n.a.

* See Florence (1957), "Reward for risk-bearing by shareholders in large companies", *J. Indust. Econ.*, 5, 81–111.
† Average for sixteen very large index companies only.
‡ Average for companies of over £3m. capital both in 1936 and 1951.
n.a. = not available.

(1951). To quote from Parkinson's earlier work (1944, pp. 5–12), the index then included among industrial and commercial companies, Austin Motors, Callenders Cables, and International Tea. Callenders Cables was merged in 1945, but the capital gain (i.e. the appreciation in the price) of the shares of Austin Motors and International Tea (both in our very large class) between 1936 and 1951 was even more unrepresentatively low than that of companies in the 1951 index list. The average dividend and capital gain of all our large industrial and commercial companies were 100 per cent. and 80 per cent., and those of the 1951 index companies 67 per cent. and 11 per cent.; but the dividend and the capital gains in the same period of Austin Motors were respectively 51 per cent. and 3 per cent., of International Tea, 43 per cent. and 36 per cent.

The companies chosen as the basis for the *Financial Times* index number are considered to be "market leaders" or "blue chips" with shares freely traded because not closely held. It is relevant, therefore, besides the matter of their size, to look into the concentration of

the companies' ownership of shares compared with large companies generally. The structure of share-holding—how closely shares were held—in all very large companies with issued capital over £3m. at each date, was analysed in 1936 along the lines sketched out in Florence (1947) and also, in 1951, the holdings of the Directors.

It appears that concentration of shares in a few hands (i.e. twenty) diminished between 1936 and 1951 and that in both years the index companies had a below-average concentration. The full report still awaits publication but provisionally Table 5 shows that all the very large companies with £3m. capital or over both in 1936 and 1951 had, on average, in 1936, 46 per cent. of their voting shares in twenty hands and in 1951, 34·5 per cent. The index companies among those very large companies had on average 29·4 per cent. and 22·6 per cent. in the two years—a considerably lower concentration.

As to the holdings of directors in 1951, just over half of all the very large industrial and commercial companies had a relatively shareless Board (all the directors between them holding less than 2 per cent. of the ordinary shares). But this divorce of control from risk ownership went even further in the very large "index" companies. Of the eighteen very large companies (including Austin Motors and International Tea) used as indices during 1948–51, 13 (or 72 per cent.) had Boards of Directors holding less than 2 per cent. of the ordinary shares.

In short, the companies used in the *Financial Times* index were not representative of the industrial and commercial companies in size, voting-share concentration or directors' holdings. A very small sample, such as the index uses, can probably never be representative since, as we have seen, companies vary so greatly in their dividend policies, and consequently in their market prices. I have certainly found (Florence, 1957*a*, p. 111) that gains over a period show wide capital dispersion, and that both capital and dividend gains show a marked (positive) skewness.

5. Variety of Dividend Policy of Individual Companies. Three Plausible Principles

The industry and the size of the company appear to some extent to affect the ratio of dividend to earnings. Judging, however, from the wide variety of dividend ratios of individual companies *within* the same size and industry group shown in Table 1, and measured summarily in Table 3, other factors besides size and industry are at work. The brewing industry was found to stand apart in its dividend policies and we will confine attention for the present to the main group of industrial and commercial companies. Can we perhaps discover certain principles connected with the dividend policy of the individual Boards of Directors that might account for this variety?

The principles underlying the policies of Boards of Directors of individual companies in allocating to dividends a proportion of the net earnings at their disposal, might be of at least three kinds:

(1) A certain definite amount or proportion of assets may be considered to be required for allocation to reserves and re-invested, in view of the technical and market conditions of different industries. Some industries, as already noted, show on average significantly different levels in the distribution of earnings between dividends and reserves, and within an industry individual companies may have to meet particular technical and market

requirements. This policy may be called the "required re-investment" type of allocation. Its allocation to reserve may act as an extension of depreciation.*

(2) A certain specific proportion may be considered "fair shares" as between dividends to shareholders and allocation to reserves, and a policy may be followed aiming essentially at earnings-sharing at a fixed proportion throughout. The principle is not just a theoretical possibility. In America, according to Graham and Dodd (1951, pp. 274–5 and 436) "10 per cent. as the proportion of total earnings to be retained in the business, reflects the average practice of independent operating electric utilities over a period of years". The authors generalize later that "in the case of industrial and railroad companies an average pay-out (in dividends) of about two-thirds of earnings may be considered as normal and appropriate, and that for soundly capitalized public utilities the desirable pay-out ratios might be set as high as 85 per cent".

(3) A stable rate of dividend on the nominal capital (or on assets) may be favoured—constituting a "dividend rate stability" type of policy. This principle, too, is not just a theoretical possibility. Many famous American and British companies have treated their ordinary shares as though they were preference shares or debentures to be paid a fixed income. The American Telephone and Telegraph Company, for instance, paid 9 per cent. for very many successive years, and the Bank of England paid its shareholders 9 per cent. for the ten successive years 1904 to 1913 and 10 per cent. for the seven years from 1914–15 to 1920–21.

In the light of the wide variety of dividend-to-earnings ratios disclosed in Table 1 it is evident that policy (2), subscribing, for a whole industry or size-group, to a common notion of a fair earnings-sharing between dividends and reserves, is not the principle mainly adopted in any crude form. If it were, a closer approximation of company results to a representative average ratio would appear, at least for each row of figures. The nearest approximation is the brewing industry and this may portend some strength in the earnings-sharing principle. In the absence, among other industries, of any approach to uniformity in the dividend-earnings ratios the predominant principles of the three suggested might be either of type (1) or (3). Evidence from various sources appears to point to the stabilizing of the rate of dividend as perhaps the principle mainly actuating directors of large companies, leading them to withhold the full benefit of profits from shareholders.

The dividend ratios displayed in Table 1 are the proportion of dividends to net earnings after prior charges (i.e. debenture interest and preference dividends) and depreciation have been deducted. They do not tell us what these net earnings amount to when compared with the capital employed. They tell us the division of the net profit, but not the rate of profit on the capital. It is often assumed, however, that dividends will be proportionately higher the higher the rate of profit; in fact, as already said, "profit" and "dividends" are often used interchangeably. To test the validity of this assumption the rate of earnings available for ordinary shares (or for short the "equity profit") in 1948–51 was calculated on the average of the net tangible assets of each company for those four years from the record, where available, given by Moody's Services for our total of 303 companies. The

* It is a possibility which must not be overlooked that of the different companies within the same industry some may allow more and some less for depreciation, according to the age of their equipment and to their policy in meeting the accounting problems of inflation, particularly the divergence of replacement from the historical cost of fixed assets. Companies allowing less and perhaps not enough for replacement and maintenance of their capital might compensate themselves by ploughing back more out of their net earnings than the other companies in the same industry.

method adopted when considering directors' year-by-year policy in distributing the profits was, as stated, to find the ratio of ordinary dividend to "earnings for ordinary" for each year and to average these yearly ratios. A different method, first totalling for the four years the dividend and the earnings respectively and then finding the ratio of the two totals to the average assets of the four years, appears more appropriate for measuring and comparing rates of dividend and profits on assets since a company's return on capital can only be judged on a period longer than one year. These "equity" profits on net tangible assets, though four-year averages, were found to vary widely among the companies, but a distinct mode was manifest at 5 to short of 8 per cent., shown by 108 out of the 303 companies. The frequency distribution is given in brackets in column (6), Table 6A.

TABLE 6

Relation of Dividend Ratios and Rates of Equity Profit for Various Sizes and All Sizes of Company 1948–51

Rate of equity profit on assets (%)	*Companies with lower ratios (0–29%) of dividend to earnings*			*Companies with higher ratios (30% +) of dividend to earnings*			*Proportion of higher to lower ratio companies*
	Very large companies	*Medium companies*	*Smaller companies*	*Very large companies*	*Medium companies*	*Smaller companies*	*All sizes of company*
0–4·9 . .	4	4	2	17	20	19	
		10			56		5·6
5–7·9 . .	12	14	5	26	35	17	
		31			78		2·5
8–10·9 . .	10	12	8	7	12	8	
		30			27		0·9
11– . .	11	23	9	4	7	17	
		43			28		0·7
All rates of profit .	37	53	24	54	74	61	
		114			189		1·7

The surprising fact emerges that the companies with the higher *rates* of equity profit to assets were on the whole the companies with the *lower* dividend to equity profit ratios. This trend is clearly brought out in Table 6 where the dividend ratios are separated into lower (below 30 per cent.) and higher (at 30 per cent. and above). At a profit rate of 0 to 4·9 per cent., companies with the higher dividend ratios greatly outnumber those with lower ratios (i.e. as 5·6 to 1); at a profit rate of 5 to 7·9 per cent. they outnumber them as 2·5 to 1. At rates of profit of 8 to 10·9 per cent. companies with higher and lower dividend ratio are as 0·9 to 1; at still higher rates of profit companies with the lower ratios are relatively yet more numerous. It appears that directors had been relatively "stingy" with the profit to shareholders in companies where the rate of equity profit was high. Thus, among companies that were successful in showing a high return on assets, the directors seem inclined to damp down the dividends the asset owners might expect, and to give them a lower ratio of the profits available for ordinary shares.

This conclusion is unexpected enough to justify going into further detail. In Table 6A the companies appearing in brackets, both generally (Col. 6) and subdivided (Cols. 3–5) into the three sizes, are classed into 14 rates of equity profit instead of the four in Table 6. The ratios of dividends to earnings, instead of being distributed into only two divisions, are, in the last column, averaged for each of the 14 classes of rates of equity profit. It is clear that the association shown in Table 6 of a lower dividend ratio with higher equity profit represents a fairly continuous fall in the dividend ratio with higher and higher rates of equity profit. All classes of rates of equity profit 8 per cent. or over have average dividend to earnings ratios of 30 per cent. or less.

TABLE 6A

Relation of Rates of Equity Profit and of Dividends on Net Tangible Assets 1948–51

(Number of Companies in brackets)

Rate of equity profit on net tangible assets		*Average rates of dividend on net tangible assets*				*Average rate left for rein-vestment*	*Dividend earnings ratio (all sizes)*
Range (1)	*Midpoint* (2)	*Very large companies* (3)	*Medium large companies* (4)	*Smaller large companies* (5)	*Weighted average all sizes* (6)	(7)= (2)−(6)	(8)= (6)÷(2)
(%)							(%)
0–0·9	0·5	(0) —	(2) 0·3	(1) 0·9	(3) 0·50	0·00	100
1–1·9	1·5	(0) —	(3) 1·2	(7) 1·2	(10) 1·17	0·33	80
2–2·9	2·5	(6) 1·1	(5) 1·1	(3) 1·8	(14) 1·26	1·24	50
3–3·9	3·5	(6) 1·3	(8) 2·0	(1) 3·5	(15) 1·84	1·66	53
4–4·9	4·5	(9) 1·5	(6) 2·2	(8) 2·9	(23) 2·18	2·32	48
5–5·9	5·5	(16) 2·2	(13) 2·3	(6) 2·5	(35) 2·29	3·21	44
6–6·9	6·5	(12) 2·5	(20) 2·5	(5) 2·4	(37) 2·47	4·03	39
7–7·9	7·5	(9) 2·4	(16) 3·0	(11) 3·2	(36) 2·91	4·59	39
8–8·9	8·5	(1) 1·7	(9) 2·4	(4) 2·9	(14) 2·53	5·97	30
9–9·9	9·5	(9) 3·4	(7) 2·2	(5) 2·8	(21) 2·87	6·63	30
10–11·9	11	(10) 2·8	(16) 2·7	(11) 3·6	(37) 3·00	8·00	28
12–13·9	13	(8) 3·4	(9) 3·6	(8) 4·8	(25) 3·91	9·09	30
14–15·9	15	(3) 3·0	(9) 2·9	(6) 3·1	(18) 2·97	12·03	20
16+	19·0*	(2) 9·1	(4) 3·2	(9) 5·0	(15) 5·06	13·94	27
Total companies:		(91)	(127)	(85)	(303)		

* Actual average for the fifteen companies involved.

In the middle columns of Table 6A a new series of calculations are presented. Dividends are here, like the equity profits, expressed as a rate on assets. It then appears that if directors take into account the book-value of these net tangible assets the rates of dividend on assets which they declared tended to rise from a certain minimum as rates of equity profits rise, but at a regularly diminishing pace.

If we subtract the rates of dividend from the midpoint of the rates of equity profit on assets we obtain the approximate rates left for re-investment on assets. As shown in column 7 they rise not only continuously with rising rates of profit but very steeply from 0 and 0·33 per cent. to 12·03 per cent. and 13·94 per cent.

Comparing first the three sizes of company, the very large are in general more highly geared than the smaller and their rates of *equity* profit (i.e. profits less preference dividend and debenture interest) on assets, as might be expected from the proportionately greater prior charges they paid, tended to be lower than those of the medium and smaller com-

panies.* Their most frequent rate of profit was 5 to 5·9 per cent. (by 16 companies), as against 6 to 6·9 per cent. (by 20 companies) for the medium, and 7 to 7·9 per cent. or 10 to 11·9 per cent. for the smaller large companies.

More noteworthy and supporting the conclusion reached earlier is the fact that in ten out of the fourteen classes of equity profit the average rates of dividend are clearly highest for the smaller companies. Apart from this greater generosity to shareholders of the smaller companies, the trends are similar for all sizes of company wherever a sufficient number of each size are averaged, and all the sub-sizes of a large company will be discussed together as a whole. Thirteen ranges of equity profit include ten or more companies each.

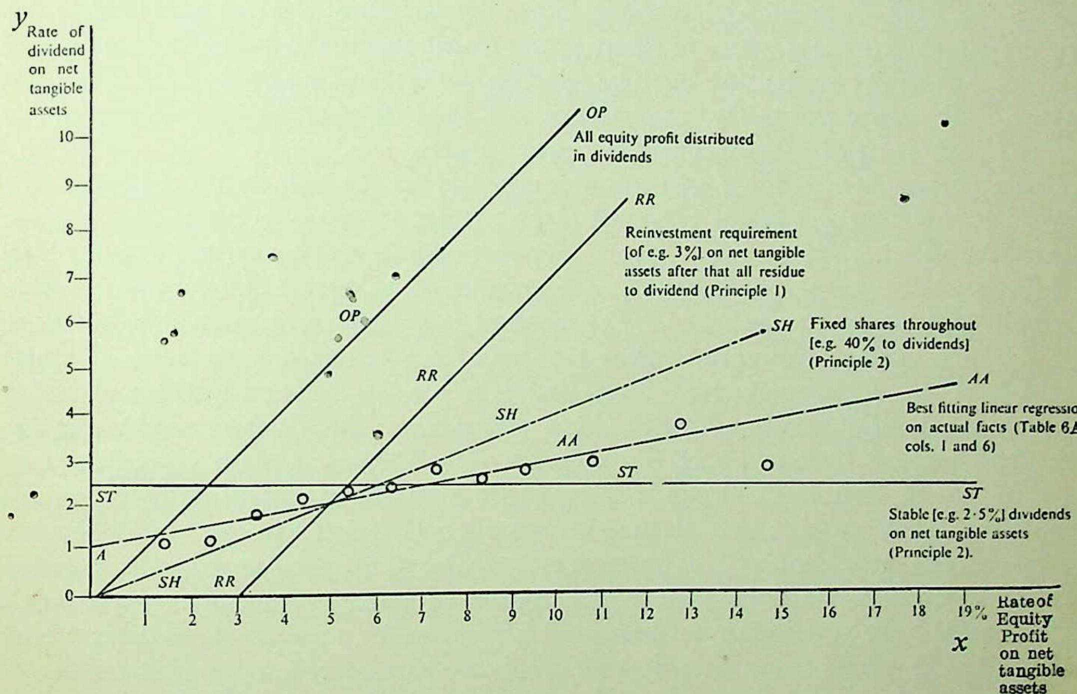

FIG. 1.—Illustrating three plausible principles in dividend policy and the actual regression line 1948–51.

In Fig. 1 the average rates of dividend along the (y) axis for the different ranges of equity profit along the (x) axis (as shown in Columns 6 and 1 of Table 6A) are marked at points 0. The points appear to lie fairly approximately along a straight line which cuts the x axis slightly above 1 per cent. The best fit for the series of points is the line marked $A\ AA$ where $y = 1 \cdot 098 + 0 \cdot 188\, x$, x being the midpoint of the ranges of equity profit.

Four lines in Fig. 1 illustrate the relation between the rate of equity profits and of dividends on assets of the various companies (*a*) if any one of the three "plausible" principles of allocation were adopted alone and (*b*) this best fit regression line for the facts during 1948–51. The first plausible principle, that of required re-investment, is illustrated

* The importance of the prior charges facing any company can be roughly measured by the ratio of the market value of their debenture and preference stocks to that of their ordinary shares. On analysis of the individual companies it appeared that their particular gearing ratio (as measured in 1951) made little difference to the dividend to earnings ratio of the medium and the smaller large companies, but among the very large companies a slight tendency appeared for those with the lower dividend ratios to be those with the higher gearing ratios. Some of the ploughing back of net earnings may here be due to the provision of reserves for the continuing liability to pay fixed charges.

by line *RR–RR* supposing that profits are re-invested up to 3 per cent. of assets and then that all the residue goes to the shareholders. The second plausible principle, that of fixed shares, is illustrated by the line *SH–SH* supposing that the sharing is about the average dividend ratio found in Table 1 for large industrial and commercial companies (of all three sub-sizes), namely, 40 per cent. or 2 shares to dividend out of 5. The third plausible principle, that of a stable rate (in this case on assets), is illustrated by the line *ST–ST* supposing that the stable rate is the (rounded) average dividend on assets for all the large companies of 2·5 per cent.

The dividend policies of the large companies indicated by the facts of 1948–51 appear to form a compromise between the application of the fixed shares and the stability principles. The fixed share principle appears in the regularly continued slope of the *A AA* line, with an 18·8 per cent. share of equity profits distributed to the ordinary shareholders at all rates of equity profit; the stability principle appears in the very low (18·8 to 100) gradient of this slope and in the flat minimum rate of assets of about 1·1 per cent. apparently assured, short of only extremely low equity profits.

The continuance of the low gradient, even when extremely high equity profits accrue, brings into question the extent to which large companies carry out investment policies planned far ahead. The picture of, say, a four-year plan in which a certain amount is to be re-invested does not seem compatible with amounts put to reserve that vary widely with the profit that happens to be made. The width of this variation is given graphically in Fig. 1 by the gaping divergence between the line of profits *OP* and the trend of actual dividends *A AA* as the equity profit increases. It is possible that some of this widening reserve went into liquid assets, though unless soon invested little excuse would exist for thus depriving the shareholder of his dividend. But whether invested immediately or soon, instead of distributed, it would appear that a certain opportunism, or let us say empiricism, ruled rather than a planned investment policy. At any rate, empiricism in contrast to long term planning of investment and sticking to the plans must not be excluded from consideration. Ploughing back and sooner or later, presumably, reinvestment often appears to be undertaken not because it is required for a specific development fixed and planned in advance, but because high profits are experienced and a high residue of funds happens to be available which, in the absence of shareholders' opposition to a stable dividend policy, is at the disposal of the directors.

6. FURTHER EVIDENCE OF THE STABILITY PRINCIPLE

The divergences toward stability from the principle of a fixed proportionate share for dividend in the equity profits, are significant enough to warrant further investigation. Dividends are publicly declared as rates on nominal shares. Assets do not change greatly from year to year—they are a gradually accumulating or decumulating fund. Stability on assets is thus likely to indicate a stable rate on nominal capital; but it is stability on the nominal ordinary capital that counts and which must now be measured directly.

Annual rates of dividend on nominal capital are normally expected to change, at least in Stock Exchange lore, mainly on three occasions; (*a*) when profits change materially enough; (*b*) when a new issue is shortly to be sold on the market, and rates of dividend may be put up in anticipation, in order to raise the market value of shares; (*c*) when a bonus issue of shares has been allotted free to existing shareholders and, with similar

profits as before the issue, the rate of dividend upon the enlarged capital is expected to fall. What in fact does happen on these occasions and in particular how far do dividend rates remain stable?

Let us follow up the two latter occasions for a change in dividend rates first, since they are the more specific.

New issues were placed on the market during 1948–51 by only thirteen of the very large companies, twenty-two of the medium large and eleven of the smaller large companies. Thus new issues only occurred in the case of about a sixth or seventh of the companies. Were their rates of dividend raised in anticipation? The thirteen very large companies floating these issues made, *in proportion to their numbers*, no more changes in rates of dividend than *all* very large companies. The medium and smaller large companies floating the new issues did, however, show the effect of new market issues in raising dividend rates. Seven of the 33 medium and smaller large companies (or 21 per cent.) floating new issues in 1948–51 made no changes in rates of dividend in 1948–51, but 68 of the 179 remaining medium or smaller companies (i.e. 38 per cent.) made no changes. In short, the current theory or lore that new issues are anticipated by a rise in dividend is not universally or indeed very clearly attested but appears truer of the medium and smaller large companies than of the very large companies.

The proportionate adjustment in rates of dividend on capital that are expected to follow the issue of freely allotted bonus shares are not very clearly attested either. Bonus shares were a more frequent occurrence than new issues, and here we collected evidence from 1946 to 1951. From a third to a half of all the large companies had issued bonus shares for one year or another during this six-year period; the proportion for the *very* large companies was just under half, for the medium large just over two-thirds, and for the smaller large companies just under two-thirds. Only a few companies made more than one issue during the period, so that half or two-thirds of the companies being involved means that in the course of the six years the annual occasions for bonus were seized to the extent of only one-twelfth or one-ninth respectively.

TABLE 7

Percentage Distribution of Bonus Shares Issued by Companies of Three Sizes 1946–51 According to Companies' Subsequent Dividend Policies

	Very large companies (%)		*Medium large companies* (%)		*Smaller large companies* (%)	
Rate of dividend on ordinary capital						
(i) *Raised*	4		11		19	
(ii) *Unchanged*	11	39	11	56	34	70
(iii) *Slightly put down*	24		34		17	
(iv) *Put down substantially*						
Nearly proportionately*	28		16		9	
Exactly proportionately*	26	61	25	44	21	30
More than proportionately*	7		3		0	
All bonus issues in the six years	100		100		100	

* Proportionately to increase in share capital.

The very large companies issuing bonus shares were more orthodox in fulfilling expectations of dividend rate adjustment than the medium, and the medium than the smaller

companies. Bonus issues are analyzed, according to various dividend policies of the companies involved, in Table 7. After 26 per cent. of their bonus issues, the very large companies adjusted rates of dividend on the ordinary (nominal) capital downward *exactly* proportionately and after 7 per cent. more than proportionately to the increase in ordinary (nominal) capital, and nearly proportionately after 28 per cent.; after 11 per cent of their issues, however, they made no adjustment of rates and after 4 per cent. actually raised rates. The medium large companies adjusted rates of dividend exactly proportionately to the increase in total capital after 25 per cent. of their bonus issues (after 3 per cent. more than proportionately), and after 16 per cent. nearly proportionately; but after 11 per cent. of the bonus issues they made no adjustment of dividend rates, and after another 11 per cent. actually raised rates. The smaller large companies adjusted rates of dividend proportionately after 21 per cent. and nearly proportionately after 9 per cent. of their bonus issues; but made no adjustment after as many as 34 per cent. and actually put up the dividend after 19 per cent. Adding the remaining percentages of bonus issues (where there was only some slight adjustment), the rate of dividend was not adjusted downwards proportionately or even nearly proportionately as orthodoxy expected after 39 per cent. of the bonus issues of the very large, 56 per cent. of the medium large and 70 per cent. of the smaller large companies.

Since the issue of bonus shares appears to have reduced rates of dividend fairly extensively (though not as much as expected) it is worth glancing forward to Table 8, where a footnote records the reductions in rates of dividend associated with a rise in profit and a bonus issue combined.

Two of the expected three reasons for a change from one year to another in the rate of dividend—new market issues and bonus shares—concern relatively few companies, and, among those companies, have not as strong an influence as often as is thought. The dividend tends towards stability. The positive evidence for the hypothesis that directors of large companies tend to adopt a principle of stabilizing the dividend rate even in the face of the third reason for a change—a substantial change in profit—may now be taken up. It consists mainly in the simple fact that rates of dividend of individual companies so often remained unchanged from one year to another over a period of generally rising profits such as 1948–51. Adding all manufacturing industries and distributive trades, the fields roughly covered by our industrial and commercial companies, net profits (after depreciation) amounted, according to the Central Statistical Office (1954, pp. 39, 41), to £1,288m. in 1948, £1,291m. in 1949, £1,510m. in 1950, and £1,776m. in 1951.

In February, 1948 the publication of the White Paper *A Statement on Personal Incomes, Costs and Prices* represented a tacit agreement to freeze dividends as well as wages, but by the end of 1949 a thaw was setting in. Dividends on *all* companies' ordinary shares, after a drop from £395m. in 1948 to £391m. in 1949, rose in 1950 to £403m. and in 1951 to £443m.

It is worth examining the facts more closely and tracing the relation, company by company, of year-to-year changes in ordinary dividend rates with the same years' changes in each company's equity profit (i.e. earnings for ordinary dividend). A methodological difficulty is that while the rates of dividends on nominal capital are not usually changed by small degrees, profits will vary by any amount and will never, except by an extraordinary coincidence, be exactly the same from year to year. To compare changes in rates of ordinary dividend with changes in the earnings for ordinary dividend we must, therefore,

look for some common measure of what constitutes "stability". In a complete review of the very large companies during 1948–51 it was found that the more regular rates of dividend with, say, five or more occurrences were 5, 7, 7·5, 8, 9, 10, 12, 12·5, 13·5, 15, 17·5, 18, 20, 22·5, 25, 35, 40 per cent. Dividends paid at these rates accounted for 292 out of a total of 352 annual rates. The average rate of dividend was at this time about 15 per cent. and it will be noticed that these more regular rates are on average spaced at intervals of rather less than 10 per cent. below and above this average rate.* This leads to the supposition that only changes in profit of at least 10 per cent. are really likely to affect the dividend rate and we will therefore take account only of rises and falls of 10 per cent. or more in the earnings for dividends. These relative changes in the earnings entail usually quite large *absolute* sums, plus or minus. Among over 90 per cent. of the companies of each size, changes of 10 per cent. in equity profits entailed year by year differences of at least £75,000 for each very large company; for medium large companies differences of at least £20,000; and for each smaller company differences of at least £5,000.

TABLE 8

Relation of Changes in Equity Profits to Changes in Rates of Dividend
1948–9, 1949–50, 1950–51

By size of company: V(ery) L(arge), M(edium) and S(maller large)

Earnings for ordinary dividends	*Ordinary dividend rate:*	*Put up*		*Same*		*Put down*		*All policies*	
Rise +10% or more:	VL . .	46	163	74	230	10	35*	130	428
	M . .	74		96		18		188	
	S . .	43		60		7		110	
Same within 10%:	VL . .	23	47	52	163	3	22	78	232
	M . .	15		68		13		96	
	S . .	9		43		6		58	
Fall −10% or more:	VL . .	1	7	38	135	15	48	54	190
	M . .	5		57		17		79	
	S . .	1		40		16		57	
Total:	VL . .	70	217	164	528	28	105	262	850
	M . .	94		221		48		363	
	S . .	53		143		29		225	

* Bonus shares were issued on the majority of these occasions: 8 of the 10 V.L., 14 of the 18 M., 6 of the 7 S.—a total of 28 of the 35 cases. Bonus shares were issued in only a small minority of the other types of cases in this table.

If, then, for comparison with changes in dividend rates we take the standard of not more than a 10 per cent. plus or minus change from year to year as constituting substantial stability in "earnings for ordinary", Table 8 shows that among the total of companies involved between the years 1948–49, 1949–50 and 1950–51, 232 occasions occurred when these earnings were thus "stable", 190 when they fell and 427 when they rose more than

* For instance the fifth regular rate of dividend *below* the rate of 15 per cent. is a rate of 9 per cent., the fifth *above* a rate of 25 per cent. Had the intervals been proportionately spaced at rates 1·5 per cent. below and above the rate of 15 per cent., the fifth below and above would have been nearly the same, i.e. 9·9 per cent. and 24·1 per cent.

10 per cent. from year to year. The rates of dividend, however, were exactly the same on 528 occasions, were put down on 105 and up on 217. In short, of the total of 850 occasions,* stability of earnings for ordinary within 10 per cent. occurred 231 times or 26 per cent., but exact stability of dividend, 528 times or 62 per cent.

Table 8 allows us to be more specific than these wide statements since, for each type of occurrence in earnings, it gives the concurrent dividend policy. By far the most frequent case, repeated 230 times in all, is the concurrence of a substantial rise (i.e. 10 per cent.) in earnings with dividends kept the same. It is the most frequent case for each of the three sizes of company though, relatively, slightly more frequent for the very large companies. On the 130 occasions when a very large company's earnings rose substantially, dividends were kept exactly the same 74 (or 57 per cent. of) times. On the 188 and 110 occasions of rising earnings, the medium and smaller large companies respectively kept dividends the same 51 per cent. and 54 per cent. of the times. And these figures of stability of dividend do not allow for the fact that on some occasions (as noted at the foot of Table 8) a rise in earnings is met by a bonus issue with consequent unstable (i.e. a lowered) dividend.

7. Summary and some Possible Inferences

1. The ratio of ordinary dividend to earnings for ordinary dividend varied very widely in 1948–51 for the different large English industrial and commercial companies. The differences were partly due to the industry, partly to the size of the company, the very large companies ploughing back more than the medium large, and the medium large more than the smaller large company. But neither size nor industry accounts completely for the very wide variation in dividend policy.

2. Owing partly to this difference in dividend policy the shares of the very large companies did not rise in price between 1936 and 1951 as much as the shares of the medium and smaller large companies. An index of share-price based, like the *Financial Times* index, on the prices of a few large companies was not, therefore, representative of prices generally.

3. The principles on which dividends were, in fact, paid out in the years 1948–51 appeared to combine a slight sharing, just less than one-fifth of the equity profit (or earnings for ordinary) to ordinary shareholders, together with a minimum flat payment of about 1·1 per cent. on the net tangible assets. The combination of a very low share of profit with a flat percentage of the relatively stable assets conformed with a policy of stability in rate of dividends on shares. The rates of dividend were found unexpectedly stable even when (*a*) new issues were being floated on the market and dividends might well have been put up, (*b*) bonus shares were granted and dividends per share might have been expected to fall proportionately and (*c*) equity profits varied greatly.

4. The indications are that in large companies shareholders have in recent times not obtained in dividend the possible benefit of high and rising profits. The larger the company the lower the rate of dividend compared with the rate of equity profit on assets and the more the rate of dividend on nominal capital is stabilized.

5. The consequence to be expected of the withholding of dividends and of large allocations to reserve is an increase in assets. The very large (giant) companies (where, according

* Since accounts before consolidation were not used some companies' policies could only be compared twice between successive years. Hence the total of "occasions" is not three times the total of companies.

to Table 2, the 1948–51 dividend ratios were on average, lowest) gave the investor the largest accounting gains (even when this "accounting gain" included the dividends paid out with the increase in assets per share), and the smaller companies (where the 1948–51 dividend ratios were on average highest) gave the lowest accountings gains. Florence (1957*b*, pp. 245–7) illustrates this disconcerting contradiction in the trend of net tangible assets and of investors' capital and dividend gains for companies of different size during 1936–51. The larger the company the lower was the investment gain, the higher the accounting gain.

6. Since we took four-year averages this plough-back cannot be considered just a year-to-year equalization to cover losses in single unprofitable years. (There were very few losses.) Shareholders seem content with comparatively fixed dividends, allowing directors to plough back most of the residual of profits fluctuating from year to year, even when profits were high and increasing over the years. The shareholders' contentment is partly due no doubt to hopes, whether justified or not, of untaxed capital gain on selling out, or of eventual bonus shares without a proportionate reduction of dividend. Shareholders remain largely passive till perhaps some financier more enterprising or ambitious then the company's directors makes a take-over bid. Failing any take-over bids the existing management may continue to put most of the big profits back to company assets as reserves. Directors may or may not consciously admit "loyalty" to the "company"; and continuance of this policy certainly serves the interests of the "company", in so far as company can be distinguished from shareholder interests. Whether this high plough-back serves the interest of the country must depend on what is done by the company, or would have been done by the shareholder, with the critical portion of the profits in dispute.

7. In a period of inflation companies might logically be expected to make provision out of profits for the increased cost of replacing assets. The evidence given in Section 5, however, shows no general trend towards a minimum proportion of assets placed to reserve before any dividends are paid: rather the contrary. First priority on profits seems, up to a point, to go to the shareholders.

8. When profits are above this (rather low) point, the high plough-back of the larger companies, higher than before 1948, may possibly be linked to my conclusion (Florence, 1953, pp. 208–9) that voting shareholdings are on the whole becoming more and more scattered with fewer very large shareholders to form a cohesive body in control of company policy, and that the directors of most companies hold only a small proportion of shares.

It is often thought that the large shareholder is likely to favour a high plough-back and low dividends because he is super-taxed on the dividends but not taxed at all on capital gains. But if, as appears, prices of shares depend so largely on the rate of dividend, the capital gain which the large shareholder wants may not be obtained without a liberal dividend policy. All shareholders, small and large, are thus probably interested in higher dividends, and with lessening shareholders' control shareholders' interests are likely to suffer as against management interests. As Samuelson (1948, p. 131) remarks: . . . "the managers of every organization have an innate tendency to try to make it grow and perpetuate itself. There is reason to question whether profits are not ploughed back into a company when the same could better be invested by the stockholders elsewhere, or be spent upon consumption."

It is worth recalling that in a relatively stationary industry like brewing, the very large companies paid out dividends on average at a significantly higher ratio on earnings than

the average of other industrial and commercial groups. The brewing companies are old-fashioned in their share structure as well as in their dividend policy, and in a high proportion of them the directors hold a considerable stake. To be precise, five-sixths of the Boards of very large brewery companies held over 2 per cent. of shares, but this percentage was held by the Boards of less than half the very large industrial and commercial companies.

Directors' shareholding and concentration of personal holdings generally are associated in this industry with little opportunity for very profitable industrial development. If the industry involved, however, is not, like brewing, rather stationary, the directors and managers of the larger companies with dispersed share holdings and low director holdings might well seize upon every opportunity offered by high profit of re-investing rather than distributing more in dividends, without effective protest from shareholders. This aspect of the so-called managerial revolution would, at any rate, repay further study.

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REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*The Advanced Theory of Statistics: Vol. I—Distribution Theory.* By M. G. Kendall and A. Stuart. London, Griffin, 1958. xii, 431 pp. $9\frac{3}{4}''$. 84*s*.

Professor M. G. Kendall's two-volume work on the *Advanced Theory of Statistics* has already been printed in several editions. It is to be found on the bookshelves of the majority of mathematical statisticians of my own generation and has become one of the main reference works for the subject. That Professor Kendall fully intends it to remain so is seen in his decision to produce a new three-volume edition and to expedite this by bringing Mr. Alan Stuart into collaboration. The first of the volumes, sub-titled "Distribution Theory", is now published. It does not differ very greatly in style and content from Volume 1 of the earlier editions. The size of page is slightly smaller, making the book more convenient to handle; the use of a smaller fount for certain notes and examples improves the general appearance of the page; the provision of a larger number of set examples enhances the value to the systematic student; and the inclusion in the text of additional material, embodying results from the statistical literature of the last ten or fifteen years, should prove of assistance to research students. But, by and large, this volume will make the same kind of friends as did the earlier editions. It is in the later volumes, perhaps, that we shall look for a development more markedly new.

"Distribution Theory" is concerned with the theoretical characterization of empirical distributions and with those methods of manipulating distributions which are needed in the discussion of the sampling fluctuations of statistical quantities. It does not seem necessary here to describe the substantial part of the work which is fundamentally unaltered, but brief mention may be made of some of the additional items. There is for instance now an account, following Dr. N. L. Johnson, of those systems of curves obtained by taking the variable x to be related linearly to another variable y and then assuming either (i) $\log y$, (ii) $\log \{y/(1 - y)\}$, or (iii) $\sinh^{-1} y$, to be normally distributed. Again, in the chapter on simple random sampling, there has been added a section on "inverse" sequential sampling. The sections on order statistics have now been expanded into a separate chapter. The treatment of methods of calculating the cumulants of k-statistics

has been clarified largely by changes in the order of presentation. The discussion of the various symmetric functions of observations is now assisted by some short tables in the appendices, expressing the relationships between the augmented symmetric functions, the power sums and the so-called multiple k-statistics. And the chapter on the multivariate normal distribution is expanded, although the systematic discussion of statistical relationship is left over to a later volume.

The total effect of these and of other emendations is, in the opinion of the reviewer, one of improvement, although, as has been stated, they do not produce an alteration in kind. Unless there is some drastic change in the interests of mathematical statisticians, therefore, it is difficult to see this book falling out of favour. There might, of course, be some such change in outlook. One does hear from time to time criticism of the statistical mathematics of the inter-war period on the grounds that it is concerned largely with problems of distribution and because there is a tendency in much of it to make exaggerated claims for the practical importance of exact sampling distributions. Not all, however, of the contributors to the advance in our knowledge of these distributions have made any such large claims. Many writers have indeed preferred to see in the consideration of such problems simply another method of developing our powers of thought and, in particular, a way of producing a "feel" for statistical variation which, indirectly, is important in almost any branch of practical statistical work. There may be other ways of developing this feeling, but my own belief is that there will always be many who find real interest in distribution theory and to whom this first volume of Kendall and Stuart will make the same appeal as Professor Kendall's *Advanced Theory* has been doing over the last ten to fifteen years.

B. L. WELCH.

2.—*Fundamentals of Statistical Reasoning*. By M. H. Quenouille. London, Griffin, 1958. 169 pp. 8½". 24*s*.

The avowed purpose of this book is to set forth the "main body of underlying concepts and ideas which are the basis of any truly statistical course". It is thus neither a book on statistical methods, nor is it one on statistical mathematics, but it aims to discuss those ideas which lie at the root of the modern statistical argument. There are seven chapters: the first two (38 pages) deal with the frequency definition of probability and carry the development of probability ideas far enough to derive, for example, Student's t-distribution. The next two chapters (36 pages) deal with basic ideas in estimation (bias, consistency, sufficiency and efficiency) and hypothesis-testing (power and likelihood). Chapter 5 (19 pages) discusses maximum likelihood; chapter 6 (37 pages), fiducial probability; and chapter 7 (23 pages) the choice of significance tests (power, minimax, etc.). Bearing in mind that the book has been photolithographed from the product of a varityper on to a small page, it will be realized that a lot of material has been condensed in a small space.

The book naturally divides itself into two parts—chapter 6 on fiducial probability, and the rest. The reason for the division is that almost all the development in the single chapter (easily the largest in the book) is new, whilst that in the rest is almost entirely a presentation of ideas with which most statisticians interested in these topics will be familiar. It is convenient to discuss the two divisions separately, and we begin with the larger.

The principal justification for a new presentation of ideas which are already expounded elsewhere must be that the new approach expounds the ideas more clearly than before, so that the reader can understand their meaning, their interrelationship and their use. The author's style is therefore important and he must take care with every sentence, indeed in definitions with every word, to make his meaning clear. The author has clearly not done this. For example events are described as independent (p. 13) when no two of them can occur simultaneously (where most writers would use the term "exclusive"), and also as independent (p. 21) when they are statistically independent in the usual probability sense. No one who has experience of teaching students and seen them wrongly apply the addition law to non-exclusive events or the product law to non-independent events will fail to appreciate the need for emphasizing the importance of these two underlying concepts and their difference. Perhaps the author does not consider these ideas important because in

deriving the F-distribution (p. 33) he fails to mention that the two χ^2 must be independent. Again the first sentence on p. 27 is false without the restriction to independence (though this is mentioned before and after the incorrect statement). The interested reader might like to set himself the amusing task of finding where, in the curious derivation of the t-distribution on pp. 46–7, the fact that the sample mean and variance are independent has been used. (The restriction is stated in the last line of the proof and a proof is suggested a few sentences later.) Despite these faults in style the author rarely makes mistakes, though sometimes one can only tell what he means by assuming he must mean that which makes the answer right (for example on p. 20 P is continuous and has a density, so the probability P_n must be a tail area; this gives the author's answer, so that is what he intended). There seems to be no reason for the publication of these six chapters.

This comment is certainly not valid for the chapter on fiducial probability. Here we have a most stimulating and brilliant development of the fiducial argument. Of course the serious deficiencies of style remain, and since this time we do not know the correct answer, we often cannot tell what is meant. But despite this there is obviously some cogent reasoning here which will influence subsequent work. The author's most important contribution, as I see it, is to emphasize that the fiducial argument in the multiparameter case requires not merely the existence of sufficient statistics, but that the statistics must be related in a special way, one to each parameter. The formal requirement (given in the chapter on estimation, p. 58) is that if $x_1, x_2, \ldots, x_m$ are jointly sufficient for $\theta_1, \theta_2, \ldots, \theta_m$ then

$$p(x_1, \ldots, x_m \mid \theta_1, \ldots, \theta_m) = p(x_1 \mid \theta_1, \ldots, \theta_m)p(x_2 \mid x_1, \theta_2, \ldots, \theta_m) \ldots p(x_m \mid x_{m-1}, \ldots, x_1, \theta_m)$$

where $p(u \mid v)$ means the probability density of u given v. Quenouille says that each of the factors on the right hand side may be manipulated to give fiducial distributions which may, in certain circumstances, be combined. This factorization has been suggested earlier by Bartlett, and I pointed out in a discussion at a recent Research Section meeting (*J. R. Statist. Soc.* B, **20**, 235) that such a factorization was at the heart of conditional tests when approached from Neyman-Pearson ideas. Whilst the factorization is important it is not clear that Quenouille's terminology is satisfactory. He calls x_1 sufficient for θ_1. Now the usual definition of sufficiency means essentially that any statistician with the sufficient statistics can do as well as any statistician with the whole of the data. But this is not true here. $\bar{x}$ is sufficient for μ, the mean of a normal distribution of unknown mean and unknown variance, according to Quenouille's definition but a statistician with $\bar{x}$ alone would need the sample variance to make some inferences (e.g. fiducial ones) about μ. It is a pity that more time was not spent on investigating the meaning of this factorization. With its aid certain paradoxes (e.g. Mauldon's and mine) with fiducial probability are resolved, though in the former no attempt has been made to prove the uniqueness of the fiducial distribution. I am unable to follow the argument here because it is stated that $\hat{c}$ is sufficient for c (for the notation see pp. 100–102 and 114–116) although it does not appear to satisfy the definition. Also there are some curious remarks about maximum likelihood and sufficiency which do not make sense to me. It seems to be suggested that with the factorization the likelihood equations will involve the parameters separately, but this is patently not so. The author presumably means something different from what he has said, or implied.

An incomprehensible, but vital, step in the argument is (p. 117) that "all integrations of parameters . . . should be equivalent to integrations of random variables". He then goes on, "For example . . . ". Now examples are useful as illustrations or (as I have used them above in criticizing the style) as counter-examples (to the thesis that the style is good). They are useless in replacing definitions, and a definition of "equivalent to" is needed here, as are other definitions throughout the book. The author has written an important and exciting chapter but he has not presented the ideas clearly nor carefully thought out their consequences. The whole of the book should have been devoted to the one topic and then there might have been enough space to deal with the thesis adequately. As it is, I am still unconvinced by the fiducial argument despite Quenouille's masterly spinning of tangled threads.

D. V. LINDLEY.

3.—*Introduction to Combinatorial Analysis.* By John Riordan. New York, Wiley, 1958. London, Chapman & Hall. xi, 244 pp. 9″. 68*s*.

This is an excellent book, delightfully readable. It is written as clearly and as simply as its contents permit so that one feels that the author has made great efforts to put the results he presents into an accessible and easily digestible form. There is very little of that unnecessary generalization and abstraction which all too often mar books from the other side of the Atlantic. Technical jargon is cut to a minimum, at least in the first six, non-specialist, chapters. Of course Combinatory Analysis, like the classical theory of numbers, which it touches closely at points, is the study of problems whose nature may be described in quite elementary and non-technical terms but whose solution is often very tortuous and tricky and can involve quite heavy mathematics. A great deal of credit is therefore due to Riordan for giving solutions in a form which seldom involves any more elaborate techniques than the algebra of generating functions, with occasional use of elementary symmetric functions. Indeed the level of mathematics necessary to follow the bulk of the arguments of the book is no more than could well be expected of an intelligent sixth former, though naturally a considerably higher degree of dexterity and stamina is required for fluent reading.

Unfortunately for the statistician the book is written for a general mathematical audience and only for the student of probability as one amongst others so that perhaps only half the content of the book has direct statistical relevance and this is often only implicit. It follows that except to the specialist in combinatorial probabilities the book will be largely one of reference, a status in any event, dictated, to some degree by its price.

It is often an advantage to consider the combinatory theory "pure" when examining a problem of combinatory probabilities so that one can dissect the combinatorial and probabilistic aspects and treat them separately but, on the other hand, the statistician likes to think in terms of probability distributions and to use the powerful apparatus of Tchebychev's and other inequalities, the normal and Poisson approximations etc., leading up to a reasonably accurate "percentage point" to use in testing an hypothesis. These applications the book does not give.

The first three chapters, dealing with (i) elementary permutations and combinations, (ii) generating functions, (iii) the "principle of inclusion and exclusion" (i.e. in probabilistic terms, the probabilities of sets of dependent events) cover ground familiar to statisticians who have read, for instance, Levy and Roth's and Feller's books on probability and Aitken's *Statistical Mathematics* but it may be refreshing for them to see it shorn of probabilistic terminology, in a form designed to reveal the underlying algebraic structure. The same is true of the fifth chapter on occupancy and parts of the last two chapters (the seventh and eighth) on "permutations with restricted position" which deal with the theory, in probabilistic terms, of "matching" and "latin rectangles". The fourth chapter, on cycles of permutations, is more purely elementary number theory but is a very useful extension of the understanding of the structure of permutations (quite apart from direct applications, e.g. to rank correlation coefficients). The sixth chapter, on Partitions, Compositions, Trees and Networks, is an exhilarating exposition of the seminal aspects of the parts of McMahon's *Combinatory Analysis* devoted to these topics together with more recent developments in this field.

Two fields of combinatory theory are apparently not treated in the book, namely "comma-free codes" and the variety of statistical problems associated with the connectedness of points on a lattice (and associated random-walk problems). And whilst it may well be that the underlying mathematical theory of these problems is contained under other heads in the book one feels that they merit explicit notice, even if only as a component of the motivation for considering the problems treated. This omission may in part be due to the fact that the book devotes its attention to problems which give reasonably explicit answers and eschews (with a few exceptions) the treatment of approximate results, inequalities or asymptotic expansions. Undoubtedly this concentration on problems which "come out" adds immeasurably to the book's very satisfying elegance.

The topics of the last three chapters are ones to which the author has made many

original contributions and his familiarity has yielded a very complete bibliography so that whilst they may seem to be given a rather undue weighting for an introductory book they give a first-class introduction to rather more specialist fields in which a fair amount of work is in progress at the present time. For the early work in Combinatory Analysis the author relies somewhat on Dickson's monumental *History of the Theory of Numbers* and whilst this is in general a wise thing to do English readers will regret that de Moivre has been robbed of the great deal of credit which is his due (he does not even get mentioned in the index).

Each chapter is followed by an extensive sequence of exercises designed to continue and develop the results of the text (a feature which enables the book to include rather more matter than might appear from its size—and which makes the reviewer's task exhausting, albeit not without its rewards). Cross references are made rather tedious by the omission of the chapter numbers from the printed pages and personal copies will soon become heavily annotated.

D. E. BARTON.

4.—*Multivariate Correlational Analysis.* By Philip H. DuBois. New York, Harper, 1957. xv, 202 pp. 9¼″. $4.50.

Perhaps the main attraction of this book is that it sets out, under a single cover, not only the formulae of multiple and partial correlations etc. together with their derivations, but also computational routines for their calculation. The general approach, which makes for computational simplicity, is to work with covariances rather than correlations.

Apart from the more standard correlational procedures, the book also deals with factor analysis. Only one kind of factor analysis is, however, treated, namely an analysis in terms of what are called "defined components". Here each factor is defined, as it were, by several of the test variables rather than by some quasi-mathematical assumptions. This could be regarded as a method which is not necessarily completely unattractive.

There are occasional slips in the book, for example, when the author avers that the correlation between the ratio of two variables and the denominator is zero, and when he implicitly accepts, as is admittedly customary, that tests of significance in factor analysis do not have to take into account the mode of derivation of the quantities which are being tested. However, the author is one of the few writers who correctly calls the customary "residuals" in factor analysis covariances and not, as most people do, correlations.

The author, a Mid-Western Professor of Psychology, does not say at whom the book is aimed. My impression is that it will not give much insight to the relative newcomer to the subject, but that it may be of interest to that curious race of beasties who spend their lives in the correlational jungle.

A. S. C. EHRENBERG.

5.—*Applied Statistics for Economists: A Course in Statistical Methods.* By P. H. Karmel. Melbourne [and London], Pitman, 1958. xi, 451 pp. 8½″. 65*s*.

As stated in the preface, this text-book may conveniently be divided into two sections. The earlier chapters provide an introduction to statistical theory. This is tailored to the needs of economics students in the topics selected for consideration and at the level of mathematical knowledge assumed. The first five chapters deal with collection of data, tables, graphs and diagrams, frequency distributions and their description (central value, dispersion and skewness). Sampling and significance are considered for the mean and a proportion but not for the variance. The normal distribution is described but not the binomial or Poisson. The χ^2 test is given for goodness of fit and independence in a contingency table. Linear regression and correlation are dealt with very thoroughly, including tests of significance and standard errors of estimates, but the correlation ratio and rank correlation are not mentioned. Multiple regression is introduced and illustrated using three variables. Special topics considered are sample surveys and time series. Less usual is a useful short introduction to quality control (but there is still no mention of the Poisson distribution).

The mathematical knowledge assumed is generally no more than school algebra but "mathematical formulations are used freely" and the student whose mathematics is weak will find the book hard going. It seems a pity that the mathematics used has not been

restricted throughout to algebra, and the proofs involving calculus relegated to the footnotes. Although the author suggests that a student may omit such proofs, their appearance without warning in the main text is likely to put him off. For instance, the use of the summation notation is explained but the integral sign is introduced without comment. In particular, the use of Lagrange multipliers (p. 137) is surely out of place in a text-book of this standard, and it would probably have been better not to attempt a proof of optimum sample allocation.

In general the exposition is straightforward and clear. In one or two places, however, it seems to me that confusion may arise. Histograms are defined but no mention is made of the vertical scale relating to a constant interval on the horizontal scale—a point which usually causes students some difficulty. The variance of a sample is defined as the sum of squared deviations from the mean divided by $N - 1$, whereas that of the population is defined with N as the divisor. The treatment within the book is consistent, but it is doubtful whether much is to be gained from the refinement at this stage. I found the notation for the mean of an array, $\mu_Y(X) = \alpha + \beta X$, rather confusing and the author himself writes it wrongly in the equation on page 183. In spite of the promise of a logical development of time series analysis, the measurement of seasonal variation is not given for the additive model and a mixed method of addition and multiplication is used for the multiplicative model.

The second part of the book deals with some topics of applied statistics in economics: social accounts and national income, index numbers of prices and product and demography. It is here that the use of Australian data for illustration becomes a disadvantage for those who have no special interest in them. This is especially true in the chapter on social accounting which follows Australian practice in preference to international (and British) procedure. Apart from this, these chapters provide a useful introduction to these subjects, dealing with many of the theoretical problems involved. Since the book is expressly for economists, the author is able to link his discussion of price index numbers to indifference curve analysis. (Another nice link with economic theory is given earlier in likening optimum sample allocation to the distribution of expenditure by consumers.) The measurement of real national product and of short-term movements in industrial production are discussed quite thoroughly but problems of labour and balance of payments statistics are mentioned only in passing. The final chapter on demography, as might be expected, provides a good introduction to the subject, but there is no mention of standardized death rates and the chapter stops short with only one paragraph on cohort fertility. And it seems unnecessary to include, even in a footnote, the proof of attaining a stable population if fertility and mortality conditions remain unchanged. RITA J. MAURICE.

6.—*Experimental Designs in Industry*. Edited by V. Chew. New York, Wiley, 1958. London, Chapman and Hall. xi, 268 pp. 11″. 48*s*.

This is a collection of nine papers given at a symposium held at North Carolina State College in November, 1956. Five other papers have been published, or submitted for publication, in technical journals. The book is divided into two parts, the first consisting of four papers reviewing work in experimental design considered useful in industrial research, and the second consisting of five shorter papers on special industrial applications. The book also contains a selected bibliography of several hundred items. On the whole the papers tend to be concerned with details of advanced methodology rather than with fundamental principles. The level of knowledge assumed of the reader varies substantially, both between and within authors. A rough indication of the standard is that an applied statistician familiar, say, with some of the later chapters of the book edited by O. L. Davies, *Design and Analysis of Industrial Experiments*, should find most of the papers reasonably understandable.

The titles and authors of the papers are as follows. In Part I; Basic experimental designs, by V. Chew; Complete factorials, fractional factorials and confounding, by R. L. Anderson; Simple and multiple regression analysis, by R. J. Hader and A. H. E. Grandage; Experimental designs for exploring response surfaces, by G. E. P. Box and J. S. Hunter. In Part II: Experiences with incomplete block designs, by W. S. Connor; Experiences

with fractional factorials, by H. W. Horton; Application of fractional factorials in a food research laboratory, by M. B. Carroll and O. Dykstra; Experiences with response surfaces designs, by R. M. DeBaun and A. M. Schneider; Experiences and needs for design in ordnance experimentation, by C. A. Bicking. The papers in Part II describe practical experience with advanced designs. These are all useful papers, particularly since detailed plans, sets of observations, and analyses are included in a number of cases. Thus Mavis Carroll and Dykstra's paper contains, among other things, quite full details of a $\frac{1}{4}$ replicate of a 4×2^6 factorial, with the analysis of variance and a graphical analysis by Daniel's half-normal plot (see later in the review for more about this method).

The remainder of the review will be devoted to the four papers in Part I. Chew's paper is in my opinion inaptly titled, and should have been called something like "The analysis of data classified in a balanced way". For, except for brief introductory remarks, the paper is concerned with the analysis of data, not with experimental design ; indeed no distinction seems to be drawn between experiments and surveys. Of course, in discussing many aspects of analysis it is quite justifiable to treat these together, but it does seem a pity not to at least mention the distinction. Chew's paper contains an account of analysis of variance for the simpler designs, including such complications as " missing values", and also reviews of recent work on the effect of heterogeneity of variance, on non-additivity, and on the analysis of residuals.

Anderson's contribution is concerned mostly with the complexities of the design and analysis of factorial experiments with factors at 2 or 3 levels, or both, when fractional replication and confounding are employed. From the non-specialist's point of view, perhaps the most interesting parts of the paper are, however, the short critiques of two subjects which have received a good deal of attention in recent American work, The first subject is that of "Multiple Comparisons"; that is, the provision of methods for making a number of statements simultaneously, when, for instance, a number of treatments are investigated and the comparisons among them to be made are decided after inspecting the data. The point at issue here is whether the probabilities should refer to the chance that *all* interval statements made are correct (error rate per experiment). Anderson argues against this. The second topic is Daniel's very ingenious method for the graphical analysis of factorial experiments (see C. Daniel, *Proc. 3rd Berkeley Symp.* (1957), Vol. 5, 87). Anderson makes two pertinent criticisms of the method; the procedure is however fairly easily modified to meet at any rate one of the criticisms (by plotting separately the points for main effects and two-factor interactions and revising the probability calculations accordingly).

Hader and Grandage's paper is a very clear account of the purely mechanical side of simple and multiple linear regression analysis, including the numerical inversion of matrices. The paper, in fact, seems to contain very little that is not adequately set out in standard text-books; I think that the paper would have been much more useful had it gone on to deal with such topics as the inclusions of additional independent variables, the deletion of independent variables, and the inclusion of additional observations.

Box and Hunter's paper is an excellent review of recent work on the design and analysis of factorial experiments with quantitatively varying factors. The following ideas are among those treated: steepest ascent to a maximum, orthogonal first-order designs (for effectively planar surfaces), elimination of trends using angular randomization of factor levels, second-order designs with symmetrical distributions of information (rotatable designs), arrangements of such designs in blocks, canonical analysis of fitted second degree equations. Most of the work has in fact already appeared in technical papers by Box and his associates, but the present review will be very welcome as a connected and lucid account of these developments.

In commenting on the book as a whole, one cannot but say that a series of rather unco-ordinated papers by different authors, writing on overlapping topics, is not a very satisfactory form for an expository book; something done collectively by at most a small group seems much more likely to produce a coherent review of a field. On the other hand, the applied statistician, in any field, not necessarily industrial, who has much to do with analysis of variance and experimental design, is likely to find useful things in the book, and hence its publication is to be welcomed.

D. R. Cox.

7.—*Studies in the Mathematical Theory of Inventory and Production.* By K. J. Arrow, S. Karlin and H. Scarf. Stanford, California University Press. London, Oxford University Press, 1958. x, 340 pp. 10″. 70*s*.

This collection of research papers, No. 1 of the Stanford Mathematical Studies in the Social Sciences, contains recent results of the analysis of business decisions concerning inventory and production. It consists of four parts: I.—Introduction (Chapters 1–3); II.—Optimal policies in deterministic inventory processes (4–7); III.—Optimal policies in stochastic inventory processes (8–13); and IV.—Operating characteristics of inventory policies (14–17). Apart from references after each chapter, there is a bibliography mainly of titles published since 1955, but no index.

The authorship is shared by six writers. One of the editors (K. J. A.) describes the historical background in Chapter 1, and all three of them write on the nature and structure of inventory problems in Chapter 2. In Chapter 3, they give summaries of the remaining chapters. Thus the first 57 pages comprise a survey of problems and results, while the detail is to be found in the remainder of the volume.

The first chapter consists of an analysis of three possible motives for holding stock, according to Keynes: the transaction motive, deriving from the cost of changing repeatedly from cash to investment and back again; the precautionary motive, i.e. the need for protection against uncertainty; the speculative motive, based on the possibility of profit through changes in demand and supply conditions. In the reviewer's experience it is mainly the second motive which concerns the medium level of managerial staff in most large institutions, and it is this which makes use of statistical ideas.

Inventory theory is concerned with constructing a mathematical model, from which numerical values for, or qualitative features of, optimal decisions can be derived. A well known policy, optimal under certain conditions, is the (s, S), or two-bin policy: whenever the stock falls below s, replenish it to S. The fundamental terms in the mathematical formulation are, *inter alia*,

$z(t)$, the rate of production,
$r(t)$, the requirement,
$y(t)$, the stock held, all these dependent on time t; and
$c(z)$, the cost of producing z per unit time,
$g(dz/dt)$, the cost, per unit time, of changing the production rate,
$h(y)$, the cost of holding stock y.

A typical problem is finding that production policy $z(t)$ (or at least some of its features), which minimizes the cost

$$\int_0^T \{c[z(t)] + g(dz/dt) + h[y(t)]\}\, dt$$

subject to the constraints

$$y(t) = y(0) + \int_0^t [z(t) - r(t)]\, dt,$$

$$y(t) \geqslant 0 \text{ for all } t,\ z(t) \geqslant 0 \text{ for all } t.$$

Assumptions are then made about the form of the relevant functions. With some very restrictive assumptions the problem becomes one of linear programming, but as a rule they are far from realistic. More suitable techniques appear to be derivable from dynamic programming.

It cannot always be assumed that the requirements are known in advance, but one might be prepared to assume probability distributions. Such cases are considered in Part III, which studies also a Minimax approach. One chapter in Part IV is concerned with describing the variations in stock level. Assuming, for the demand, independent and

identically distributed random variables in all periods, one obtains a renewal problem, and some new results are communicated here.

This book is the first of its type in this field, and if managers read it, then it should convince them of the usefulness of a rigorous approach to decision problems which are still, all too frequently, considered to be solvable by intuition. S. VAJDA.

8.—*Sampling Opinions: An Analysis of Survey Procedure.* By Frederick F. Stephan and Philip J. McCarthy. New York, Wiley, 1958, London, Chapman & Hall. xxi, 451 pp. 9″. 96*s*.

The title of this book is misleading. One expects a discussion of the problems specific to opinion surveys. One would expect, for example, to find a lengthy discussion of the influence of the interviewer on the answers obtained from opinion questions, but this is scarcely mentioned. Indeed, if one looks through the tables one finds they deal mainly with factual items.

The book would seem to lack any clearly thought-out plan. It is divided into three main sections, the first one hundred pages dealing with "The Nature and Role of Sampling", the next two hundred with "Empirical Studies", and the last hundred with "The Design of Sample Surveys".

The first section consists of a rather long-winded non-technical introduction to sampling. It could be excused if what followed was more useful.

The middle section consists of a rather ill-digested selection from studies already published elsewhere. In many cases the original authors' conclusions are merely quoted verbatim. A number of papers by the Division of Research Techniques at the London School of Economics, which have appeared in this *Journal*, are treated in this way. It would certainly seem that the authors of this book have not read the discussions of those papers. Considerable space is devoted to quota sampling, but although the evidence presented is sufficient to condemn the practice the authors seem very reluctant to do so. In contrast to the considerable amount of space devoted to quota sampling, a mere twenty pages deal with the estimation of variance for probability samples. A further chapter deals with non-response. There are also two chapters emphasizing the difficulties in comparing survey data either with the known population or with the results of other surveys. These are perhaps the most interesting parts of the book.

In introducing the final section the authors would appear to have begun to have doubts about the value of their book. They say "Many readers will find the discussion boring; they are well advised to skip it. Others will find it insufficiently concrete and technical". Your reviewer found it both. Anyone wishing to design a sample survey would do well to consult other books. It is difficult to recommend this book for any class of reader.

P. G. GRAY.

9.—*Further Contributions to the Solution of Simultaneous Linear Equations and the Determination of Eigenvalues.* National Bureau of Standards. Washington, Superintendent of Documents, 1958. iii, 81 pp. 10¼″. 50*c*. (Appl. Math. Ser. 49)

The readers of recent American literature on numerical analysis will have noticed a spate of papers making what we have always thought of as a very simple problem of solving a set of equations, into a very complex one by studying alternative iterative procedures in place of the usual elimination methods. At first sight these studies seem rather pointless, but with this latest contribution to these techniques it is clear that the effort has been worth while, since the theory developed here unites a whole range of techniques that have been evolved for both the problems of solving simultaneous equations and finding eigenvalues and eigenvectors.

A typical iterative process determines the correction $\Delta \mathbf{x}$ as a multiple of the residual vector $\mathbf{r}$, and previous iterative procedures have attempted to define the multiplier to give some local improvement. It does not follow, however, that the succession of local improvements in successive iterates will necessarily give the best overall improvement, and Stiefel's paper " Kernel Polynomials in Linear Algebra and their Numerical Applications" shows

how to evolve a strategy for determining the whole set of multipliers in such a way as to achieve both local maximum improvement and overall maximum improvement. He has moved the problem from one of tactics applied at a given instant in the calculation to a strategy in the whole calculation. He defines a weighted mean square error with an arbitrary density function, $\rho(\lambda)$, and shows that the coefficients with the multipliers needed are intimately related to the coefficients in the recurrence relation of the Kernel polynomials with respect to the density $\rho(\lambda)$. The density $\rho(\lambda)$ has to be defined over a range enclosing the eigenvalues of the matrix under consideration and, according to the type of bounds that can be placed on the eigenvalues, different density functions are used which give rise to a series of different iterations. Of these, the Lanczos procedure is derived as a special case.

The paper then goes on to discuss the use of Kernel polynomials to define a matrix transformation of a given matrix into one in which eigenvalues will be suitably separated so that the power method will be more effective.

It shows that the recurrence relations for any orthogonal polynomials and their successive Kernel polynomials can be arranged in an array which is formally identical to the array used in the quotient difference algorithm introduced by Rutishauser.

The second paper approaches the quotient-difference algorithm from a more classical standpoint using Hankel determinants and traces the connection between this algorithm and the technique introduced by Aitken as a modification of the Bernoulli recurrence method to determine the roots of equations. It then shows how the practical computing procedures can avoid the evaluation of the Hankel determinants and shows its applications to the finding of eigenvalues of matrices, zeros of polynomials and the derivation of continued fraction experiments from a given Taylor expansion.

The last section of the paper gives an interpretation of the quotient-difference algorithm as a set of Jacobi matrices $\mathbf{J}_n$, which can be factored $\mathbf{J}_n = \mathbf{L}_n\mathbf{R}_n$ such that $\mathbf{R}_n\mathbf{L}_n = \mathbf{J}_{n+1}$.

This serves as an introduction to the third paper "Solution of Eigenvalue Problems with the LR-Transformation", in which this relationship is generalized to forming a sequence of matrices from a given matrix $\mathbf{A}$ by taking the triangular solution $\mathbf{A} = \mathbf{LR}$ and forming $\mathbf{A}_1 = \mathbf{RL} = \mathbf{L}_1\mathbf{R}_1$ and $\mathbf{A}_2 = \mathbf{R}_1\mathbf{L}_1$ etc. This succession of transformations leaves undisturbed the latent roots, and it can be shown in a wide range of circumstances $\mathbf{A}_r$ converges to an upper triangular matrix of which, therefore, the diagonal elements will be the latent roots.

This is not a practical procedure for general matrices, but is particularly powerful for the striped matrices that occur as a numerical solution of partial differential equations.

In spite of the theoretical nature of these three papers, which on careful study give a deep understanding of the way in which the behaviour of matrices is governed by their eigenvalues, the papers do not neglect practical considerations of numerical procedures and all three consider and establish stability of numerical processes and give numerical illustrations.

All serious students of numerical analysis should read this book. K. D. TOCHER.

10.—*Integrals of Airy Functions.* National Bureau of Standards. Washington, Superintendent of Documents, 1958. iii, 28 pp. $10\frac{1}{4}''$. 25*c*. (Appl. Math. Ser. 52.)

The Airy function is of no interest to statisticians, but since so many statisticians today are also interested in numerical analysis these tables may interest readers of this *Journal*.

The Airy function $Ai(x)$ is useful in finding approximate solutions to second order differential equations in the neighbourhood of a turning point, and is defined as the normed finite solution of a differential equation $y'' - xy = 0$. It has been extensively tabulated by J. C. P. Miller and these new tables give:

$$f(x) = \int_0^x Ai(-t)\,dt$$

and

$$F(x) = \int_0^x f(t)\,dt$$

by numerical integration of the Miller tables. An accuracy of $\frac{1}{2}$ place in the 8 decimals is claimed and justified over the range $x = -2{\cdot}0\ (0{\cdot}1)\ 5{\cdot}0$. The second central differences are also tabulated.

An integral form for $Ai(x)$ is:

$$\frac{1}{\pi}\int_0^\infty \cos(t^3/3 + xt)\,dt = \mathscr{R}\left\{\frac{1}{\pi}\int_0^\infty e^{i(xt+t^3/3)}\right\}dt.$$

The modified Airy function $A_0(x)$

$$= \int_0^\infty e^{-(xt+t^3/3)}\,dt$$

is useful in solving the differential equation $y'' + xy = 0$, and more generally $y'' + xy = f(x)$. A table of this function was formed by solving numerically the former differential equation using an adaptation of the Kutta-Runge formulae on the SEAC, and also directly by the numerical integration of the integral. The range of the table is $x = 5{\cdot}0000\ (0{\cdot}0001)$ $10{\cdot}0000$ (16 d.p.). A skeleton table of $\int_0^\infty A_0(t)\,dt$ is also tabulated. K. D. TOCHER.

11.—*Table of Natural Logarithms for Arguments between Five and Ten to Sixteen Decimal Places.* National Bureau of Standards. Washington, Superintendent of Documents, 1958. xiii, 506 pp. $10\frac{1}{4}''$. \$4. (Appl. Math. Ser. 53.)

This is a re-issue of the old Volume IV, the table of logarithms prepared under the Mathematical Tables Project, and needs no introduction to our readers. It tabulates the natural logarithms in the range five and ten, to sixteen decimal places at an interval of 0·0001, and in addition to the original Foreword has a further Preface in which four misprints in the old tables, corrected in the new edition, are noted. K. D. TOCHER.

12.—*Finite Queueing Tables.* By L. G. Peck and R. N. Hazelwood. New York, Wiley, 1958. London, Chapman & Hall. xvi, 210 pp. 11″. 68*s*.

This book is No. 2 in a series of monographs sponsored by the Operations Research Society of America. The first book in the series* was a review of some of the mathematical theory of queueing systems. This second book considers a special but not uncommon kind of queue in which the potential customers form a closed group. If at any time there are h customers being served, l customers waiting and j customers "idle" (i.e. not requiring service), then

$$h + j + l = N,$$

where N is the total number of customers in the group.

The principal application of the theory is to problems of machine interference, where the customers are automatic machines and the servers are the attendant operatives. In terms of this application, "idle" is a curious word to describe the productive state of the machines.

In the short introduction, the unlimited queue is discussed first to introduce the ideas. For this and the finite queue, the standard Erlang methods of analysis are used, with the assumptions of negative exponential distribution of "idle" time and service time. The tables were obtained by calculating the state probabilities from the "steady-state" equations using UNIVAC 1. The results of the calculations were printed from the tape with the UNIVAC high-speed printer and the book produced by photo-offset from the machine printing.

* P. M. Morse, *Queues, Inventories and Maintenance.*

The terms and symbols used are:

N = population (number of machines, etc.).
M = service channels (number of operatives, etc.).
T = average service time.
W = average waiting time.
U = average "idle" time (running time, etc.).
H = average number of units being serviced.
L = average number of units waiting for service.
J = average number of units "idle" (running, etc.).
F = efficiency factor $= (H + J)(H + L + J)^{-1}$.
X = service factor $= T(T + U)^{-1}$.
D = probability of not having to wait for service.

The tables give F and D in terms of N, M and X. The other quantities required can be obtained from the formulae: $H = FNX$, $L = N(1 - F)$, and $J = NF(1 - X)$.

The tables cover the ranges: $N = 4(1)\ 26(2)\ 70(5)\ 170(10)\ 250$, and $X = 0{\cdot}001(0{\cdot}001)$ $0{\cdot}026(0{\cdot}002)\ 0{\cdot}070(0{\cdot}005)\ 0{\cdot}170(0{\cdot}01)\ 0{\cdot}34(0{\cdot}02)\ 0{\cdot}60(0{\cdot}05)\ 0{\cdot}95$. The range of values of M differs with N, but is always more than adequate.

It is not clear to the reviewer what advantages are gained by the use of X as argument rather than the Palm servicing factor $x = TU^{-1}M^{-1}$ or, perhaps, Mx.

It is convenient to have tables in this form but I suspect that there are few potential readers for whom the complete range is necessary. Naor (1956) has shown how the values tabulated here could be obtained from existing tables (Molina (1942) and Pearson & Hartley (1954)) of the Poisson functions

$$p(k, \xi) = e^{-\xi}\, \xi^k/k!$$

and

$$P(k, \xi) = \sum_{i=k}^{\infty} p(i, \xi).$$

The arithmetic involved in Naor's approach is admittedly heavy when M is large but this cannot be a common case in practical problems. Naor (1957) has given an approximate solution using tables of the normal distribution for use when M is large.

Since these alternative methods of obtaining numerical values are available and since there is no discussion of modifications to the mathematical model for queue discipline, heterogeneous populations, intermittent availability of servers, etc., this book can only be of value to a limited number of specialists.

Following a suggestion by Dr. D. R. Cox, I have checked a number of values in the tables. In each case, while the tabulated value of F is correct, the value of D refers to a population size smaller by one than that in the tables.

F. BENSON.

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MOLINA, E. C. (1942), *Poisson's Exponential Binomial Limit*. New York: Van Nostrand.
NAOR, P. (1956), "On machine interference", *J. R. Statist. Soc.* B, **18**, 280–287.
—— (1957), "Normal approximation to machine interference with many repairmen", *J. R. Statist. Soc.* B, **19**, 334–341.
PEARSON, E. S. & HARTLEY, H. O. (ed.) (1954), *Biometrika Tables for Statisticians*, Vol. 1. Cambridge University Press.

13.—*A Comprehensive Bibliography on Operations Research, through* 1956 *with Supplement for* 1957. New York, Wiley, 1958. London, Chapman & Hall. xi, 188 pp. 11″. 52*s*.

This Bibliography, which is the fourth in the series of monographs sponsored by the Operations Research Society of America, contains about 3,000 entries. The papers are given in alphabetical order of authors. Foreign language publications appear separately after the English contributions under each letter of the alphabet. The articles are classified

by a ten-digit numerical code. The information in the code is given in accordance with the following scheme.

Position(s)		*Information*
1	.	General classification of entry.
2–3	.	Type of organization involved.
4–5	.	Function or type of activity involved.
6–7	.	Techniques used or described.
8–9	.	Aspects of research discussed.
10	.	Aspects of practice discussed.

In addition to the printed volume, the entries are also available on IBM punched cards. Some special short bibliographies have been included in the volume.

The compilers of the bibliography have attempted to make it comprehensive and we find the following entry:

Potter, Stephen. "The Theory and Practice of Gamesmanship".

The technique used, according to the classification, is the theory of games! The reviewer, however, searched in vain for the following entry:

2 35 16 06 04 6 Ellis, H.F. "Written in a Queue," *Punch*.

More seriously, there are some surprising omissions amongst the papers on the theory of queues. The many contributions of A. J. Khintchine and L. Takács receive no mention whatever.

Some of the publications listed in the foreign language section are in fact available in English. For example, the paper by R. Kronig in *Physica* is written in English.

It is, however, very easy to criticize a bibliography, but the authors have made an extremely useful contribution to the work of Operational Research groups.

F. Benson.

14.—*Selected Studies of Migration since World War II.* New York, Milbank Memorial Fund, 1958. 244 pp. 9″. $1.

This volume contains papers read at the Thirty-Fourth Annual Conference of the Milbank Memorial Fund held at New York in October, 1957. It invites comparison with the symposium on the same subject in 1946—*Post War Problems of Migration* (1947).

In the opening essay Dr. Dudley Kirk gives an admirable survey of major migrations since the war. He shows that the vast uprooting of refugees in Western Europe, though entailing intense human suffering, was coped with successfully because it went with the grain of economic opportunity, either from East to West or from Europe overseas. This was not true of the forced movement of 17 million people caused by the partition of India in 1947; similar displacements followed the establishment of the State of Israel, the victory of the Communists in China and the Korean War. It is significant that there are more Arab refugees under international care today than there were just after the conflict; in contrast, 90 per cent. of the Hungarian refugees who fled to Austria and Yugoslavia have been resettled in new homes. There has been a resumption of normal emigration from Europe, with agencies such as the Intergovernmental Committee for European Migration taking a prominent part in organizing it, and it has nearly all gone to oversea areas already occupied by Europeans. On the whole, economic adjustments are now made to an increasing extent through *internal* migration, as shown, for example, in Irene Taeuber's instructive paper on "Continuities in Internal Migration in Japan".

In a critical assessment of U.S. policy Dr. Ernest Rubin suggests that the present quota be replaced by a quantity which would be related to the existing annual population of the United States. The most substantial analytical essays are by Professor Joseph J. Spengler on "The Economic Effects of Migration" and Professors Kuznets and Dorothy S. Thomas on "Internal Migration and Economic Growth". The latter show for the United States that " . . . whereas in earlier decades, opportunity as measured by general state levels of service income per worker was acting quite selectively upon native and foreign-born

whites and was scarcely touching the Negro component, during the later decades all classes of the population tended to move toward many of the same areas of greater and away from the same or similar areas of lesser opportunity" (p. 210). As a possible explanation of the persistence over many decades of a big difference between real income per person in agriculture and in industry, it is suggested that the growth of industrial opportunities in the United States was so fast that the internal migration processes could not keep pace. Hence "... a kind of paradox in which the internally mobile developed countries appear to be insufficiently mobile in terms of their economic opportunities whereas the apparently immobile underdeveloped countries have too much internal migration judged in terms of their *economic* framework" (p. 203). In an earlier discussion Professor Philip M. Hauser also stressed that much of the urbanisation going on in Asia is simply a movement from rural poverty to urban poverty. The paper on "Models in Migration" is disappointing. It is confined to work done by sociologists on "intervening opportunities", and one is inclined to agree with Donald J. Bogue's description of this approach as "a very trivial part of the whole analysis" (p. 171). Surely some attention should have been given to economic models.

Space has allowed only a brief mention of some of the topics dealt with in this volume. It contains excellent samples of the statistical research on migration now being done in the United States, and it is good to have the summaries of the discussions provoked by the papers. BRINLEY THOMAS.

15.—*The Older Population of the United States.* By Henry D. Sheldon, with Introductory and Summary Chapters by Clark Tibbitts. New York, Wiley, 1958. London, Chapman & Hall. xiii, 223 pp. 9". 48*s*.

This interesting volume illustrates the virtues of two features of recent census procedure in the United States of America: the thoroughness of the questioning undertaken in the enumeration schedule, and the allocation to individuals of the duties of reporting verbally on various aspects of the census data and related statistics. Because so many items of information were sought—some on a sample basis—it is now possible to discuss the characteristics of the older population in relation to a large number of factors: age-structure, geographic distribution, extent of employment, occupations followed, marital status, living arrangements, housing and income. Thus a multidimensional representation of the life of the aged can be built up. By not confining his attention entirely to the census, the author has been able to enrich his account with some results of recent non-censal surveys. The text is further enlivened by the inclusion of introductory and summary chapters written by a different author and in a broader manner that sets the enumerated details in an appropriate context. In this way, and also by transferring much technical material to a series of lengthy appendices, a pedestrian style has been avoided—a difficult achievement in this class of work.

Chapter 1 is largely concerned with the growth of awareness in the United States of the "burden of the aged", or perhaps rather of the fact that this burden was not being borne sufficiently by the active part of the population. It shows what political and social action has been taken, more especially during the last two decades, to try to solve the problems of older people and meet their needs. Readers will find it interesting to compare these developments with corresponding changes in Great Britain. The opening pages refer also to a tendency that is perhaps more specifically American, namely the increasing degree of separation of parents from the children once the latter have married and set up homes of their own. The use of demographic data in the study of this trend towards isolation in old age arises from the fact that it is associated with such contributory factors as family size, increased longevity, housing, urbanization and retirement from employment. On these matters the 1950 census has been particularly revealing, and the succeeding chapters deal with each item in turn.

In a highly mobile population such as that of the United States, it is not surprising to learn that the evidence shows that young people tend to migrate more than the old. One reason is, perhaps, that economic development brings about new types of occupation open

only to the young, who move to find suitable opportunities. Studies are made of jobs according to the degree to which people follow them in old age. This work is reminiscent of the investigations of Le Gros Clark and Dunne (1955) in Great Britain*. The separation of the elderly is associated also with the combined effects of early marriage and increasing survival—"the typical husband and wife may now expect . . . 14 years of life together after the last child has left home and before the appearance of . . . old age". Incomes of the old vary widely but relatively few have any substantial savings and one in five needs to have recourse to national assistance. The absence of an American national health service is partly responsible for penury in old age, as only one elderly person in three there has any form of insurance protection against hospital and surgical expenses.

Although it is shown that old people are now receiving forms of State help and are also learning to organize themselves socially in "sunset" clubs, the extent to which their needs are catered for is not fully clear. Nevertheless, this is a remarkably comprehensive volume, well stocked with valuable information.

P. R. Cox.

16.—*The Efficiency of the Coal Industry*. By James M. Henderson. Cambridge (Mass.), Harvard University Press, 1958. London, Oxford University Press. xiii, 146 pp. 8½". 36*s*.

This book, which is primarily directed to economists, is one of the first to describe an application of the technique of linear programming to the solution of a national distribution and production problem in the United States coal mining industry. The reviewer has devoted most of his remarks to the uses of linear programming.

A general discussion of the method of analysis is given in the first chapter and the idea of using linear programming as a tool for finding "perfectly competitive norms" (minimum cost solutions) is introduced. The second chapter is devoted to the development of linear programming equations for the general case, detailed consideration being given to the relationship between the ordinary and dual transportation problems. The dual is used to discover prices and royalties for a perfectly competitive system. This chapter contains a description, using a small scale example, of the method of solution. Despite the fact that a number of improvements in the arithmetical procedures have been made, Dantzig's original methods are largely used.

In the application of the technique data are taken for 3 years: 1947 (record), 1949 (bad year), 1951 (average year). For a number of reasons the country is divided into areas following as far as possible U.S.A. state boundaries. Area receipts are taken as the demands, and capacities are estimated on the assumption of a 280-day working year for underground mining and 260 days for surface deposits. Using average transport costs, the author solves a transportation problem for each year. Much space has been devoted to describing the information set down in 40 tables, some of which could have been combined.

Owing to the lack of cost information, certain routes have been ignored, but perhaps it has not been realized that transport costs do not necessarily become prohibitive for shipments over long distances. It is quite easy to construct examples in which expensive rather than cheap routes are used. The policy of ignoring routes over which "deliveries have not been made since World War II" is regretted. Extra information would be more quickly provided if there were a rearrangement of the costs in the tables used for the calculations. Such rearrangement would show the cost of the "unused capacity" routes as minus the extraction costs and actual transport costs used for the other routes. The programmer could then see immediately what further cost information might improve the solutions.

The optimum solutions are calculated and errors that may occur in the original data are discussed. Certain adjustments are made to counteract the effects of these errors. The next three chapters are devoted to efficiency. The actual allocation is compared with the optimum, and the differences regarded as a measure of efficiency. A comparison is

* *Ageing in Industry*. By F. le Gros Clarke and Agnes C. Dunne. The Nuffield Foundation, 1955.

made between the average difference between total cost of the optimum and actual. These work out at 153 (for 1947), 258 (for 1949) and 230 (for 1951) (all in dollars/ton). These three figures are divided into two parts called secular and cyclical inefficiency. The figure for 1947 is taken as the secular inefficiency for each year. From these three figures two conclusions are drawn, namely that "the level of inefficiency appears to vary inversely with the level of total demand" and "the costs of secular inefficiency appear to exceed those of cyclical inefficiency even in years of low demand". These conclusions are repeated in the summary, so they seem to have some importance. However, because of the limited number of results they cannot be taken seriously (e.g. suppose secular inefficiency is really 120 dollars/ton).

In the later chapters comments are made on the distribution of costs of these inefficiencies, the reasons for them, and methods of control and improvements. In referring to the National Coal Board the author states that "The prices set by the N.C.B. are based on fuzzy concepts of average and maginal costs". It must be pointed out that the pricing policy of the N.C.B. is controlled by the Government who consider the optimum for the country and not the mining industry alone.

Two appendices give the sources and rectification of data and the formulation of a model for the agricultural industry. Finally there is a long bibliography of mainly economics books and articles on linear programming, industry and coal mining. There are a number of small errors, mostly incorrect signs, which must be attributed to printing mistakes as the results obtained are accurate.

The purpose of the book would seem somewhat vague. It is certainly not a planning tool, although a suggestion that Federal control or a concentration of private control might be able to reduce capacity is made. The idea of nationalization is dismissed. It is, however, a survey of historical data to find some measure of the efficiency of the industry. Whether such an approach will be able to improve the industry only time will tell.

K. B. HALEY.

STATISTICAL NOTES

(1) British Official Statistics

The index of retail prices compiled by the Ministry of Labour and National Service which was 108 in August (prices at 17th January, 1956 = 100) remained at that figure in September and rose to 109 in October and to 110 in November and December. Calculated to one place of decimals the figures were 108·3, 108·4, 109·4, 109·8 and 110·2. The detailed figures of the weights used in calculating the index and the indices for different commodity groups were as follows:

	Food	*Alcoholic Drinks*	*Tobacco*	*Housing*	*Fuel and Light*	*Durable Household Goods*	*Clothing and Footwear*	*Transport and Vehicles*	*Miscellaneous*	*Services*	*All Items*
Weights:	350	71	80	87	55	60	106	68	59	58	1,000
Aug. 13th, 1958	105·2	105·8	107·8	123·3	110·7	100·1	102·7	113·1	113·3	114·9	108·3
Sept. 16th, 1958	105·6	105·8	107·8	123·3	110·8	100·0	102·8	112·7	113·2	115·0	108·4
Oct. 14th, 1958	108·1	105·8	107·8	123·8	110·9	100·0	102·8	112·7	113·5	115·2	109·4
Nov. 18th, 1958	108·4	105·8	107·8	124·2	116·5	99·9	102·7	112·9	113·5	115·4	109·8
Dec. 16th, 1958	109·2	105·8	107·8	125·1	116·6	99·9	102·7	113·1	113·6	115·4	110·2

The Ministry of Labour index of weekly wage rates, calculated on the basis of January 31st, 1956 = 100, showed that in August the level was 114·1. It rose to 114·7 in September, 115·9 in October, 116·1 in November and 116·2 in December. In manufacturing industries alone the figures were 113·5, 113·8, 115·7, 115·9 and 116·0. The principal classes of workpeople whose rates of wages rose in the period included coal miners, agricultural workers and employees in port transport, engineering, shipbuilding and heavy chemical manufacture.

The estimated total working population and the numbers in civil employment in the five months ended November, 1958, were as follows:

(*Thousands*)

Date	*Total Working Population*			*Numbers in Civil Employment*		
	Males	*Females*	*Total*	*Males*	*Females*	*Total*
July 1958	16,180	7,912	24,092	15,303	7,791	23,094
August 1958	16,207	7,951	24,158	15,318	7,821	23,139
September 1958	16,190	7,944	24,134	15,292	7,805	23,097
October 1958	16,194	7,955	24,149	15,271	7,807	23,078
November 1958	16,168	7,958	24,126	15,238	7,805	23,043

The numbers of persons on the unemployment registers of the Employment Exchanges rose by 30,400 in September, by 37,900 in October and by 22,200 in November. In December the figure fell by 4,300. The total for December represented 2·4 per cent. of the number of employees in Great Britain. The percentages in the different Regions ranged from 1·4 in London and the South-East to 4·1 in Wales and 4·4 in Scotland.

The following is the sex analysis of the figures:

Numbers of Unemployed Persons on the Registers of Employment Exchanges

Date	*Men and Boys*	*Women and Girls*	*Total*
August 11th, 1958	310,907	134,696	445,603
September 15th, 1958	331,751	144,235	475,986
October 13th, 1958	359,755	154,087	513,842
November 17th, 1958	377,666	158,361	536,027
December 8th, 1958	377,075	154,652	531,727

Of the total of 531,727 in December 89,840 had been unemployed for less than two weeks, 141,792 for two to eight weeks and 248,282 for over eight weeks while 51,807 were temporarily stopped. In the three weeks ended 3rd December 81,941 vacancies were filled by the Employment Exchanges. The number of vacancies unfilled at this date was 162,583.

In the week ended November 22nd, 1958, it is estimated that in manufacturing industries 159,300 manual workers were on short time. This was about 5,000 less than in October. In November 1,364,300 operatives were reported to be working on overtime.

The number of insured workers absent from work owing to illness, including self-employed as well as employed, was 789,600 in August, 822,900 in September, 900,100 in October, 910,900 in November and 915,200 in December. The numbers of employed persons absent owing to industrial injuries were 56,200, 61,800, 63,600, 61,900 and 57,800.

Figures published in the *Ministry of Labour Gazette* for December, 1958, show the number of members of trade unions in 1956 as 9,752,000 in 653 unions. Of this total 6,575,000 were members of 17 unions, each with a total of 100,000 or more while another 1,320,000 were in unions with a membership of 50,000 to 100,000.

The same issue of the *Gazette* contains figures relating to the earnings and hours of part-time women workers in manufacturing industries. Between October 1948 and April 1958 the total number of all workers covered by the enquiries rose by 13 per cent. In the same period the number of women rose by 9·7 per cent., and that of part-time women by 36·8 per cent. In April 1958, the average weekly earnings of part-time women were 68*s.* 3*d.* for an average of 22·0 hours. For full time women workers the average was 131*s.* 2*d.* for 40·9 hours.

(2) Other Statistics

The International Labour Office has recently issued the *International Standard Classification of Occupations* (as developed and approved by the International Conference of Labour Statisticians at its 7th (1949), 8th (1954) and 9th (1957) Sessions. It is a basically five-digit classification to serve as a basis for international comparison of occupation statistics. The classification is on the basis of work performed—the job itself rather than the degree of skill, ability and knowledge brought to it. A unit group of I.S.C.O. is a group of occupations related to each other by the general similarity of the characteristics of the work they entail. In the classification structure—all civilian occupations are divided into ten major groups (e.g., professional, managerial, clerical, sales, etc.). Armed Forces are in a separate group. These major groups are divided into 73 minor groups and further into 201 unit groups. The unit groups are finally subdivided into occupations of which altogether 1,345 are listed. A decimal coding system is employed, e.g. 7–35.60 = major group 7/8 (craftsmen, production process workers), minor group 35 (moulders and coremakers), and occupation 60 in this group (coremaker, hand, metal foundry). The published volume (I.L.O. Geneva, 1958, 3.50 dollars, or 21*s.* 0*d.*) contains full definitions of the occupations grouped serially in the minor groups within each major group, a list of titles used, and an alphabetical index of occupation titles. Its size and binding make it suitable for desk use.

The *Year Book of Labour Statistics* 1958 (I.L.O., Geneva. 627 pp. 30*s.* paper, 36*s.* cloth) follows the lines of recent years, the only change in the scope of the volume being the inclusion of a table on the receipts and expenditure of social security institutions for 1956 and 1957. The number of countries and territories covered has again increased—to

126 (119 in 1957)—and more details have been obtained for certain countries. These changes have increased the size of the volume to 627 pages (534 in 1957). The most comprehensive table is that on consumer price indexes which covers 109 countries: the least, that on general level of hours of work which covers only 16 countries.

The International Statistical Institute has published a further volume in its series "International Statistics of Large Towns" (see this *Journal*, 1957, p. 103) namely *Economic Data of Large Towns* (1958, 144 pp., 15 florins). A series of tables is given showing the total economically active population and its distribution by sex, by economic division, and by social status, and the unemployed, distinguishing those in receipt of unemployment payments; another series on licenses, for motor vehicles, for telephones, radio, and television sets; and a series on visitors (accommodation available, nights spent etc.) and on the daily movements into and out of the area. Two hundred and twenty-six large towns (i.e. over 100,000 population) collaborated in this enquiry, in 22 countries (all in Europe except Alexandria and Cairo): of these about one-third are in the United Kingdom. The data are almost complete for the series on economically active and telephone licences but only about two-thirds of the towns gave information on the other topics. Towns were asked to estimate the number of "radio pirates" and figures of 10 per cent. and over are given for some towns in Germany, Austria, Denmark; for several Spanish towns the estimates are 30 to 50 per cent., and even 80 per cent. in one case.

CURRENT NOTES

The Institute of Mathematical Statistics and the University of Chicago have established a series of publications entitled *Statistical Research Monographs*. The primary purpose of this series is to provide a medium of publication for material of interest to statisticians not ordinarily provided for by existing media. It will help fill the gap between journal articles and textbooks or treatises. Among the kinds of publications envisaged are: new research results too lengthy for the usual journal article, research results of interest in both theoretical and applied statistics, expository monographs in particular areas of statistics and discussions of statistical problems and techniques in particular areas of application.

Every attempt is to be made to maintain the highest standards of scholarship.

The Editorial Board consists of David Blackwell (University of California), William G. Cochran (Harvard University), Henry E. Daniels (University of Birmingham), Leo A. Goodman (University of Chicago), Wassily Hoeffding (University of North Carolina), Jack C. Kiefer (Cornell University), and William H. Kruskal (University of Chicago).

Authors are invited to send manuscripts and correspondence concerning the series to Leo A. Goodman, Department of Statistics, University of Chicago, Chicago 37, Illinois.

The British Conference on Automation and Computation (a standing committee of societies, of which the Royal Statistical Society is a member) now issues the *B.C.A.C. Bulletin*, a bi-monthly publication the first number of which appeared in September-October 1958. It carries a list of forthcoming meetings to be held by member organizations, which are of interest in this field, and reports other matters of interest such as forthcoming international conferences.

The cost of the Bulletin to individual members of member organizations (e.g. Fellows of the Royal Statistical Society) will be 1*s*. 0*d*. per copy (post free), or 6*s*. 0*d*. per annum. The cost to other purchasers will be 2*s*. 0*d*. per copy (post free), or 12*s*. 0*d*. per annum. Requests for individual copies, or subscriptions, should be sent to *B.C.A.C. Bulletin*, c/o The Institute of Mechanical Engineers, 1 Birdcage Walk, Westminster, London, S.W.1.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Accounting Research—

Vol. 9 (1958), No. 4—Working capital: its role in the short-run liquidity policy of industrial concerns; *M. H. B. Abd El-Motaal.* Electronic data-processing literature; *R. H. Gregory* and *B. Nanus.* Composition of the corporate sector. II. A comparison of public and private companies; *R. Ma.* Wages and salaries in the expenditure of county boroughs, 1951/2 and 1955/6; *E. B. Butler.*

Annals of Human Genetics—

Vol. 23 (1958), Part 1—The rate of approach to homozygosity ; *P. A. P. Moran.* An analysis of Geissler's data on the human sex ratio; *A. W. F. Edwards.* The repeat frequency of twinning; *M. G. Bulmer.*

Biometrika—

Vol. 45 (1958), Parts 3 and 4—Studies in the history of probability and statistics. IX. Thomas Bayes's *Essay towards solving a problem in the doctrine of chances*; with a biographical note by *G. A. Barnard.* The properties of a stochastic model for two competing species; *P. H. Leslie* and *J. C. Gower.* A problem in the combination of accident frequencies; *J. C. Tanner.* The multiple-recapture census. I. Estimation of a closed population; *J. N. Darroch.* Confidence intervals for distance in the analysis of variance; *M. G. Bulmer.* The efficiencies of alternative estimators for an asymptotic regression equation; *D. J. Finney.* The use of autoregression in fitting an exponential curve; *H. D. Patterson.* Two queues in parallel; *F. A. Haight.* Moment estimators and maximum likelihood; *L. R. Shenton.* Effect of non-normality on the power function of *t*-test (with a note by E. S. Pearson); *A. B. L. Srivastava.* A quick estimate of the regression coefficient; *D. E. Barton* and *D. J. Casley.* On the choice of the best amongst three normal populations with known variances; *A. Zinger* and *J. St.-Pierre.* Approximate formulae for the statistical distributions of extreme values; *J. J. Dronkers.* The sampling variance of correlation coefficients under assumptions of fixed and mixed variates; *J. W. Hooper.* The mean deviation, with special reference to samples from a Pearson Type III population; *N. L. Johnson.* An approximation to the distribution of non-central *t*; *M. Merrington* and *E. S. Pearson.* Upper percentage points of the generalized beta distribution. III; *F. G. Foster.* Estimation of parameters of mixed exponentially distributed failure time distributions from censored life test data; *W. Mendenhall* and *R. J. Hader.* A bibliography on life testing and related topics; *W. Mendenhall.* A two sample distribution free test for comparing variances; *B. V. Sukhatme.* The mean difference and the mean deviation of some discontinuous distributions; *T. A. Ramasubban.* The mean deviation of the Poisson distribution; *E. L. Crow.* Note on the characteristic function of a serial-correlation distribution; *R. Leipnik.* Two further applications of a model for binary regression; *D. R. Cox.* A note on a series solution of a problem in estimation; *I. Guttman.* On Nair's transformation of the correlation coefficient; *M. Sankaran.* Short proof of Miss Harley's theorem on the correlation coefficient; *H. E. Daniels* and *M. G. Kendall.* Runs in a ring; *D. E. Barton* and *F. N. David.* Some applications of Meijer-G functions to distribution problems in statistics; *D. G. Kabe.* Selection of the population with the largest mean when comparisons can be made only in pairs; *R. J. Maurice.*

British Journal of Preventive and Social Medicine—

Vol. 12 (1958), No. 3—Congenital malformations of the central nervous system in Scotland; *J. H. Edwards.* Post-neonatal infant mortality in Birmingham between 1947 and 1956; *G. Knox* and *J. Mackintosh.* Tuberculosis in immigrants in Birmingham, 1956–1957; *V. H. Springett, J. C. S. Adams, T. B. D'Costa* and *M. Hemming.* A note on the interpretation of $n \times 2$ tables; *J. H. Edwards.* Coronary and other heart disease in a group of Irish males aged 65–85; *R. M. Acheson* and *E. D. Acheson.* A new method of standardizing death rates; *D. Kerridge.*

British Journal of Sociology—

Vol. 9 (1958), No. 4—The measurement of housing need; *J. B. Cullingworth.*

Bulletin of the Oxford University Institute of Statistics—

Vol. 20 (1958), No. 3—Wage differentials in a large steel firm; *S. W. Ostry, H. J. D. Cole* and *K. G. J. C. Knowles.* Price stability in the United Kingdom; *C. R. Ross.* Inflation and the earnings gap; *H. F. Lydall.* The problem of adequate nutrition; *T. Schulz.*

Vol. 20 (1958), No. 4—Aspects of competition in manufacturing industry; *H. F. Lydall.* Growth and structure of Iraq's population, 1867–1947; *M. S. Hasan.* Middle-class families in France and in England; *T. Schulz.* Measuring Soviet industrial growth; *L. R. Klein.* Measuring Soviet industrial growth: a rejoinder; *F. Seton.* Fresh fruit and vegetable statistics in Britain: a reply; *G. Allen.*

Economica—

Vol. 25 (1958), No. 100—The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957; *A. W. Phillips.* Liquidity preference and loanable funds: stock and flow analysis; *D. Patinkin.*

Economic Journal—

Vol. 68 (1958), No. 272—Concentration of exports and imports: an international comparison; *M. Michaely.* Inflation, the terms of trade and the share of wages in national income; *G. Maynard.*

Eugenics Review—

Vol. 50 (1958), No. 3—Three surveys of promising families; *C. O. Carter.*

Incorporated Statistician—

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BANK OF ENGLAND

For Wednesday in each Week,

(000's omitted)

ISSUE DEPARTMENT

1	2	3	4	5	6	7	8		9
Liabilities			Assets						
Notes in Circulation	Notes in Banking Department		Govt. Debt. (£11,015) and Other Govt. Securities	Other Securities	Coin other than Gold Coin	Total of cols. 4, 5 and 6 (Fiduciary Issue)	Gold Coin and Bullion†		Total of Liabilities or Assets
£	£		£	£	£	£	£	per oz. fine *s. d.*	£
2,092,130	58,228	Jan. 1	2,146,280	706	3,014	2,150,000	358	*249/1*	2,150,358
2,033,662	66,696	,, 8	2,096,235	755	3,010	2,100,000	358	*249/0*	2,100,358
1,994,774	55,584	,, 15	2,046,282	705	3,013	2,050,000	358	*248/10*	2,050,358
1,966,360	33,998	,, 22	1,996,248	740	3,012	2,000,000	358	*248/8*	2,000,358
1,957,158	43,200	,, 29	1,996,200	786	3,014	2,000,000	358	*248/9*	2,000,358
1,959,303	41,055	Feb. 5	1,996,200	787	3,013	2,000,000	358	*249/1*	2,000,358
1,960,375	39,983	,, 12	1,996,279	707	3,014	2,000,000	358	*248/10*	2,000,358
1,959,124	41,234	,, 19	1,996,283	707	3,010	2,000,000	358	*249/0*	2,000,358
1,962,115	38,244	,, 26	1,996,231	756	3,013	2,000,000	359	*249/5*	2,000,359
1,973,768	26,591	Mar. 5	1,996,281	705	3,013	2,000,000	359	*249/3*	2,000,359
1,979,623	20,735	,, 12	1,996,281	704	3,015	2,000,000	359	*249/3*	2,000,359
1,982,553	17,806	,, 19	1,996,215	771	3,014	2,000,000	359	*249/3*	2,000,359
1,992,741	57,618	,, 26	2,046,285	703	3,013	2,050,000	359	*249/3*	2,050,359
2,018,011	32,348	April 2	2,046,238	749	3,014	2,050,000	359	*249/6*	2,050,359
2,025,094	25,264	,, 9	2,046,283	704	3,013	2,050,000	359	*249/2*	2,050,359
2,016,524	33,834	,, 16	2,046,285	704	3,011	2,050,000	358	*249/1*	2,050,358
2,007,987	42,372	,, 23	2,046,237	748	3,015	2,050,000	359	*249/4*	2,050,359
2,010,411	39,947	,, 30	2,046,247	740	3,013	2,050,000	359	*249/2*	2,050,359
2,015,829	34,530	May 7	2,046,287	702	3,010	2,050,000	359	*249/7*	2,050,359
2,020,005	30,354	,, 14	2,046,287	702	3,010	2,050,000	359	*249/5*	2,050,359
2,022,124	28,235	,, 21	2,046,204	784	3,012	2,050,000	359	*249/4*	2,050,359
2,033,908	16,451	,, 28	2,046,283	703	3,014	2,050,000	359	*249/5*	2,050,359
2,038,032	12,327	June 4	2,046,184	803	3,013	2,050,000	359	*249/3*	2,050,359
2,033,522	16,837	,, 11	2,046,217	769	3,013	2,050,000	359	*249/6*	2,050,359
2,033,991	16,369	,, 18	2,046,284	702	3,013	2,050,000	359	*249/7*	2,050,359
2,038,309	12,051	,, 25	2,046,233	755	3,013	2,050,000	360	*250/1*	2,050,360
2,047,600	52,760	July 2	2,096,283	702	3,015	2,100,000	360	*250/4*	2,100,360
2,060,339	40,021	,, 9	2,096,238	749	3,013	2,100,000	360	*250/4*	2,100,360
2,078,892	21,468	,, 16	2,096,240	748	3,012	2,100,000	361	*250/10*	2,100,361
2,105,657	44,703	,, 23	2,146,236	751	3,014	2,150,000	360	*250/6*	2,150,360
2,128,869	21,491	,, 30	2,146,288	702	3,010	2,150,000	360	*250/3*	2,150,360
2,123,184	27,176	Aug. 6	2,146,286	702	3,012	2,150,000	360	*250/2*	2,150,360
2,087,698	12,662	,, 13	2,096,223	767	3,011	2,100,000	360	*250/3*	2,100,360
2,058,861	41,499	,, 20	2,096,278	712	3,010	2,100,000	360	*250/1*	2,100,360
2,046,987	53,373	,, 27	2,096,219	769	3,013	2,100,000	360	*250/5*	2,100,360
2,037,221	13,140	Sept. 3	2,046,194	794	3,012	2,050,000	361	*250/7*	2,050,361
2,029,712	20,648	,, 10	2,046,283	704	3,013	2,050,000	360	*250/6*	2,050,360
2,020,290	30,070	,, 17	2,046,281	707	3,013	2,050,000	360	*250/5*	2,050,360
2,013,126	37,234	,, 24	2,046,285	703	3,012	2,050,000	360	*250/1*	2,050,360
2,017,510	32,850	Oct. 1	2,046,205	784	3,011	2,050,000	360	*250/2*	2,050,360
2,018,455	31,906	,, 8	2,046,209	778	3,012	2,050,000	360	*250/3*	2,050,360
2,014,115	36,244	,, 15	2,046,234	754	3,012	2,050,000	360	*250/1*	2,050,360
2,008,641	41,719	,, 22	2,046,284	704	3,012	2,050,000	360	*249/11*	2,050,360
2,011,957	38,403	,, 29	2,046,282	704	3,014	2,050,000	360	*250/1*	2,050,360
2,020,951	29,409	Nov. 5	2,046,219	770	3,011	2,050,000	360	*250/1*	2,050,360
2,026,375	23,985	,, 12	2,046,534	702	2,764	2,050,000	360	*250/2*	2,050,360
2,031,010	19,350	,, 19	2,046,470	766	2,764	2,050,000	360	*250/1*	2,050,360
2,044,219	56,141	,, 26	2,096,452	785	2,763	2,100,000	360	*250/3*	2,100,360
2,078,512	21,849	Dec. 3	2,096,533	702	2,764	2,100,000	360	*250/5*	2,100,360
2,121,555	28,805	,, 10	2,146,452	787	2,762	2,150,000	360	*250/5*	2,150,360
2,164,361	35,998	,, 17	2,196,537	703	2,760	2,200,000	360	*250/1*	2,200,360
2,172,906	27,453	,, 24	2,196,508	730	2,762	2,200,000	360	*250/2*	2,200,360
2,134,831	65,529	,, 31	2,196,457	778	2,764	2,200,000	360	*250/3*	2,200,360

WEEKLY RETURNS*

during the year 1958

(000's omitted)

10	11	12	13		14	15	16	17	18	19
BANKING DEPARTMENT										
Liabilities					Assets					
Capital (£14,553) and Rest	Deposits: Public†	Deposits: Bankers	Deposits: Other Accounts		Govt. Securities	Discounts and Advances	Other Securities	Reserve (Notes and Coin)		Total of Liabilities or Assets
£	£	£	£		£	£	£	£		£
18,150	14,641	192,802	78,801	Jan. 1	194,906	26,727	22,166	60,595		304,395
18,188	10,462	213,380	75,234	,, 8	207,021	19,137	22,031	69,076		317,264
18,292	10,444	209,623	74,494	,, 15	200,475	32,660	21,755	57,962		312,853
18,309	12,825	206,372	73,481	,, 22	227,520	25,575	21,516	36,376		310,987
18,315	12,233	205,142	74,553	,, 29	217,990	25,305	21,375	45,574		310,244
18,348	11,248	204,758	73,381	Feb. 5	197,395	45,870	21,039	43,431		307,735
18,385	12,630	219,307	73,771	,, 12	197,355	65,465	20,919	42,353		326,092
18,424	13,035	222,260	74,540	,, 19	244,105	19,525	21,021	43,608		328,259
18,453	11,401	223,985	72,999	,, 26	237,460	27,920	20,857	40,600		326,837
18,483	11,757	233,045	73,562	Mar. 5	273,800	12.460	21,636	28,950		336,847
18,493	14,076	215,180	73,806	,, 12	250,950	25,920	21,596	23,089		321,554
18,509	15,214	222,814	74,511	,, 19	263,630	25,836	21,424	20,159		331,048
18,529	17,184	214,578	79,462	,, 26	231,445	16,580	21,726	60,003		329,754
18,551	11,405	218,690	75,029	April 2	247,955	19,685	21,301	34,733		323,674
17,711	18,933	207,272	75,294	,, 9	249,630	22,425	19,500	27,656		319,210
17,772	9,747	238,068	75,144	,, 16	262,765	22,448	19,293	36,225		340,732
17,786	11,981	213,957	73,980	,, 23	230,905	22,814	19,225	44,760		317,704
17,818	9,940	220,405	73,059	,, 30	230,890	29,068	18,927	42,337		321,222
17,855	9,459	214,602	70,878	May 7	243,170	13,019	19,688	36,917		312,794
17,863	11,172	216,708	71,294	,, 14	248,225	16,744	19,333	32,735		317,038
17,925	11,114	224,592	70,310	,, 21	246,360	27,600	19,374	30,607		323,941
17,972	12,895	209,872	71,630	,, 28	246,115	28,666	18,773	18,815		312,368
17,999	9,922	217,669	73,765	June 4	261,345	24,603	18,712	14,695		319.354
18,008	10,224	208,932	71,899	,, 11	251,545	19,559	18,746	19,214		309,063
18,083	12,401	207,632	72,547	,, 18	242,165	30,910	18,840	18,748		310,663
18,119	13,305	230,470	72,079	,, 25	287,725	12,910	18,907	14,432		333,973
18,162	10,532	224,851	73,331	July 2	212,030	40,900	18,808	55,138		326,875
18,202	12,008	215,320	73,449	,, 9	227,405	30,310	18,859	42,406		318,980
18,220	10,718	212,108	72,892	,, 16	258,500	12,695	18,950	23,792		313,937
18,238	12,286	204,120	71,309	,, 23	224,730	15,090	19,105	47,028		305,952
18,307	16,541	204,623	72,090	,, 30	257,380	11,105	19,267	23,810		311,562
18,343	20,888	192,483	70,183	Aug. 6	247,095	6,420	18,898	29,485		301,898
18,349	9,560	204,628	71,269	,, 13	259,670	10,216	18,994	14,927		303,806
18,418	12,020	214,995	73,689	,, 20	246,190	10,039	19,271	43,622		319,122
18,449	12,033	206,005	70,127	,, 27	224,205	8,176	18,938	55,297		306,615
18,486	15,747	210,555	70,801	Sept. 3	264,140	16,116	20,356	14,977		315,589
18,488	10,984	217,246	70,922	,, 10	265,850	9,114	20.366	22,312		317,641
18,510	12,476	231,107	71,364	,, 17	271,080	10,642	20,165	31,569		333,456
18,517	13,735	241,549	73,367	,, 24	270,235	17,956	20,406	38,572		347,168
18,550	12,797	239,746	69,462	Oct. 1	261,220	24,045	21,252	34,038		340,556
17,678	12,876	219,050	68,895	,, 8	247,015	17,330	21,230	32,924		318,498
17,718	11,095	246,812	69,539	,, 15	270,225	16,760	21,041	37,137		345,163
17,723	13,939	237,371	70,178	,, 22	266,605	8,775	21,269	42,562		339,211
17,795	11,582	238,608	70,896	,, 29	266,435	11,841	21,376	39,231		338,881
17,799	14,295	229,164	70,831	Nov. 5	267,005	13,580	21,281	30,221		332,089
17,874	10,197	237,969	67,911	,, 12	283,255	4,250	21,479	24,967		333,951
17,908	11,210	236,721	68,896	,, 19	289,505	3,800	21,159	20,270		334,734
17,952	12,119	241,038	69,500	,, 26	255,670	6,576	21,310	57,055		340,610
17,961	13,803	215,429	71,202	Dec. 3	264,005	10,516	21,122	22,754		318,396
17,990	12,177	207,000	69,411	,, 10	245,475	9,621	21,761	29,723		306,579
18,076	14,172	208,534	73,939	,, 17	240,245	16,126	21,444	36,907		314,721
18,107	13,746	181,482	72,638	,, 24	215,365	20,936	21,325	28,349		285,975
18,146	12,378	215,499	76,570	,, 31	214,800	20,176	21,209	66,409		322,593

* Bank rate was 7% at 1st Jan., 6% from 20th March, 5½% from 22nd May, 5% from 19th June, 4½% from 14th Aug., 4% from 20th Nov.

† Valuation, per oz. fine, given in italics.

‡ Including Exchequer, Savings Banks, Commissioners of National Debt, and Dividend Accounts.

Vol. 122.] [Part II

Journal of the Royal Statistical Society

SERIES A (GENERAL)

PART II, 1959.

The Determinants of Wage Inflation: United Kingdom, 1945–56

By L. A. Dicks-Mireaux and J. C. R. Dow

National Institute of Economic and Social Research

[Read before the Royal Statistical Society, December 18th, 1958, the President, Sir Harry Campion, C.B., C.B.E., in the Chair]

1. Introduction

1.1. This study is an attempt to explain, by multiple regression techniques, the varying pace of the rise in wage-rates, and to estimate among other effects the effect of the changing pressure of demand for labour. Since no single study is likely to provide a definitive answer to these questions, it may be well to explain how this one originated, and why we adopted the approach we did.

1.2. The general impression left by previous studies (in particular Brown, 1955, diagrams 14 and 17) was that before 1939 the rate of change of wage-rates was associated with the level of unemployment; but that since the war the relation has been less obvious, and the rate of change of wage-rates has been more closely related to that of prices. It seemed possible, therefore, that the relation between wage-changes and unemployment had been masked by the relation between wage- and price-changes; and that an attempt to take both into account at the same time would reveal that a relationship of the former type persisted.

1.3. A previous study by one of the present authors (Dow, 1956) proposed as a hypothesis that "full compensation for price increases is something which trade unions aim at and which both sides to wage negotiations accept as a standard of reference; and that variations of wage-earnings about this norm can be treated as a variable affected by employers' demand for labour, or the strength or pushfulness of trade unions" (paragraph 38). Estimates were there given of annual changes for the years 1948–54 in the average wage-salary deflated by lagged price-changes (Table XII); and the effect of the discussion was to suggest that the pressure of demand had had a small effect only on the rate of change of wages and salaries. It was later pointed out (Pounce, in an unpublished study) that the change of wages-salaries, so deflated, was closely correlated with the level of unemployment in each year; and that to judge by this association, a small change in unemployment made a startlingly large difference to the rate of wage and salary increase.

1.4. It seemed necessary therefore to examine the relation much more closely. This study deals with part of the field only, and relates only to the behaviour of wage-rates. In the course of the work it appeared necessary to relax the assumption that apart from the

influence of other factors, wage-earners obtained *full* compensation for price increases. The hypothesis tested thus was that the rate of wage-change was a function both of the pressure of demand and of the rate of price-change. The results proved to be technically satisfactory, in the sense that much of the variation in the rate of wage-change is shown to be "explained" in terms of the hypothesis.

1.5. The use of price-change as part-explanation of wage-change necessitates great care in interpreting the results. For wage costs are an important constituent of prices, and wage-changes must probably in turn entail price-changes. The rate of price-change (used as an explanatory variable) might therefore contain a substantial element of "feed-back" i.e. an element itself to be explained in terms of wage-changes, and not therefore an independent variable. For reasons given later, we think that this element is small; and that the statistical association can be taken as evidence of the causal relationship under investigation (see further 5.3–5.9 below).

1.6. The basic data used are:

(i) *Changes in wage-rates* calculated from the Ministry of Labour index for all workers in all industries. This index refers in general to nationally negotiated minimum or standard rates for time-workers, piece-workers and shift-workers. Local changes in *actual* time-rates and piece-rates, except in so far as they are affected by nationally agreed changes, are not taken into account. The series for average wage-rates in the seven broad industry-groups are based on confidential data kindly made available by the Ministry of Labour. There are many difficulties in constructing indices for separate industries. Estimates for separate industries have recently been presented by Devons and Ogley (1958), where the various difficulties are fully discussed. Such indices are inevitably somewhat less reliable than an average for all industries, but the data are probably sufficiently accurate for the kind of analysis here used.

(ii) *Changes in retail prices* calculated from the Ministry of Labour indices, except that alternative estimates (based on Allen, (1949)) were made for the period before June 1947 when the official index understated the rise in prices.

(iii) *The pressure of demand for labour* measured by an index, described elsewhere, based on the official statistics of unemployment and unfilled vacancies (Dow and Dicks-Mireaux, 1958, particularly Fig. 4 and Table A3). In general the index roughly follows the inverse of percentage unemployment. Similar results were obtained when some of the calculations were repeated using unemployment as the explanatory variable in place of the demand index; and this would probably generally be true.

The general course of wage-rates and retail prices is shown in Fig. 1.

1.7. Quarterly averages were used throughout. For the greater part of the study, the change in wage-rates was calculated as the percentage change each quarter over the same quarter in the previous year. The course of wage-rates being always upward, all changes were positive, varying between 0·9 per cent. and 10·7 per cent. For most of the analysis it was thought appropriate (see Section 3) to use four-quarter moving averages of the other variables. Thus the annual percentage change in the moving average of retail prices was used as the explanatory price variable. This proved to have little correlation with the other explanatory variable, the index of demand (as a four-quarter moving average), so

that it was possible to obtain significant estimates of the separate effects of price-changes and the level of demand respectively.

1.8. There are special difficulties in explaining changes in an index of average wage-rates in all industries, for this reflects discontinuous changes in the wages of successive groups of workers; and the interval between increases for any one group cannot be inferred from the index. It is convenient for exposition, therefore, to analyse first (Section 2) wage-changes in two industries, engineering and building, in which a wage-change affects most workers at the same point of time. Section 3 develops a model to deal with the difficulty of using average data for all industries, which is tested in variant forms. Section 4 repeats the analysis for seven broad industry-groups. Results are discussed in Section 5.

2. Wage-changes in Engineering and Building

2.1. A preliminary study was first made of wage-changes in two industry-groups, engineering and building, where the data readily available enable the intervals between wage increases to be distinguished. In the engineering industries (S.I.C., groups V–IX) national negotiations are made between the Confederation of Shipbuilding and Engineering Unions and the Engineering and Allied Employers' National Federation. In the building industry (S.I.C., group XVII) national negotiations are made through the National Joint Council for the Building Industry, in which all relevant trade unions and groups of employers participate.

2.2. For each industry, the percentage wage-change was calculated *over the varying intervals between each successive settlement*. An attempt was made to explain these wage-changes in terms of (i) the price-increase over the varying interval since the previous wage-change, or over a period of the same length but lagged by various amounts; and (ii) the pressure of demand for labour in the quarter of settlement, or with lags of various amounts. The index of labour demand in the particular industry-group was used. The analysis was repeated replacing this by the index of overall demand; but since the two demand indices are in these industries themselves highly correlated (Table 10), the results were substantially the same, and are not reported. The hypothesis can thus be expressed formally as:

$$\frac{w_t}{w_{t-\theta}} = A \cdot \left(\frac{p_t}{p_{t-\theta}}\right)^{\alpha} \cdot d_t^{\beta} \cdot \epsilon_t \tag{1}$$

where: w = the wage-rate index, June 30th, 1947 = 100,
p = the retail price index, June 17th, 1947 = 100,
d = a measure of the excess demand for labour,
A = some constant,
t = time subscript,
θ = the interval between successive wage settlements, which is not necessarily constant,
ϵ = error variable.

The index of excess demand for labour as given in Dow and Dicks-Mireaux (1958) varies about zero according as demand, in relation to supply, is excessive or deficient. If d' is the

original index, the index d which has been used throughout this analysis may be written as:

$$d = \frac{100 + d'}{100}.$$

For calculation the series were transformed into logarithmic form (see Table 1). Though a non-linear formulation was adopted here and throughout this paper, the range of values fitted is such that a linear formulation would have yielded similar results. This restriction is discussed further in Section 5.

2.3. It was thought best to exclude the period of wage-restraint from 1948 to 1950; and the equation was fitted, by the method of least squares, to data for 1950–56. The estimates of α and β thus obtained are set out in Table 1. For each industry six observations

TABLE 1

Engineering and Building Industries: Explanation of Changes in Wage-rates

Time lag for price effect (months)	*Price Effect* (α) 0	3	6	*Demand Effect* (β) 0	3	6	*Percentage of Variation Explained* ($100 R^2$) 0	3	6
				Time lag for demand effect (months)					
Engineering (October 1949 to March 1956)									
0 .	0·33* (0·07)	0·16	0·24	3·33* (0·74)	3·54	2·83	96·1**	93·9*	83·1
3 .	0·27*	0·14	..	3·44*	3·53*	..	96·9**	94·8*	..
6 .	0·35*	..	..	4·92**	..	..	94·8*	..	..
Building (February 1950 to April 1956)									
0 .	0·21 (0·16)	0·14	0·11	2·18 (1·22)	0·99	1·15	66·6	38·8	33·9
3 .	0·17	−0·01	..	2·70	1·53	..	58·1	34·0	..
6 .	0·18	..	..	2·99	..	..	58·8	..	..

The value of the constant term (not shown above) was about $4\frac{1}{2}$ per cent. and $5\frac{1}{2}$ per cent. respectively for the two industries. To simplify the table, standard errors are only shown for unlagged results; those for lagged results were somewhat larger. * and ** indicate estimates significantly different from zero at the 5 per cent. and 1 per cent. probability levels respectively.

only were available. For the engineering industry the multiple correlation (as measured by R) was high and significantly different from zero. For the building industry none of the correlation coefficients was significantly different from zero. In both cases the estimate of the price coefficient is strikingly small and suggests that a change in prices of 10 per cent. induces a wage-change of 3 per cent. or less. The influence of the level of demand for labour, *per contra*, appears marked; the presence of 1 per cent. excess demand being sufficient to account for about a 3 per cent. change in wages. In each of the estimated equations there is a constant term which suggests that wage-rates increase at an annual rate of about 5 per cent., over and above that induced by price-changes and the level of demand.

2.4. The evidence regarding time-lags is inconclusive. It is true that as the lag is

increased the explanation of wage-change (R^2) diminishes. This is particularly true as regards the demand effect: but the results do not exclude the possibility of distributed lags, i.e. that demand two quarters previous has some effect. A distributed lag for the price effect seems a less plausible hypothesis. The results for engineering suggest there may be a lag of some months, as is plausible in view of the various delays involved in wage negotiations (see 5.4 below); and those for building hardly suffice to rule this out.

2.5. Building wage agreements contain a cost-of-living sliding scale, as set out in the *Constitution, Rules and Regulations* of the National Joint Council for the Building Industry. Broadly speaking, each increase of three points in the yearly average of the retail price index results in an increase in February each year of ½*d.* in wage-rates. Thus it is possible to calculate the percentage increase in rates due to the sliding scale, which, when compared with the corresponding percentage increase in prices, suggests a price coefficient falling between 0·6 and 0·7 (see Table 2). Under such arrangements the remainder of the increase

TABLE 2

Changes in Craftsmen's and Labourers' Wage-rates in the Building Industry

		Annual Percentage Changes			
Period of Change	*Retail Price Index*[a] (1)	*Total Wage-rate* (2)	*Cost-of-living Adjustment* (3)	*Negotiated Change*[b] (4)	(3) ÷ (1) (5)
1. Craftsman (Grade A)					
1950–51 . .	6·2	8·7	1·4	7·3	0·23
1951–52 . .	12·1	8·0	4·0	4·0	0·33
1952–53 . .	6·2	4·9	4·9	—	0·79
1953–54 . .	1·4	5·9	2·4	3·5	1·71
1954–55 . .	3·6	6·7	1·1	5·6	0·31
1955–56 . .	6·2	7·3	2·1	5·2	0·34
Average 1950–56 .	6·0	—	3·0	—	0·62
2. Labourer (Grade A)					
1950–51 . .	6·2	10·3	1·7	8·6	0·27
1951–52 . .	12·1	9·4	4·7	4·7	0·39
1952–53 . .	6·2	5·7	5·7	—	0·92
1953–54 . .	1·4	6·8	2·7	4·1	1·93
1954–55 . .	3·6	5·1	1·3	3·8	0·36
1955–56 . .	6·2	8·4	2·4	6·0	0·39
Average 1950–56 .	6·0	—	3·1	—	0·71

[a] Generally from December to December preceding February in each case.
[b] Obtained by difference from the changes in the total rate and the cost-of-living adjustment.

Sources: *Ministry of Labour Gazette; Constitution, Rules and Regulations* (as adopted by the Council and Adherent Bodies, January 1949), of the National Joint Council for the Building Industry. Information kindly supplied by the Ministry of Labour.

in wages (the "negotiated" change in Table 2) might be expected to bear little systematic relation to price-changes; and this appears to be the case. The sliding scale effect might therefore be expected to approximate to our estimated price effect (Table 1). Though it is in fact three times as large the difference is hardly outside the margin of error.

3. CHANGES IN AVERAGE WAGE-RATES IN ALL INDUSTRIES

(a) *The Model*

3.1. We here attempt to explain the rate of change in the index of average wage-rates, the change being calculated as the percentage change between the average in one quarter and the average in the same quarter of the preceding year. The percentage change so calculated will reflect adjustments in wage-rates made not only in the current quarter but in each of the preceding three. The wages of various groups of workers contained in the average change in discontinuous jumps, each group tending to obtain increases in rotation at intervals of a year or so. It follows that, if wages are influenced by price-changes, only a fraction of all workers will in any quarter be affected by the price-changes over the year up to the current quarter; and that the price-change over the year up to each of the preceding three quarters will also be relevant to the explanation of our percentage change in wage-rates. Thus it seems that the explanatory variable should be something like a four-quarter moving average of price-changes. In other words, price-changes must be supposed to operate on *average* wage-rate changes with distributed lags; and the average length of the lag must be, quite apart from any further institutional delays, about six months.

3.2. To simplify the problem we assume:

(i) That the number of workers obtaining a wage increase is the same each quarter;

(ii) that the period between successive settlements is in all cases twelve months. These assumptions are only roughly realistic. Table 3 shows that the distribution of wage settlements by quarters has been erratic: in some years half the settlements have

TABLE 3

The Distribution of Wage Settlements, 1946–56

	Workers Affected by Settlements (millions)	*Distribution Through Year (Percentages)*			
		First Quarter	*Second Quarter*	*Third Quarter*	*Fourth Quarter*
1946 .	8·3	24	41	24	11
1947 .	5·5	14	14	29	43
1948 .	7·8	25	18	23	34
1949 .	5·9	49	16	15	20
1950 .	6·4	30	7	19	44
1951 .	19·1	31	16	21	32
1952 .	13·7	29	15	28	28
1953 .	10·6	28	15	33	24
1954 .	13·6	26	39	14	21
1955 .	15·7	50	28	10	12
1956 .	16·8	53	28	11	8

Workers affected by more than one settlement in a year are counted more than once for that year. The hypothesis of an even distribution throughout each year is unsupported by the χ^2 test.

Source: *Ministry of Labour Gazette*

come in one quarter and in some quarters there have been few settlements. With regard to the second assumption, although many of the large settlements (i.e. covering many workers) have occurred at intervals of about a year, this has not always been so: for example, the engineering settlement in April 1954 came 17 months after the previous

settlement in November 1952. We consider later the distortions these assumptions may have introduced.

3.3. Fig. 1 shows three indices: (A) the retail price index; (B) a distributively lagged (i.e. moving average) index of prices; and (C) the wage-rate index for all workers. We propose to use changes in (B) as one of the variables to explain changes in (C). As already explained, an alternative to the official price index was constructed for the period before June 1947, and Fig. 1 shows this index. The effect of using the official index is also tested.

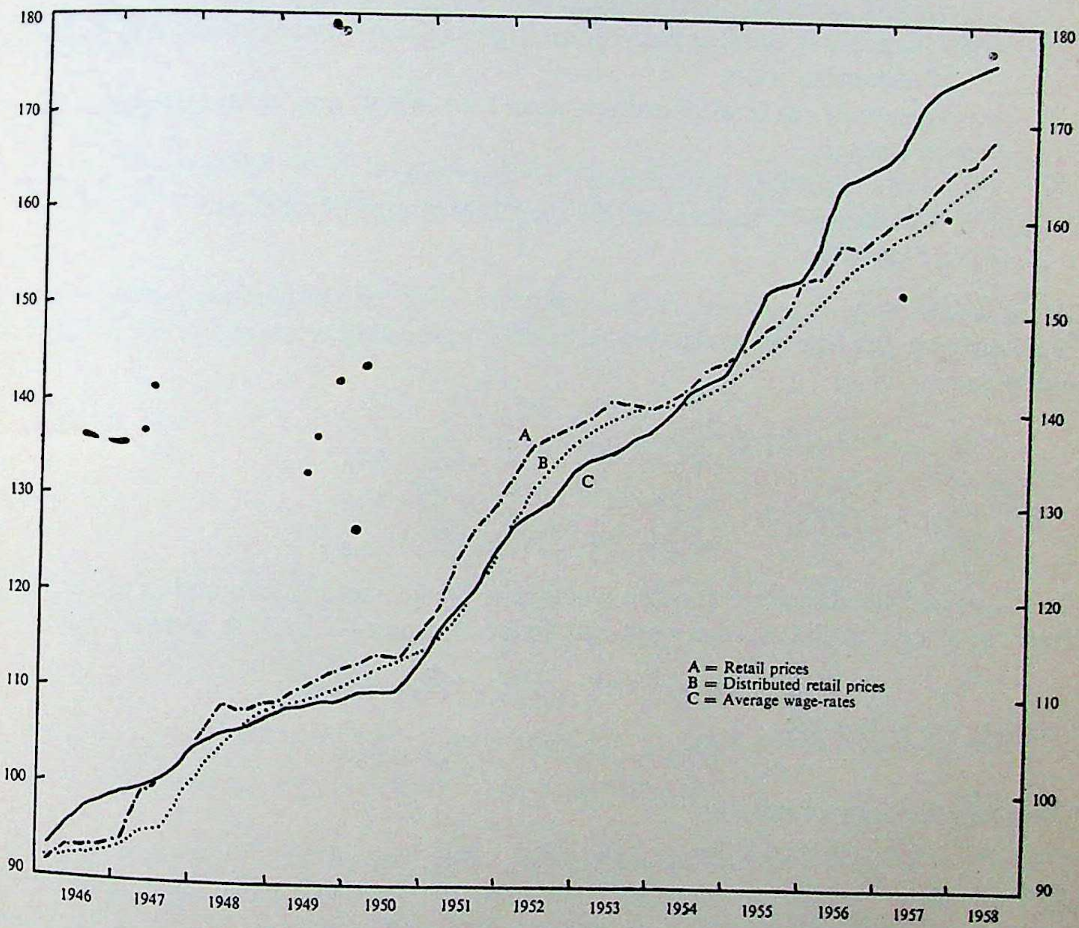

FIG. 1.—Quarterly wage-rate and retail prices, 1946 to 1956

Quarterly index numbers, June, 1947 = 100

3.4. In general we assume that it is the level of demand in the current quarter that affects the wage settlements reached in that quarter. The reasons already given for the use of moving averages apply here also. We therefore relate the change in the average wage-rate index in a given quarter (since the same quarter in the previous year) to a moving average of our index of demand in that quarter and the three preceding quarters. Later we test the assumption that wage settlements are not affected by demand in the current quarter but by demand in periods preceding the settlement.

3.5. We further assume that all wage settlements are influenced by the level of demand for labour *in the economy as a whole*. Later, in Section 3, we disaggregate into seven major

industry-groups; but the results given there and also in Section 1 suggest that little is gained by explaining wage-changes in each group by demand conditions in that industry rather than by demand conditions in the economy as a whole.

3.6. The general model adopted may be set out as below using the following notations:

W = ratio expressing the change in the average wage-rate index between corresponding quarters of successive years,

w = ratio expressing the change in the wage-rate index for a given quarterly group of workers between corresponding quarters of successive years,

P = ratio expressing the change in the retail price index between corresponding quarters of successive years,

d = a measure of the level of excess demand for labour in a given quarter,

A = some constant,

t = subscript measuring time at quarterly intervals,

$i, j, k, l,$ = superscripts for each of the quarterly groups of workers,

ϵ = error variable.

The basic data are given, as used, in the Appendix. For each of the four groups of workers the relationship between wage-changes, price-changes and the average level of demand may be written as:

$$\begin{aligned} w_t^{(i)} &= A^{(i)} \,.\, P_t^{\alpha} \,.\, d_t^{\beta} \,.\, \epsilon_t \\ w_{t+1}^{(j)} &= A^{(j)} \,.\, P_{t+1}^{\alpha} \,.\, d_{t+1}^{\beta} \,.\, \epsilon_{t+1} \\ w_{t+2}^{(k)} &= A^{(k)} \,.\, P_{t+2}^{\alpha} \,.\, d_{t+2}^{\beta} \,.\, \epsilon_{t+2} \\ w_{t+3}^{(l)} &= A^{(l)} \,.\, P_{t+3}^{\alpha} \,.\, d_{t+3}^{\beta} \,.\, \epsilon_{t+3} \,. \end{aligned}$$

If we disregard the difference between changes in the geometric mean and in the arithmetic mean the change in the average wage-rate index at time $t+3$ may be expressed as:

$$W_{t+3} = (w_t^{(i)} \,.\, w_{t+1}^{(j)} \,.\, w_{t+2}^{(k)} \,.\, w_{t+3}^{(l)})^{\frac{1}{4}}.$$

This may be written as:

$$W_{t+3} = A \,.\, \bar{P}_{t+3}^{\alpha} \,.\, \bar{d}_{t+3}^{\beta} \,.\, \nu_{t+3}, \tag{2}$$

where for simplicity of notation:

$$\bar{P}_{t+3} = (P_t \,.\, P_{t+1} \,.\, P_{t+2} \,.\, P_{t+3})^{\frac{1}{4}}$$

and

$$\bar{d}_{t+3} = (d_t \,.\, d_{t+1} \,.\, d_{t+2} \,.\, d_{t+3})^{\frac{1}{4}}.$$

Expressing equation (2) in logarithmic form estimates of the parameters were then obtained by the method of least squares. The geometric moving averages of the basic data become arithmetic moving averages of their logarithms.

3.7. The previous remarks about assumptions of non-linearity (2.2 above) apply with equal force here. For the values fitted the results are little different from what would have been obtained with a linear form of equation. The assumed form of equation is thus an important restriction of the results, which will be further considered in Section 5.

3.8. The use of moving averages leads to serial correlation in the dependent and determining variables. Consecutive quarterly observations of all the variables in equation (2) (e.g. the moving averages of the price-changes) will contain three common quarterly observations of the more basic data (e.g. changes in the retail price index). What is more

important is the induced autocorrelation in the error term (ν) of equation (2). This term is formally a moving average of the unspecified error terms (ϵ) of the component equations of equation (2) and as such will be positively autocorrelated. This should not introduce a bias in the estimates of α and β; but their standard errors will be affected so that standard tests of significance will be invalid (cf. Cochrane and Orcutt, 1949). In an attempt to remove this autocorrelation all variables, dependent and independent, were transformed by the operator $(1-\frac{3}{4}E^{-1})$, where E is the conventional forward shift operator. A test (Durbin and Watson, 1951) was then made to see whether autocorrelation in the residuals had been removed. In those cases where this was so standard significance tests were applied.

(b) *General Results*

3.9. Three applications of equation (2) to the data were tried, with the results shown in Table 4. First it was applied to data for the whole period 1946–56, using our alternative

TABLE 4

Explanation of Wage-rate Changes

$$W_t = A \,.\, \bar{P}_t^{\alpha} \,.\, \bar{d}_t^{\beta} \,.\, \nu_t$$

Data Fitted	Estimates of Parameters in Equation (2): Constant[a] (log A)	Price Coefficient (α)	Demand Coefficient (β)	Percentage of Variation Explained (100 R^2)
Period 1946.IV–1956.IV using alternative price index[b]	0·013 (0·003)	0·38 (0·12)	1·04 (0·91)	23·6
Period 1946.IV–1956.IV using official price index	0·008 (0·003)	0·54 (0·11)	2·62 (0·84)	41·9
Period 1950.IV–1956.IV	0·011 (0·001)	0·52 (0·05)	3·64 (0·40)	89·3

[a] The constants were estimated in their logarithmic form: as antilogarithms they read 1·030, 1·019, 1·026 respectively.
[b] The alternative price index was used for the period until June 1947.

price index for quarters up to mid-1947. Inspection showed that the twelve calculated wage-changes obtained for the years 1947–49 were well below the actual values, probably partly because of the policy of wage restraint then being followed; and perhaps partly also because wage negotiations were influenced by the published official price index, misleading though it was believed to be. In the second estimate the published official price index was used for quarters before mid-1947. The results are much better than the first set, though the low percentage of variation explained probably still reflects the abnormal reaction during the wage restraint period. The third estimate is based on the period 1950–56 thereby excluding entirely the influence of wage restraint and the dubious price indices. The explanation is considerably improved.

3.10. The general conclusion is similar to that reached in the analysis of wage-changes in engineering and building (Section 2), namely:

(i) Price changes appear to be an important influence, although the price effect is well below unity;

(ii) the demand for labour appears to have a considerable influence, a one-point change in the percentage level of excess demand* being associated with a change of 3 or 4 per cent. in the percentage annual rate of wage-change;

(iii) there is a constant factor which, in the absence of price-changes and of excess demand for labour, either positive or negative, implies an annual wage increase of 2½ per cent.

These values, though not identical to those of Section 2, suggest that the additional assumptions of the aggregative model have not led to great distortion.

3.11. The significance of the results is best judged after an attempt at removing the autocorrelation in the residuals discussed in 3.8 above. For the third of the variants (i.e. for the period 1950–56) the transformation of the variables left the residuals uncorrelated (see Table 5). Though reduced slightly, the correlation now appears significant. The

TABLE 5

Estimates of Regression Coefficients After Correction for Autocorrelation in the Residuals

Sample Period: 1950.*IV to* 1956.*IV*	*Number of Observations* (n)	*Constant Term* (*log A*)	*Price Coefficient* (α)	*Demand Coefficient* (β)	*Percentage of Variation Explained* ($100\ R^2$)	*Test for Auto-Correlation of Residuals*[a] (d)
1. No transformation made to eliminate autocorrelation	25	0·011 (0·001)	0·52 (0·05)	3·64 (0·40)	89·3	1·25[b]
2. Transformation of variables by $(1 - \frac{3}{4}E^{-1})$	24	0·004 (0·001)	0·44 (0·11)	3·77 (0·90)	62·5**	1·77[c]
3. Changes between the *first quarter* of each year only	6	0·011 (0·002)	0·52 (0·07)	3·38 (0·54)	97·2**	..
4. Changes between the *second quarter* of each year only	6	0·016 (0·002)	0·36 (0·07)	3·39 (0·68)	93·7*	..
5. Changes between the *third quarter* of each year only	6	0·013 (0·002)	0·48 (0·08)	3·67 (0·63)	95·2*	..
6. Changes between the *fourth quarter* of each year only	7	0·005 (0·003)	0·71 (0·13)	3·93 (0·95)	92·0*	..

[a] The Durbin-Watson statistic as developed by Durbin and Watson (1951).
[b] For this value of d the Durbin-Watson test is inconclusive.
[c] Autocorrelation not detectable at the 1 per cent. probability level.
** Significantly different from zero at the 1 per cent. probability level, and
* Significantly different from zero at the 5 per cent. probability level.

estimates of the price and demand coefficients are not materially affected; and though their standard errors are more than doubled, the estimates differ significantly from zero.

3.12. Further to test these conclusions, the sample observations were divided into four groups, the first containing changes between first quarters in each year, the second between second quarters, etc.; and regression equations were fitted to each group separately. In this way successive error terms in a given group may be considered to be mutually exclusive and hence independent. Taking due account of sampling errors, none of the estimates of the price and demand coefficients in Table 5 is significantly different (at the 5 per cent. level) from another, nor from the original estimates. Each of the correlations differs significantly from zero. The general conclusion is that the estimates of the constant

* A one point change in percentage excess demand corresponds to a change of about one point in percentage unemployment (see charts in Dow and Dicks-Mireaux, (1958)).

and regression coefficients appear to be unaffected by the autocorrelated residuals. The possibility of bias arising out of "feedback" is discussed in Section 5.

3.13. It is difficult to test for error introduced by our assumption that settlements are made at regular twelve-month intervals (cf. 3.2). Since 1951, the *average* frequency of settlements has been considerably shorter than for the years 1946–50 (this can be deduced from Table 3); and, as already noted, some large settlements have occasionally been postponed well beyond a twelve-month interval. But it seems plausible to suppose that the wage-change over a two-year span, for example, will be subject to the same determining factors, whether, in fact, the change occurs in one or two jumps. In this case our assumption would not introduce systematic error into the estimates of the price and demand effects, though high residuals (between actual and calculated wage-changes) might be expected whenever the interval between settlements contracted or expanded. Examination of the

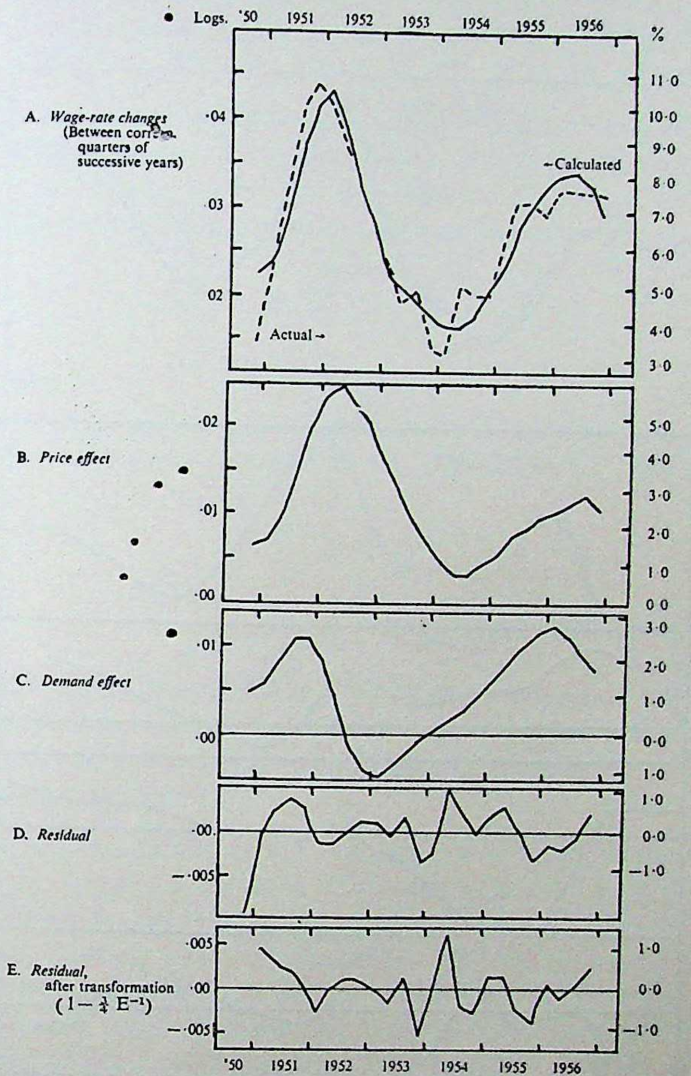

FIG. 2.—Explanation of wage-rate changes, 1950.IV–1956.IV

residuals for 1953.III, 1953.IV and 1954.I suggests that they were affected by the shift that then occurred of the important engineering settlement from the autumn to the spring of each year. This shift accounts for most of the variation over the period 1950–56 in the average interval between settlements (as indicated by the numbers of workers obtaining awards each year: Table 3).

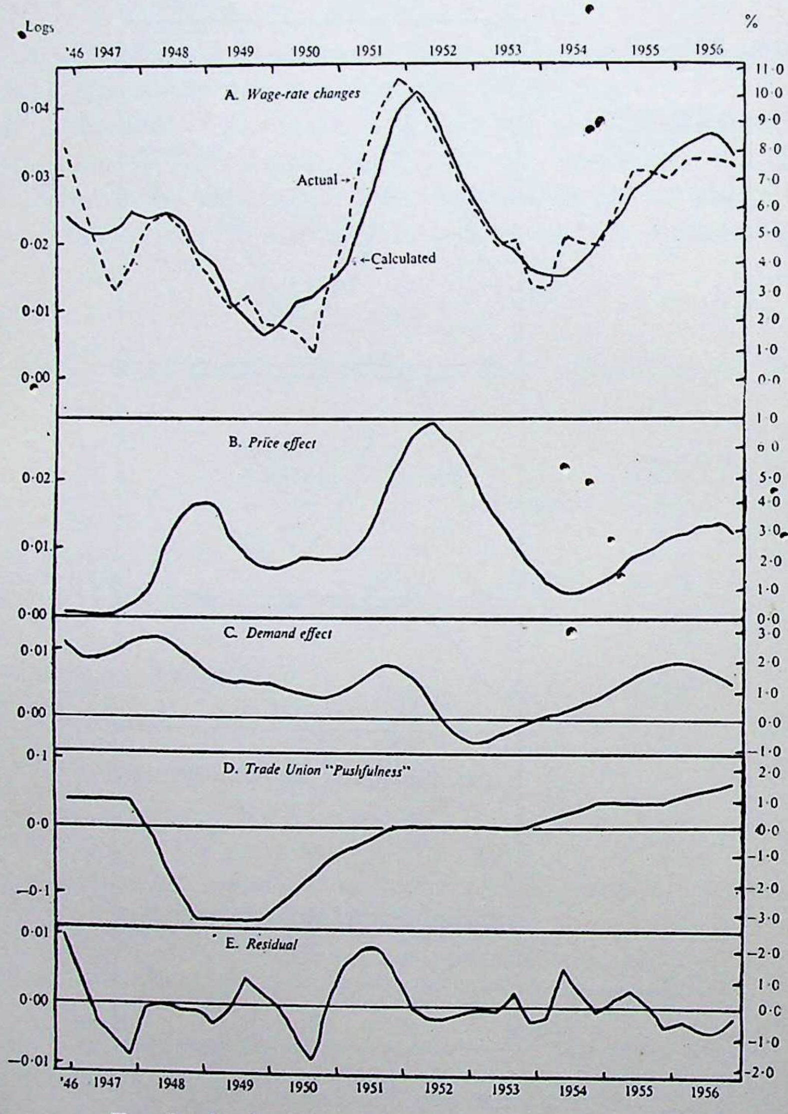

FIG. 3.—Explanation of wage-rate changes, 1946–1956

3.14. The results, set out graphically in Fig. 2, thus seem to show that both price-changes and the level of demand are important influences on wage-rates. After elimination of autocorrelation the percentage of variation explained is:

	Explanatory Variables Used		
Sample Period	*Price-changes Alone*	*Level of Demand Alone*	*Both Price-changes and Demand*
1950.*IV*–1956.*IV* .	31·3	35·7	62·5

We have so far succeeded in accounting for two-thirds of the variation in the rate of wage-change.

(c) *Variants of the Basic Hypothesis*

3.15. In this section we consider the effect of varying the hypothesis in different ways; and of introducing certain additional explanatory variables which might be supposed to affect wages.

3.16. At the outset of this enquiry we started by assuming that since trade unions are likely to demand full compensation for price-increases, the price coefficient could be expected to be about unity. It may be worth reporting the result of testing this hypothesis. In terms of equation (2) the assumption may be expressed formally as:

$$\frac{W_t}{\bar{P}_t} = B \,.\, \bar{d}_t^{\beta'} \,.\, v_t \qquad (3)$$

where B is some constant.

Correlating the composite variable on the left-hand side of equation (3) with the demand variable, the best fit was obtained for the sample period 1950–56, the estimated equation being:

$$\frac{W_t}{\bar{P}_t} = \bar{d}_t^{3.92}$$

$$B = 1{\cdot}000 \qquad R^2 = 0{\cdot}44.$$

The residuals showed a marked correlation (—0·90) with changes in the distributed price index, clearly suggesting that the price coefficient was indeed more nearly one-half. We are, as a result, driven back to the previous formulation of the model. The stability of the demand coefficient is noteworthy.

3.17. We have already shown in 3.1 that discontinuity of wage-changes makes it appropriate to explain changes in an index of *average* wage-rates in terms of the changes in a four-quarter moving average of prices, i.e. with a distributed lag of mean length six months between price-changes and wage-changes. But there are reasons (Peacock and Ryan, 1953) to believe that additional delays occur between the presentation and the settlement of wage claims; and the analysis of wage-changes in engineering (2.4) suggests that a lag of the order of three months might well be possible. We therefore repeated the analysis introducing into equation (2) additional monthly lags of up to nine months in the price variable and additional quarterly lags of up to two quarters in the demand variable.

3.18. In all cases the introduction of additional lags worsens the explanation (R^2 in Table 6). This suggests that additional lags are small; but as before (2.4) it does not conclusively rule out a combination of distributed lags. The estimate of the price coefficient is hardly affected by the introduction of lags in the price variable except when demand is lagged six months, but is reduced by the introduction of lags in the demand variable. The demand coefficient on the other hand tends to increase with the introduction of lags in the price variable; but to be slightly reduced by lags in the demand variable. These results inevitably throw doubt on the precise values to be assigned to the parameters.

TABLE 6

Wage-rates in All Industries: Effect of Additional Time Lags

$$W_t = A \cdot (\bar{P}_{t-\theta})^{\alpha} \cdot (\bar{d}_{t-\pi})^{\beta} \cdot \nu_t$$

Time lag (θ) for price effect (months)	Price Effect (α)			Demand Effect (β)			Percentage of Variation Explained (100 R^2)		
	Time lag (π) for demand effect (months)								
	0	3	6	0	3	6	0	3	6
0 .	0·52 (0·05)	0·39 (0·06)	0·31 (0·09)	3·64 (0·40)	3·46 (0·51)	3·18 (0·74)	89·3	83·3	72·4
1 .	0·51 (0·05)	N.C.	N.C.	4·07 (0·45)	N.C.	N.C.	87·1	N.C.	N.C.
2 .	0·51 (0·06)	N.C.	N.C.	4·55 (0·49)	N.C.	N.C.	85·5	N.C.	N.C.
3 .	0·53 (0·06)	0·33 (0·07)	0·18 (0·09)	5·04 (0·54)	4·38 (0·60)	4·03 (0·78)	83·7	75·8	62·9
6 .	0·58 (0·10)	0·30 (0·09)	0·10 (0·09)	6·56 (0·82)	5·20 (0·72)	4·45 (0·80)	74·9	70·3	58·6
9 .	0·62 (0·16)	0·33 (0·11)	0·08 (0·10)	7·73 (1·39)	6·06 (0·93)	4·66 (0·89)	60·1	67·3	57·5

N.C. = Not calculated.
The sample period with no lags in either variable ($\theta = 0$, $\pi = 0$) was 1950.IV to 1956.IV. As lags were introduced the same number of observations was maintained by extending the sample period backwards. No adjustment was made to eliminate autocorrelation.

3.19. Although most economists are agreed that the level of demand must have some influence on the rate of wage-change, there is wide disagreement about how critical this influence is. The results so far suggest that a relatively small change in the demand for labour bears strongly on the rate of wage-change. This result is worth scrutinizing since our analysis is designed to throw light precisely on this question.

3.20. One possibility which seemed worth considering was that while the level of demand might affect the strategy of trade unions in the *timing* of their wage claims, it would have less effect on the ultimate outcome. Even if unions postponed claims when circumstances were unfavourable, they might make good their claims in later years. The previous equations attempted to account for wage-changes over twelve-month periods. But a year is a relatively short period; and the high value of the demand coefficient might reflect the influence of demand on union tactics rather than a longer-run influence.

3.21. The original model (equation (2)) was therefore recast in terms of wage-changes over successive *two-year intervals*. The dependent variable now became the percentage change in wage-rates in a given quarter compared, not with the same quarter in the previous year but in the year previous to that. Correspondingly longer periods were taken in calculating the explanatory variables, i.e. the distributed indices of price-changes and of the level of demand. In terms of equation (2) the new model may be written as:

$$W_{t-4} \cdot W_t = A\,(\bar{P}_{t-4} \cdot \bar{P}_t)^{\alpha} \cdot (\bar{d}_{t-4} \cdot \bar{d}_t)^{\beta} \cdot \nu_t. \tag{4}$$

Wage changes over intervals longer than two years were not considered, as autocorrelation in the error term (ν) would have increased prohibitively. Equation (4) was fitted to data

for 1950–56 but, as might be expected, positively autocorrelated residuals were obtained. However, it was found that transformation by the operator $(1 - \frac{7}{8}E^{-1})$ left the residuals uncorrelated. The multiple correlation coefficient obtained after this transformation is significantly different from zero (although less than that obtained for twelve-month changes). The estimates of both the price and demand coefficients, α and β, are much as equation (2) (Table 7). This test fails, therefore, to provide evidence that the apparent influence of demand stems in part from its bearing on the *tactics* of negotiations.

TABLE 7

Estimated Parameters from One- and Two-year Changes in Wage-rates

Period 1950.*IV to* 1956.*IV*

Dependent Variable	*Trans-formation to Eliminate Auto-correlation*	*Constant Term (log A)*	*Price Coefficient* (α)	*Demand Coefficient* (β)	*Multiple Correlation Coefficient* (R^2)	*Durbin-Watson* (d)
Changes in wage-rates over 12-month intervals	$(1 - \frac{3}{4}E^{-1})$	0·003 (0·001)	0·44 (0·05)	3·77 (0·90)	0·63	1·8
Changes in wage-rates over 24-month intervals	$(1 - \frac{7}{8}E^{-1})$	0·003 (0·001)	0·58 (0·14)	3·78 (1·28)	0·56	2·1

All estimates are significantly different from zero at the 1 per cent. probability level. The Durbin, Watson test shows no evidence of autocorrelation in the residuals at the 1 per cent. probability level.

TABLE 8

The Effect of Additional Explanatory Variables

Additional Explanatory Variable	*Coefficient of the Explanatory Variable Added* (γ)	*Price Coefficient* (α)	*Demand Coefficient* (β)	*Percentage of Variation Explained* (100 R^2)	*Durbin-Watson Statistic* (d)
Sample Period 1950.*IV to* 1956.*IV*					
1. Original equation (2)	—	0·52 (0·05)	3·64 (0·40)	89·3	1·25†
Transformed: $(1 - \frac{3}{4}E^{-1})$	—	0·44 (0·11)	3·77 (0·90)	62·5	1·77*
2. Wage change lagged 12 months (equation 5)	−0·02 (0·08)	0·53 (0·06)	3·55 (0·53)	89·3	N.C.
3. Time trend (equation 6)	0·003 (0·006)	0·53 (0·05)	3·59 (0·42)	89·4	1·26†
4. Rating of trade union "push" (equation 7)	0·007 (0·006)	0·53 (0·05)	3·47 (0·43)	89·8	1·28†
Transformed: $(1 - \frac{3}{4}E^{-1})$	0·006 (0·02)	0·44 (0·12)	3·66 (1·03)	62·6	1·76*
Sample Period 1946.*IV to* 1956.*IV*					
5. Original equation (2)	—	0·54 (0·11)	2·62 (0·84)	41·8	0·25
Transformed: $(1 - \frac{3}{4}E^{-1})$	—	0·51 (0·13)	3·26 (1·05)	41·4	1·07
6. Time trend (equation 6)	0·009 (0·005)	0·47 (0·11)	3·43 (0·92)	46·8	0·27
7. Rating of trade union "push" (equation 7)	0·030 (0·003)	0·60 (0·06)	2·63 (0·46)	82·6	0·78
Transformed: $(1 - \frac{3}{4}E^{-1})$	0·024 (0·007	0·57 (0·11)	3·10 (0·93)	55·4	1·40†

* Indicates that the test showed no autocorrelation at the 1 per cent. probability level. † Indicates that the test was inconclusive. For all other values the term showed positive autocorrelation. N.C. indicates not calculated.

3.22. We also considered, as an alternative form of the above hypothesis, the possibility that a small wage settlement in one year would lead to dissatisfaction and to a larger settlement in the year after. The wage-change twelve months previous was therefore introduced as an additional explanatory variable in equation (2). The equation thus becomes:

$$W_t = A \,.\, \overline{P}_t^{\alpha} \,.\, \bar{d}_t^{\beta} \,.\, W_{t-4}^{\gamma} \cdot \nu_t. \tag{5}$$

The estimated coefficient of this variable is very small and offers no support to the hypothesis tested (Table 8, top).

3.23. A further question investigated was whether trade unions exerted an *independent* influence upon wage settlements. To explain wage-changes as an effect of price-changes and of the level of demand is to explain the reactions of negotiators in terms of changes in their objective environment. But the "pushfulness" of trade unions is often considered an independent force in wage determination. This factor, if important, might be embodied in the constant term estimated in earlier equations. To determine the scale of its effect requires that there should be variations in the degree of "pushfulness" and that these should be measurable.

3.24. It could be argued that in the course of continuous inflation from 1946 to 1956 trade unions have progressively realized their power to secure repeated wage-increases, and have become less and less wary of using it. We therefore inserted a time trend variable into equation (2) and fitted this equation to the data for the periods 1946–56 and 1950–56:

$$W_t = A \,.\, \overline{P}_t^{\alpha} \,.\, \bar{d}_t^{\beta} \,.\, \bar{t}^{\gamma} \,.\, \nu_t \tag{6}$$

where

$$\bar{t} = \{(t-3)\,(t-2)\,(t-1)t\}^{\frac{1}{4}}$$

and t is measured at quarterly intervals numbered from the first quarter of 1946. This new variable does not contribute greatly to the explanation in either period and it is clear that for the shorter of the two, 1950–56, it may be neglected (see Table 8). Over the period 1946–56 the coefficient of the time trend is sizeable, i.e. it suggests that towards the end of the period some 4 points in the annual percentage wage-rate change is the result of a slowly increasing rate of change. The previous coefficients (equation (2)) of price-change and demand are changed by the introduction of this time variable: in particular the demand coefficient now resembles that obtained for the period 1950–56.

3.25. For a more refined test of the hypothesis, we attempted to construct our own rating of the degree of trade union "pushfulness" during the years 1946–56. This rating is not an attempt to measure the success of trade unions in obtaining wage-increases, for

TABLE 9

Rating of Trade Union "Pushfulness"

5-point Scale

Ratings	1946	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957
1. "Marked restraint" .	—	—	×	×	—	—	—	—	—	—	—	—
2. "Some restraint" .	—	—	—	—	×	—	—	—	—	—	—	—
3. "Neutral" . .	—	—	—	—	—	×	×	×	—	—	—	—
4. "Some pushfulness" .	×	×	—	—	—	—	—	—	×	×	—	—
5. "Marked pushfulness"	—	—	—	—	—	—	—	—	—	—	×	×

A quarterly series was obtained by interpolation from the above ratings.

(as our previous results suggest) this must have been strongly influenced also by the rate of price-change and the level of demand. Rather it is intended to measure changes in their attitude additional to their reaction to such objective changes. The ratings shown in Table 9 suppose that in the war period 1939–45, trade unions were, for patriotic reasons, fairly restrained in their demands; that this disappeared in the first years of peace, partly because the overtime earnings declined; that appeals for wage restraint were effective in the years 1948–50; and that in subsequent years restraint gradually gave place to a positive pushfulness. We have tried to check these ratings against others' impressions; and they seem to concur with the remarks of Ross (1958): but they remain subjective. With this additional variable equation (2) may now be written:

$$W_t = A \cdot \bar{P}_t^{\alpha} \cdot \bar{d}_t^{\beta} \cdot \bar{f}_t^{\gamma} \cdot v_t \tag{7}$$

where f_t = our rating of trade union "pushfulness" at time t measured in quarters,

and $\bar{f}_t = (f_{t-3} \cdot f_{t-2} \cdot f_{t-1} \cdot f_t)^{\frac{1}{4}}$.

3.26. Adding a variable for "trade union attitude" removed the need to exclude the period of wage restraint. The equation was therefore first fitted to the whole period 1946–56 (Fig. 3, p. 156). The addition of "pushfulness" as a variable resulted in a considerable apparent gain in the explained variation of wage-rate changes. The price and demand coefficients were little different from those obtained from equation (2) (see Table 8, bottom). Unfortunately, however, transformation of the variables by the method previously adopted did not remove autocorrelation in the residuals. The significance of the correlation can not therefore be statistically tested, although it seems plausible enough on general grounds.

3.27. Equation (7) i.e. including "pushfulness" as an explanatory variable, was also fitted to the shorter period 1950–56. In this case the transformation appears effective in removing autocorrelation, and the multiple correlation is significant at the 1 per cent. probability level. The contribution of the new variable is here, however, negligible and the estimate of its coefficient (γ) does not differ significantly from zero (Table 8, top). The values of the other coefficients are little changed from previous results for this period.

3.28. Any contribution to the explanation of wage changes which our variable for "pushfulness" could be shown to have thus chiefly reflects the "wage restraint" experience of 1948–50. There are good grounds for believing that wage restraint did affect the course of wage-rates in this period, so that our estimates of its effect may have some interest notwithstanding the inconclusive result of the tests noted above (3.26). On the basis of the estimates it would seem that trade union attitude may have affected the rate of wage-change within a range of about 5 per cent.

Rating of "Pushfulness"	*Effect on Annual Wage-change*
1. Marked restraint . . .	$-3\frac{1}{4}$
2. Some restraint . . .	$-1\frac{1}{4}$
3. Neutral	0
4. Some "pushfulness" . .	1
5. Marked "pushfulness" . .	$1\frac{1}{2}$

Little significance can be attached to the non-linearity of the response, which results from the manner in which the variable was inserted into the equation. It will be noted (Table 8,

bottom) that neither the price coefficient (now 0·60) nor the demand coefficient (now 2·6) appear to be much affected by the introduction of this new variable.

4. WAGE-CHANGES IN MAJOR INDUSTRY GROUPS

4.1. One reason for the imperfect success of the aggregative model in describing the behaviour of average wage-rates might be that the various influences affect wages differently in different industries. The same method of analysis was therefore applied to wage changes in separate industries. Indices of the demand for labour are available (Dow and Dicks-Mireaux, 1958) for seven major industry-groups. Since we wished to test among other things the influence of "local" demand for labour (as distinct from the average demand for labour in all industries) the analysis was confined to these seven industry-groups (defined as in Table 10). The grouping is still broad, and further refinement would

TABLE 10

Description of Major Industry-groups and Correlations Between Variables

Period: 1950.*IV to* 1956.*IV*

Industry Group (*Short Title in Italics*)	S.I.C. Orders	Weight in Total	*Correlation Coefficients*: Wage-rates and Average Wage-rates	Wage-rates and Prices	Wage-rates and Average Demand	Wage-rates and Local Demand	Local and Average Demand	Prices and Local Demand
1. *Metals*, engineering and vehicles	V–IX	291	0·89	0·63	0·49	0·75	0·85	0·38
2. *Textiles*, leather and clothing	X–XII	79	0·65	0·25	0·52	0·39	0·64	−0·68
3. *Other manufacturing* .	III, IV, XIII–XVI	128	0·95	0·81	0·45	0·49	0·96	−0·02
4. *Mining*, quarrying and public utilities	II and XVIII	98	0·73	0·41	0·65	0·59	0·64	0·59
5. *Building* and contracting	XVII	97	0·89	0·73	0·35	0·62	0·87	0·36
6. *Agriculture*, forestry and fishing	I	54	0·66	0·60	0·27	0·50	0·53	0·39
7. *Services* . . .	XIX–XXIV	253	0·89	0·66	0·43	0·53	0·94	0·06
All industries . .	I–XXIV	1,000		0·70	0·58	0·58[a]		−0·07[a]

[a] Correlation with *average* demand.

be necessary to evaluate the different forces affecting wages in different industries, or to explain changes in relative wages.

4.2. Data only for the period 1950–56 were used in this analysis. Over this period average wage-rates increased by 47 per cent., the highest and lowest increases shown by the separate industry-groups being 52 per cent. and 43 per cent. respectively. The rate of wage-change was measured, as before, as the percentage increase between one quarter and the same quarter of the previous year, so that there are 25 observations (1950.IV–1956.IV) for each industry group.

4.3. A first indication of the forces at work is provided by the simple correlation coefficients set out in Table 10. Since the rate of wage-change in different industries was fairly highly correlated, there is limited scope only for separate explanations. In every

industry group, wage-change is positively correlated with the average level of demand. But here again there are low values (*Building* and *Agriculture*); and in these, as in two other cases (*Metals* and *Services*), there is a stronger relation with the "local" level of demand. The scope for distinguishing between the separate effects of average and "local" demand is, however, greatly restricted by the correlation between these two variables, which even where lowest (*Textiles*, *Mining*, and *Agriculture*) is still appreciable. Moreover in two cases (*Textiles* and *Mining*) there is appreciable correlation between price-change and the local level of demand. Too much should therefore not be hoped for from the attempt to explain separate wage-changes in the broad industry-groups.

4.4. The equations fitted were of the same form as for the all-industries analysis. Three variants were tested, the explanatory variables being respectively:

(i) rate of price-change; and *average* (i.e. all-industry) demand for labour;

(ii) rate of price-change; and *local* demand for labour;

(iii) rate of price-change; and *both average and local* demand for labour.

Thus formally the equations may be described for (i) as equation (2); for (ii) as equation (2) with the substitution of the index of local demand for overall demand; and for (iii) as equation (2) incorporating local demand as an additional explanatory variable, i.e.

$$W_t = A \, . \, \bar{P}_t^{\alpha} \, . \, \bar{d}_t^{\beta} \, . \, \bar{l}_t^{\gamma} \, . \, \nu_t \tag{8}$$

where l_t = the index of local demand for labour at time t and $\bar{l}_t = (l_{t-3} \, . \, l_{t-2} \, . \, l_{t-1} \, . \, l_t)^{\frac{1}{4}}$.

4.5. The assumption that wage settlements are reached evenly through the year (cf. 3.2) is even less justifiable for separate industry-groups than for all industries taken together. A rough indication of how the facts diverge from this assumption is provided by a comparison of the rate of wage-change in each of the four quarters of the year (Table 11).

TABLE 11

Wage-changes in Major Industry-groups: Average Changes Between Successive Quarters 1950–56

Percentages

Industry-group	*First Quarter*	*Second Quarter*	*Third Quarter*	*Fourth Quarter*
1. *Metals*	2·3	2·5	0·3	1·4
2. *Textiles*	1·4	1·9	1·5	1·4
3. *Other manufacturing*	1·9	1·8	2·0	1·1
4. *Mining*	2·6	2·6	0·4	1·1
5. *Building*	2·8	4·1	0·2	0·0
6. *Agriculture*	2·4	0·4	1·2	2·2
7. *Services*	1·8	1·4	1·2	1·9
All industries	2·3	1·9	1·0	1·3

Changes are dated by the final quarter: thus under "first quarter" are shown changes from the fourth to the first quarter.

Only in *Textiles*, *Other Manufacturing* and *Services* were wage settlements fairly evenly spread throughout the year. In other industries the distribution was uneven, with a clear tendency to bunch in two adjacent quarters of the year. As already argued in 3.13 the effect should be, not to distort the estimates of the parameters, but to reduce the goodness-of-fit. In fact, the fit appears definitely poor only in *Agriculture* (Table 12).

TABLE 12

Wage-changes in Major Industry-groups: Percentage of Variation Explained (100 R^2) *on Alternative Hypotheses*

Period: 1950.*IV to* 1956.*IV*

	Explanatory Variables Employed		
	Price-change and Average Demand (*Equation* 2)	*Price-change and Local Demand* (*Equation* 2′)	*Price-change, Average Demand and Local Demand* (*Equation* 8)
1. *Metals*	69·1	70·5	70·1
2. *Textiles*	35·9	65·2	66·6
3. *Other manufacturing*	91·3	90·8	91·7
4. *Mining*	63·1	35·5	69·8
5. *Building*	69·6	68·5	69·6
6. *Agriculture*	45·9	44·6	47·6
7. *Services*	66·1	67·0	67·3
All industries (from Table 4)	89·3	—	—

No correction has been made to remove induced correlation in the residuals. If we assume that residuals are serially independent, all results are significantly different from zero at the 5 per cent. probability level.

4.6. As might be expected the explanation of wage-changes in most of the separate industry-groups (Table 12) is inferior to that for the all-industries model. Since elimination of autocorrelation in the residuals was not attempted, no strict test of the significance of the results is available. In the four cases where average and local demand are themselves highly correlated (groups 1, 3, 5 and 7: see Table 10), the substitution of local for average demand makes little difference to the degree of explanation obtained and little is gained by the inclusion of both. This does not enable us to reject the hypothesis that local demand was an important influence. In the remaining three industries, *Textile* wages seem to have been more influenced by local than general demand; and *Mining* wages more by general demand; while agricultural wages are explained well neither by either variable taken singly nor by both in combination.

TABLE 13

Wage-changes in Major Industry-groups: Estimates of Demand Coefficients (β)

Period: 1950.*IV to* 1956.*IV*

	Explanatory Variables Employed			
			Price-change and Both Average and Local Demand (*Equation* 8)	
	Price-change and Average Demand (*Equation* 2)	*Price-change and Local Demand* (*Equation* 2′)	*Local Demand*	*Average Demand*
1. *Metals*	3·51	4·94	*3·64*	*1·00*
2. *Textiles*	3·45	1·68	2·02	*−1·26*
3. *Other manufacturing*	4·38	4·18	*1·66*	*2·74*
4. *Mining*	6·55	11·52	−12·62	10·42
5. *Building*	2·29	2·01	*0·06*	*2·23*
6. *Agriculture*	2·12	*3·27*	*1·80*	*1·46*
7. *Services*	2·57	2·34	*1·59*	*0·89*
All industries (from Table 4)	3·64	—	—	3·64

No standard errors are given but those estimates not significantly different from zero are shown in italics. See also the note to Table 12.

4.7. The estimates of the demand coefficients in Table 13 show less than they seem because the standard errors are high, especially when both demand variables were employed. The "average demand" coefficients do not differ significantly among themselves, or from that previously estimated for all industries. The values for the "local demand" coefficients for *Mining* and *Agriculture* are especially unreliable. Apart from them, the values are little different from those estimated for the "average demand" coefficient in that industry, though that for textiles (to which some special interest attaches: see 4.6) is somewhat lower.

4.8. Estimates of the price coefficients are shown in Table 14. In *Textiles* (if one rejects the first estimate) and in *Other Manufacturing* the coefficient is about 0·8. In other industries it is about one half, as estimated previously.

TABLE 14

Wage-changes in Major Industry-groups: Estimates of Price Coefficients (α)

Period: 1950.*IV to* 1950.*IV*

	Explanatory Variables Employed		
	Price-change and Average Demand (Equation 2)	*Price-change and Local Demand (Equation 2′)*	*Price-change, Average Demand and Local Demand (Equation 8)*
1. *Metals*	0·53	0·33	0·38
2. *Textiles*	*0·22*	0·74	0·84
3. *Other manufacturing*	0·87	0·85	0·86
4. *Mining*	0·53	*0·10*	0·97
5. *Building*	0·53	0·40	0·53
6. *Agriculture*	0·50	0·39	0·44
7. *Services*	0·45	0·40	0·42
All industries (from Table 4)	0·52	—	0·52

Estimates not significantly different from zero are shown in italics.

4.9. Each of the equations provides for a constant term (Table 15). The estimates are little different from that obtained in the all-industries model, which implies a constant annual rate of increase in wages of between 2 per cent. and 3½ per cent. In *Other Manufacturing* the constant is negligilbe.

TABLE 15

Wage-changes in Major Industry-groups: Estimates of Constants (*log A*)

Period: 1950.*IV to* 1956.*IV*

	Explanatory Variables Employed		
	Price-change and Average Demand (Equation 2)	*Price-change and Local Demand (Equation 2′)*	*Price-change, Average Demand and Local Demand (Equation 8)*
1. *Metals*	0·012	0·017	0·015
2. *Textiles*	0·015	0·011	*0·011*
3. *Other manufacturing*	0·003	0·003	0·003
4. *Mining*	0·008	0·012	*0·009*
5. *Building*	0·014	0·020	0·014
6. *Agriculture*	0·012	0·012	*0·011*
7. *Services*	0·012	0·011	0·012
All industries (from Table 4)	0·011	—	—

Estimates not significantly different from zero are shown in italics.

4.10. The results for *Metals* and *Building* compare very satisfactorily with those previously obtained by the more refined methods employed in Section 2. The only important difference is that the price coefficient in *Building* now seems to be about 0·5 instead of 0·2 as earlier estimated—a result much more in accord with what would be expected from the cost-of-living sliding scale in *Building* (cf. 2.5).

4.11. Two main conclusions may be drawn from this study of wages in separate industry-groups. First the results may throw some light on the kind of demand influence at work. There is considerable correlation between wage-changes in different industries; and also between the demand for labour in different industries (Table 10). But this is not complete. In three industries (*Textiles*, *Mining* and *Agriculture*) wage-changes have been rather different from the national average, and also demand has moved rather differently. But in only *Textiles* does local demand definitely help to explain wage-changes. Though the evidence is very incomplete, this might suggest that the level of demand affected the general "climate" of wage-negotiations, and that the economic situation of different industries was of much less importance. Alternatively, the effect could be explained by supposing that one or more industries were "wage-leaders". Engineering wages for instance can be explained as well by local as by general demand (Table 12); and the general level of wage-rates is well correlated with those in engineering (Table 10), perhaps because other industries follow the engineering workers' lead. Neither of these explanations is strictly in terms of "economic forces".

4.12. The second conclusion is that explanation of *average* wage-rates in terms of overall demand seems a fairly adequate procedure as a first approximation, so that the estimates of overall price and demand coefficients (Section 3) are probably unaffected by the neglect of the difference between industries. Nevertheless, room for improvement remains: since there is some evidence that values vary between industries, a more elaborate, less aggregative, approach would probably improve the fit.

5. DISCUSSION OF RESULTS

5.1. We now discuss the statistical and economic significance of the previous results, which are first briefly summarized.

(a) *Summary of Main Results*

5.2. (i) Movements of the index of the average wage-rates may be reasonably well explained in terms of price-changes and the pressure of demand for labour. Though any simple explanation of changes in average wage-rates is bound to be incomplete, these two factors alone appear to explain 89 per cent. of the variation in the rate of change of wage-rates in the period 1950–56 (Table 8).

(ii) The price coefficient appears well below unity and probably of the order of 0·5.

(iii) Where the explanation is in terms of the above two variables, the estimated demand coefficient is of the order of 3·5, i.e. a change of a 1 per cent. point in excess demand (or, roughly, of unemployment) is associated with an increase or decrease in the percentage rate of wage-rate change of 3·5 per cent.

(iv) In all cases there is a constant term which implies that with zero price-change and zero excess demand, wage-rates will rise by 2–2½ per cent. a year.

(v) The results seem compatible with the belief that the attitude of trade unions has a considerable effect on the rate of wage-change, i.e. with zero price-change and zero excess demand the rate of wage-change might be −1 per cent., if unions were as "restrained" as in 1948–50, or 3½ per cent., if they were as "pushful" as in recent years.

(vi) In general these results are confirmed by separate analysis of wage-changes in industry-groups. Only in one industry does the incomplete evidence show that "local" demand conditions were as important as the demand for labour in the country as a whole.

(b) *Identification of the Causal Relationships*

5.3. We must now deal with a question hitherto deferred, namely whether the above observed statistical associations can be taken as evidence of the causal relationships posited by our hypothesis. Price-changes may cause wage-changes, but these in turn cause further price-changes; and the question is whether these two effects can be disentangled and separately estimated. We rely on two arguments for believing that this is possible.

5.4. Separate identification would in the first place be possible if there were sufficient time lag between cause and effect and further effect. There are three elements in the total lag, in all considerable. We have already argued that to explain changes in *average* wage-rates it is appropriate to use a *moving average* of prices as an explanatory variable. Since the workers in a given group are likely to obtain an award only once a year, on the average they are likely to wait about six months before obtaining any award. There is therefore a distributed lag with a mean length of about six months built into the system. The immediate effect on wage costs of any rise in prices will be small, because only a fraction of workers is immediately due for a wage increase, and the effect will only begin to become important after several months have passed. Second, there are in addition some institutional delays. Our analysis, not surprisingly, has been inconclusive about their length. But the reasons given by Peacock and Ryan (1953) suggest that they are of the order of a month or so. The total price-wage lag arising both through institutional delays and through the discontinuity of wage settlements may thus be reckoned to be over six months.

5.5. There is also reason to believe that there is a delay between an increase in wage cost and an increase in prices. The matter is not well documented. Rough estimates can be made of the period of through-put, i.e. the average time taken for input to emerge as output. This period is equivalent to the ratio which stocks and work in progress bear to the flow of input: but it varies according to the type of input. One calculation by Parkinson (1955), from the known values of stocks and input, suggests that for imported materials it is of the order of 6–12 months. Another approach is to estimate both the historical and replacement costs of industry as a whole; comparison of these series for the post-war years suggests a period of throughput of this length (Dow, 1956, paragraph 29 and Chart 1). Many economic processes can be conceived as an initial purchase of material input, and then *continuous* processing until the output is sold: the average period of throughput for labour input might therefore be shorter than for imported materials (perhaps 3–6 months). The period of physical throughput need not be the same as the cost-price lag, i.e. prices may vary more with replacement costs than with historical costs (Dow, 1956, paragraphs 59–65 and Table IX). More enquiry is needed, particularly of cost and price relationships in different industries, to put the matter on a firm basis. But

at a guess the average delay between changes in wage-costs and changes in prices might be put at about three months. The time taken for an increase in prices to affect wage-costs appreciably and for this to be subsequently reflected in a further increase in prices, though of the same order, is probably therefore less than the twelve-month intervals in terms of which this analysis has generally been conducted.

5.6. This being so, there is a general possibility of biased estimates. Wise (1956) has shown that in recursive systems of the type here in question, autocorrelation in the residuals can lead to substantial bias in the estimates if least square estimates are used. But the transformations we made to eliminate autocorrelation seem generally to have succeeded in so doing (Table 8 above), which suggests that the estimates obtained after such transformations should be unbiased.

5.7. A second reason, of a different order, for this belief is as follows. Over the period 1946–52, changes in import costs were almost as important a constituent of price-changes as were changes in labour costs; after 1952 this was not the case (Dow, 1956, particularly Table V). Most of the variation in the rate of price-change was therefore due to variation in the rate of rise of import costs (which were erratic), not that of labour costs (which increased more steadily). If therefore price-changes (over twelve-month intervals) contained an element of feed-back, this may not have greatly affected our results which are based on correlations not of price-changes but of changes in the rate of change of prices.

5.8. This argument, however, really requires to be tested further. Another way of putting this is to say that price-changes can be explained as a function of changes in import costs and of wage-changes; that wage-changes are a function of price-changes and demand; and that the separate effects may (in this period) be estimated by solving both equations simultaneously, the independent variables being import-cost changes and demand. An attempt to do so by Ball and Klein, in a paper to be published, yields in the wage equation estimates of the price and demand coefficients somewhat different from ours.

5.9. It has been suggested to us that our estimate of the demand effect may also contain an element of feed-back. We have treated wage-changes as a function of the level of demand for labour. But the latter depends on the demand for final goods and services, of which consumers' demand is an important part. If therefore wages increase in relation to prices the rise in real incomes is likely to cause an expansion in consumers' demand. This, however, is a reason for expecting an association not between the rate of wage-increase and the level of demand (which we examined), but between the rate of wage-increase and *changes* in the level of demand. If the process started at a low level, it would progressively raise the level of demand without immediately causing it to be high; and, as one would expect, there is no correlation between the level of demand and changes in the latter as measured by our index. This seems sufficient ground for rejecting the suggestion that a *demand-inducing* effect is confused in our estimates with the *demand-induced* effect on wages.

5.10. These general arguments do not imply that our estimates of the demand and price effects are necessarily the best that could be made. It will be remembered that we have assumed (3.4) that the wage increase obtained by any group of workers in any quarter is influenced by the pressure of demand ruling *in that quarter*. We might have assumed that it was influenced by demand conditions over a longer past period. As will be seen by inspection of Figs. 2 and 3, this would have introduced some correlation between the price

and demand variables, which in our formulation is negligible (−0·07); perhaps altered the estimates of their respective effect; and certainly increased their standard errors. Time has not permitted recalculation, but we do not think that any plausible reformulation on these lines would introduce such correlation between the explanatory variables as to destroy our method.

5.11. It may be worth noting that the kind of relation in time between the dependent and explanatory variables depicted in Figs. 2 and 3 throws some light on an important economic question, namely the point at which demand "impinges" on the economic system (cf. Dow, 1958, particularly Part 1). The lags indicated in the diagram suggest that wage-change is affected by *present* demand and that it is only after an interval that prices move in sympathy, i.e. that prices as such are less sensitive to demand. An hypothesis of this sort is a basic assumption of our method of treating price-change and demand as separate explanatory variables. But fully to establish its validity requires further work on the behaviour of other elements of prices that may be sensitive to demand (e.g. wage-drift, salaries, and profit margins) which this paper does not touch on.

(c) *Previous Studies of Wage-Determination*

5.12. It is convenient next to refer to the results of such previous studies as have attempted to give quantitative estimates of the factors affecting wage-changes. Some previous estimates of the price coefficient have been similar to ours; while previous estimates of the demand coefficient (relating, however, to periods when unemployment was much higher and demand thus lower than during the last ten years in this country) have shown a much lower value than ours.

5.13. In his statistical study of business cycles in the United States, Tinbergen (1939*b*) estimates a wage fixation equation for the period 1919–32. Hourly earnings are explained by (i) volume of production (a proxy for employment); (ii) cost of living; and (iii) a time trend. The price coefficient translated into our terms is between 0·4 and 0·5. The demand coefficient cannot be directly compared because Tinbergen here explains the level of wages by the level of production (or employment). His results indicate a lag of about five months between wages and the determinant variables, a result not unlike our own.

5.14. In his study of business cycles in the United Kingdom for the period 1871–1914, Tinbergen (1951) (particularly Table IV 1, page 51) tests nine alternative equations "explaining" wage-rates, the determinants being as in his previous study, except that change of wage-rates is now expressed as a function of employment. The estimated influence of prices on wage-rates is very small (the largest price coefficient is 0·126). Precise comparison with our estimated demand coefficient is not possible, but it is clear that the magnitude of our estimated demand effect far exceeds Tinbergen's estimates for the period 1871–1910. Tinbergen himself rejects both price and demand effects examined as being too small to be of importance. In one equation the cost of living appears unlagged and lagged by one year: from the estimated coefficients of each of these variables it is possible to deduce an approximate lag of five months between wage-rates and prices.

5.15. Both of the econometric models describing the economy of the United States with which Klein has been associated contain a wage-change equation. The first relates to the period 1920–41 (Klein, 1950, particularly page 121). The change in the annual average wage-earnings is treated as a function of unemployment, the level of previous wage-earnings and a time trend. The estimate of the influence of demand on the rate of

wage-change is much smaller than ours: if we adjust for units of measurement, the equation implies that an absolute fall in unemployment—i.e. an increase of demand—in the *current* year of 1 per cent. goes with an increase of $\frac{3}{4}$ per cent. in average wage-earnings. They suggest too that there is a delayed effect also, so that if unemployment had fallen 1 per cent. in the *previous* year this would raise wages by almost $\frac{1}{4}$ per cent. From these two estimated coefficients it would seem that the lag between wage-changes and unemployment is roughly three months. Price-changes are not included as an explanatory variable.

5.16. In the second of these models (Klein and Goldberger, 1955, page 52), price-change is included as one of the independent variables and a price-wage lag of one year is assumed. Apart from a time trend, the only other explanatory variable is unemployment in the current year. The sample period included not only 1920–41 but also 1946–50. Here again the estimated demand effect was small: an absolute fall in unemployment of 1 per cent. was associated with a rise in average wages of about $\frac{1}{2}$ per cent. The price effect was not materially different from ours.

5.17. Pre-war data for this country have also been examined by Brown (1955) who, however, confined himself to inspection of scatter diagrams for the periods 1881–1914 and 1920–51, rather than formal correlation. He does not discuss the size of the demand influence in our sense. But his diagrams suggest that the demand coefficient has declined over time, and that, again in contrast to our results, it was well below unity. An attempt has been made by Phillips (1958) to analyse the relation between unemployment and the rate of change in wage-rates since 1861. At the beginning of this period recorded unemployment was very low. His analysis obtains a markedly non-linear relationship: only at high levels of demand is his demand coefficient similar to ours.

(d) *Economic Interpretation of the Results*

5.18. In assessing the economic significance of our results, the limitations imposed by the form of the equations used in fitting the data must be kept in mind. We posited essentially linear relationships between the variables (2.2, 3.7), a procedure to be justified only as providing a first way into disentangling the main relationships, but not to be accepted uncritically.

5.19. The first question is whether our estimate of the price coefficient (i.e. about 0·5) should be taken to imply that workers ordinarily receive much less than full compensation for price increases. Nothing in our calculations can destroy the fact that wage-rates have gone up as fast as prices; but our formulation seems to imply that this was something of an accident. It has, however, been suggested to us that our estimate of the price coefficient seems more acceptable if it is considered along with our constant term, itself an accident of the calculation which requires to be given some sense in economic terms. The modal price-change in our period is about 4 per cent. per year, the inter-decile range being 2 to 8. Neglecting the effect of demand, the degree of price compensation becomes:

	Annual Percentage Changes		
Price change	2	5	8
Effect of price coefficient	1	$2\frac{1}{2}$	4
Effect of constant	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$
Wage change	$3\frac{1}{2}$	5	$6\frac{1}{2}$

It is perhaps not contrary to common impression to say that workers obtain rather less

than full compensation for price increases when prices rise more rapidly than usual, and rather more than full compensation when the price-rise is unusually small. This way of putting it, however, destroys the apparent simplicity of our hypothesis; and there must be other formulations, little more complicated, which are equally valid.

5.20. Our assumption that demand operates upon wage-change in a linear fashion also calls for scrutiny. We started off this study with the idea that so many things had changed since 1939 that only post-war experience was likely to throw light on current behaviour-patterns. This, however, means that we had a very limited range of experience to work with: unemployment, for instance, varied in our period only within a range of 0·8 and 2·2 per cent. For a description of post-war experience to date we think the linearity assumption adequate: somewhat different values would have been obtained with other assumptions, but the important results would have remained much as we have stated them.

5.21. This does not mean that our results can be extrapolated beyond the range of experience encountered. In particular it is almost certainly not valid to project them into the zone of under-full-employment. Previous studies already referred to suggest, as might indeed be expected, that in such conditions the effect of demand is altogether smaller in degree; and that if a general description is possible it will be found to be non-linear. Our results therefore provide no basis for calculating what level of demand would have been required to stop wages or prices rising; and indeed experience may always be insufficient to allow of this being done with great accuracy or confidence (see Fig. 4).

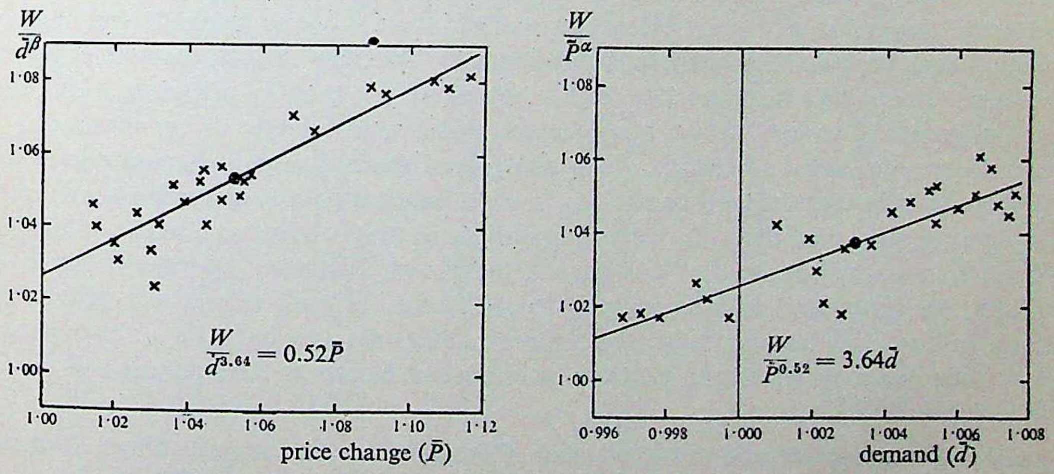

FIG. 4.—Wage-changes, price-changes and demand, 1950.IV–1956.IV

a) Wage-change (W) adjusted for demand (d) with price-change (P)

(*b*) Wage-change (W) adjusted for price-change (P) with demand (d)

5.22. What we think is the important result of our analysis is the conclusion that a level of demand much in excess of the (still high) minimum encountered in the period had a major effect on the rate of wage-change. Two remarks may be in order on the scale of this effect, which, as already made clear, turned out much higher than the present authors had anticipated. First, though the excess demand for *labour* has varied within very narrow limits in the post-war period, it may be that the excess demand for *goods and services* has undergone much greater variation (cf. Dow and Dicks-Mireaux, 1958). It may be therefore

that if the demand coefficient could have been measured in terms of final demand it would have been much smaller, i.e. a change in the rate of wage-change of 1 per cent. may, in the post-war period, have been associated with a variation in final demand, not of a fraction of 1 per cent., but of several per cent.

5.23. Second, it may be desirable to comment on the longer-run implications of our results. At first glance it might be thought that the large demand coefficient implied a continuous shift to wages during a demand-induced wage inflation, and vice versa in deflation; which would conflict with the observed rough stability of factor shares. This, however, is not the case. It is for instance quite possible, if lags are assumed, to construct a cost-price model embodying the assumption that percentage profits are *constant*: this implies that the share in national income both of profits and "other incomes" is constant also.

5.24. Full consideration of these questions would require more elaborate analysis of the working of cost-price models; and these would have to include elements not examined in this paper. It might be found that there were additional demand effects, e.g. on "wage-drift" and on profit margins; and perhaps also that there was some offsetting effect, stemming from an association between high demand and increases in output-per-head. An analysis of wage-earnings along lines similar to the present analysis of wage-rate changes is one obvious next step to which we hope to proceed.

5.25. There is one general question which the present study can only pose, not answer —namely what type of influence must we suppose the demand influence to be? Economists argue that "market forces" cause prices to rise when there is excess demand; but anything less rapid than an instantaneous rise is put down to "frictions" which themselves are not explained. Economics therefore has little to say about the *speed* of response, which must presumably depend on institutional arrangements and habits of mind: things not necessarily very uniform, and liable to change. Our analysis of wage-changes in separate industries may seem to hint that demand conditions in each industry do not determine the outcome in any precise way; and that it is demand conditions in the economy as a whole which have influenced, in a way not strictly "economic", the general "climate" of wage negotiations. Similarly, our experiment with treating the "pushfulness" of trade unions as a determining variable at least does not contradict the belief that attitudes may in fact have changed and that the changes may have been a factor as important as any in determining the pace of wage inflation.

5.26. These considerations suggest that if practical conclusions are drawn from this type of analysis, parameters such as we have estimated should not be applied mechanistically to any given situation. Apart from the margin of error attaching to any such estimates, an attempt, for instance, to utilize a demand effect might itself affect attitudes. This certainly leaves economic policy where it was, as much political as economic. Econometric analysis should help; but it can hardly hope to establish an exact connection between the means and ends of policy.

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Appendix

*Wage-rates, Retail Prices and Demand for Labour**

1946–58

Year and Quarter		Wage-rates: Index June 30th 1947 = 100	Wage-rates: Change Between Corresponding Quarters of Successive Years (Ratio)	Retail Prices: Index June 17th 1947 = 100	Retail Prices, Four-quarter Geometric Moving Average: Index June 17th 1947 = 100	Retail Prices, Four-quarter Geometric Moving Average: Change Between Corresponding Quarters of Successive Years (Ratio)	Demand: Index of Excess Demand for Labour	Demand: Four-quarter Geometric Moving Average
1946	I	93·6	1·071	100·0	100·5	—	1·0143	—
	II	95·7	1·075	100·3	100·5	—	1·0089	—
	III	97·7	1·083	101·3	100·4	—	1·0077	—
	IV	98·7	1·082	100·3	100·5	1·003	1·0080	1·0098
1947	I	99·3	1·061	100·7	100·7	1·002	1·0075	1·0080
	II	99·9	1·044	100·0	100·6	1·000	1·0080	1·0078
	III	100·7	1·031	100·7	100·4	1·000	1·0116	1·0088
	IV	102·7	1·041	102·7	101·0	1·005	1·0118	1·0098
1948	I	104·7	1·054	105·3	102·2	1·015	1·0102	1·0104
	II	105·7	1·058	108·7	104·3	1·037	1·0073	1·0102
	III	106·0	1·053	108·0	106·2	1·057	1·0055	1·0087
	IV	107·0	1·042	108·7	107·7	1·066	1·0044	1·0069
1949	I	108·0	1·032	109·0	108·6	1·063	1·0041	1·0053
	II	108·3	1·025	110·3	109·0	1·045	1·0044	1·0046
	III	109·0	1·028	111·3	109·8	1·035	1·0052	1·0045
	IV	109·0	1·019	112·3	110·7	1·028	1·0034	1·0043
1950	I	110·0	1·019	113·0	111·7	1·029	1·0027	1·0039
	II	110·0	1·016	114·0	112·7	1·034	1·0022	1·0034
	III	110·0	1·009	113·7	113·3	1·031	1·0027	1·0028
	IV	112·7	1·034	115·7	114·1	1·031	1·0036	1·0028
1951	I	116·0	1·055	118·0	115·3	1·032	1·0057	1·0036
	II	118·3	1·076	123·3	117·6	1·044	1·0086	1·0052
	III	120·7	1·097	127·0	120·9	1·068	1·0086	1·0066
	IV	124·7	1·107	129·3	124·3	1·089	1·0046	1·0069
1952	I	127·7	1·101	132·7	128·0	1·110	0·9999	1·0054
	II	129·0	1·091	136·0	131·2	1·116	0·9952	1·0021
	III	130·0	1·077	137·0	133·7	1·106	0·9966	0·9991
	IV	133·0	1·067	138·0	135·9	1·093	0·9975	0·9973
1953	I	134·7	1·055	139·0	137·5	1·074	0·9980	0·9968
	II	135·0	1·047	140·7	138·7	1·057	0·9991	0·9978
	III	136·3	1·049	140·3	139·5	1·043	1·0006	0·9988
	IV	137·3	1·032	140·0	140·0	1·030	1·0009	0·9997
1954	I	139·0	1·032	140·7	140·4	1·021	1·0004	1·0023
	II	141·7	1·050	141·7	140·7	1·014	1·0022	1·0010
	III	142·7	1·047	144·0	141·6	1·015	1·0041	1·0019
	IV	143·7	1·047	144·7	142·8	1·020	1·0048	1·0029
1955	I	147·3	1·060	146·0	144·1	1·026	1·0057	1·0042
	II	152·0	1·073	148·0	145·7	1·036	1·0070	1·0054
	III	153·0	1·072	149·7	147·1	1·039	1·0086	1·0065
	IV	153·7	1·070	153·3	149·2	1·045	1·0081	1·0074
1956	I	158·7	1·077	153·7	151·2	1·049	1·0068	1·0076
	II	163·7	1·077	157·3	153·5	1·054	1·0051	1·0071
	III	164·7	1·077	156·7	155·2	1·055	1·0038	1·0060
	IV	165·3	1·076	158·3	156·5	1·049	1·0029	1·0047
1957	I	166·9	1·052	160·0	158·1	1·046	0·9994	1·0028
	II	171·4	1·047	161·0	159·0	1·036	0·9996	1·0014
	III	173·8	1·055	163·2	160·6	1·035	1·0010	1·0007
	IV	174·7	1·057	165·2	162·3	1·037	1·0003	1·0001
1958	I	175·5	1·052	165·7	163·8	1·036	0·9970	0·9995
	II	176·3	1·029	168·2	165·6	1·042	0·9934	0·9955

* Averages of wage-rates and retail prices have been calculated from the published whole numbers.

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DISCUSSION ON PAPER BY MR. DICKS-MIREAUX AND MR. DOW

Professor D. G. CHAMPERNOWNE: It is a particular pleasure to me to introduce this paper, which deals with a very important subject by the application of statistical methods. By examining the movements of four statistical indices over a period of twelve years, the authors have attempted to explain variations in the movements of money wages in terms of variations in the three conditions, (i) the rate of increase of prices; (ii) the level of demand; (iii) the pushfulness of trade unions. They also examine wage-movements in particular industries, but I find most interesting their estimates relating to industry as a whole and it is to these estimates that I will confine my remarks.

They claim to explain about 80 per cent. of the variation in the rate of wage-increase in terms of these three variables: a larger part was, as one would expect, attributed to variation in the rate of price increase, but the level of demand was found to be equally important and the degree of trade union pushfulness also made some contribution to the variation.

An important conclusion, to which I will return later, is that each increase in the rate of price-climb results in an increase in the rate of wage-climb only about one-half as great: the break-even point is a 5 per cent. rate of increase of prices, which at the average level of demand and trade union pushfulness corresponds to the same rate, 5 per cent. of increase of wages: a faster climb of prices than 5 per cent. leaves wages behind: a slower price-climb allows wages to rise faster than prices.

One's first reaction to a piece of research like this inevitably is that it attempts to conclude too much from too few data. One knows that much of the correlation between

prices and money-wages is due to the effect of money-wage increases on prices: it is therefore dangerous to try to measure the effect of prices on wages from the same set of data.

The authors have put up quite a strong defence against this line of criticism. They point out that there is likely to be a delay of many months both in the effect of wages on prices and again in the effect of prices on wages. By taking due account of these lags it may after all be possible to disentangle the effects of the two causes of correlation, and by examining annual changes we may escape the worst effects of feed-back operating with a lag of more than one year. They further point out that changes in import prices are *not* influenced by money-wage increases and that much of the variation in the rate of increase in prices during the period has in fact been due to variations in import price-increases. This strengthens the case for attributing the observed correlation between wages and prices mainly to the effect of prices on wages and not vice versa.

The results, if true, will I suspect lead to much moralizing. *What* morals will be drawn I hesitate to predict. On the one hand I suppose it will be claimed that if only trade union pushfulness could be reduced we could enjoy equilibrium with steady demand, steady prices and real wages going up in exact proportion to productivity: or more crudely, that it is up to the trade unions to choose between pushfulness and full employment. On the other hand it will be claimed that the low elasticity of wage-movements to price-movements reflects the moderation of the trade unions in the face of price-increases and stabilizes the system at a quite moderate rate of increase of prices: thus if productivity is rising at 2 per cent. per annum, a price increase of 1 per cent. per annum is all that is required to allow real wages and profits each to increase in line with productivity—thus money wages would rise at 3 per cent. while money prices rose at 1 per cent. per annum.

I suppose these two morals may be drawn by people with different viewpoints, but I do not wish to draw either moral. I am more concerned to cast some doubt on the low figure of about one-half found for the elasticity of wage-movement to price-movement. Even if this is a fair measure of the short-term elasticity, I believe that there is a larger long-term elasticity which is of great practical importance. I do not believe, for example, that if prices went up by 9 per cent. year after year money wages would only go up by 7 per cent. year after year: this might happen for one year or possibly two, but after that they would go up by at least 9 per cent.: in my opinion the long-term elasticity of wages to prices should be nearer unity than one-half.

There is another reason why I think that the estimate of the elasticity of wages to prices may be too low. Price increases have been taken as the independent variable and wage-increases as the dependent variable. It may well be that the measure of the rate of price increase contains some errors as a measure of those aspects of price increase which affect wages: in particular there may well be some year to year fluctuations in the rate of increase of the price index which are not fully relevant to changes in the rate of increase in wages: in this case the slope of a regression line of wage-increases on price-increases would under-estimate the elasticity of wage-increases to price elasticities. In this case a better measure of the rate of increases in prices than the one-year-change of the price index would be some estimate of the trend value of that annual rate of change.

In order to allow for both these effects, I related the wage increases of column 3 of the Appendix not to the price-increases of column 6, but to the centred three-year moving average of these price-increases: that is, to the average rate of price-increase over a period of about three years, centred on a date some twelve months earlier. I did the calculation for the figures relating to the first quarter of each year and as a check I repeated it for the figures for the third quarter of the year. In each case I obtained as the value of the elasticity of wage-increase to price-increases a figure of 0·82. This is much nearer to unity than the figure of 0·5 obtained by the authors for the short period elasticity. To some extent this confirms the suspicion that by properly allowing for the delayed effects of price-increases on wages and by smoothing away part of the irregularity of the movement of prices, one would get a figure of nearly one, suggesting that extra price-increases *eventually* result in almost as much extra wage-increases. It is true that the authors give some discussion of lagged effects of prices on wages, and reach opposite conclusions, but

I feel they have not tried long enough lags and I hope they will look more deeply into this aspect of the question than I have been able to do. I believe it is one of considerable practical importance.

I will not pursue the further possibility that the wage statistics used understate the actual increase in hourly wages paid, but this possibility should perhaps be borne in mind when the results are interpreted.

I have great pleasure in moving the vote of thanks for a most interesting and suggestive paper.

Professor A. W. PHILLIPS: I have great pleasure in seconding the vote of thanks to the authors for this extremely interesting paper. One of the important policy problems of our times is that of maintaining a high level of economic activity and employment while avoiding a continual rise in prices. The results obtained in the paper provide valuable information on the possibility of reconciling these objectives.

There are formidable difficulties in statistical analyses of this sort. A wide range of alternative hypotheses is possible and the series available for testing them are short. The hypotheses used in this paper seem very reasonable, though non-linear forms of relationships might also have been considered.

The authors have chosen to work mainly with four-period moving averages of quarterly data. If there is anything in the notion of an "annual wage round" a straightforward use of annual figures may have been more appropriate. It is of interest to see from Table 5 that the estimates obtained using annual figures for the first quarter of each year are similar to those obtained using quarterly data, and the fit appears to be extremely good.

If we start with an error term which is a four-period moving average of independent errors an autoregressive transformation of the form $1 - \frac{3}{4}E^{-1}$ cannot reduce the autocorrelations for all lags to zero. The Durbin-Watson test applies only to the first autocorrelation coefficient whereas the validity of significance tests depends on the autocorrelations of the error terms being zero for all lags. The results of the significance tests may therefore not be completely reliable. Autocorrelation of the error terms will also cause bias in the estimates of causal relationships even though there is a distributed lag in the "feed-back" relationships. Bias will be avoided only if there is a delay or "dead-time" in the feedback greater than the interval over which the error terms are autocorrelated. The case for believing that there is negligible bias must therefore rest on the argument implicit in para. 5.7, that the variance of the error term in the equation being estimated is small relatively to the variance of the explanatory variables.

The introduction as an additional explanatory variable of a subjective judgment of the "pushfulness" of trade unions is an interesting device. I personally would have put foward a somewhat different subjective judgment. I feel that changes in the attitudes of trade unions have had little effect on the outcome of wage bargains except in the years 1948 and 1949 when the Cripps policy of wage restraint seems to have been effective. I believe also that in 1951, when prices were rising rapidly following the outbreak of the Korean war, trade unions recovered most of what they had lost during the period of wage restraint. I would therefore have made the variable negative in 1948 and 1949, positive in 1951 and zero in other years. From an inspection of Fig. 3 it appears that this would also have produced a better statistical fit than that obtained on the authors' hypothesis, since it would have raised the calculated wage change in 1951 and lowered it in 1955 and 1956.

The results obtained in this paper suggest that if the authors' index of excess demand were zero, which seems to correspond to an unemployment figure of a little under 2 per cent., and if price changes were zero, wage rates would rise at about $2\frac{1}{2}$ per cent. per year. This rate of wage change would be consistent with zero price change if productivity per man hour were rising at $2\frac{1}{2}$ per cent. per year. It may be of interest to compare this result with one which I obtained on the hypothesis that the rate of change of wage rates depends only on the level of unemployment and the rate of change of unemployment, except in years of very rapid increase of import prices. This relation was estimated from data for

the years 1861–1913, but also fits very well for the years 1953–1957. With unemployment constant at 2 per cent. the rise in wage rates predicted from this relation is about $2\frac{1}{2}$ per cent. per year. Assuming a rise in productivity of $2\frac{1}{2}$ per cent. per year, this result would agree closely with the result given in the present paper. The relation I obtained is highly non-linear, but a linear approximation in the range between 1 per cent. and 2 per cent. unemployment gives a demand coefficient of about $4\frac{1}{4}$ as compared with the coefficient of about $3\frac{1}{2}$ obtained in the present study.

The vote of thanks was put to the meeting and carried unanimously.

Mr. K. J. Burton: I should like to congratulate the authors on their work and the clarity of their exposition, also on their caution in drawing general conclusions from their results and in attempting to forecast the future.

I shall not say much about methodology, but will make a few remarks based on my own experience, having had an opportunity of watching post-war wage movements from fairly close at hand.

I believe that circumstances have changed sufficiently over the post-war period to make the fitting of a single formula connecting wage rates with "demand" and "prices" a very dubious operation, particularly as I believe that "demand" and "prices" are themselves correlated variables. While not wishing to go into much detail, I should need to be convinced by more than the happy outcome of a significance test that a particular formal relationship had been established. What I have in mind is that all the series with which we are concerned are time series and that over the period considered a common inflationary nexus links them.

I think too much significance is often attached to wage rates. Manual workers' earnings are certainly related to current basic wage rates. In some industries the relationship has become a rather distant one. Not only are overtime, shift, and straight piecework payments involved but also sometimes so-called bonuses which are little more than disguised local wage increases designed to help to recruit particularly scarce classes of labour from other employers or industries.

What some people call the wage-drift has, I believe, changed its character over the past decade in its relationship to the basic wage bargaining arrangements of some industries. At the beginning of the period most negotiations were directed to establishing basic wage rates for particular categories of worker, and the so-called wage-drift was, if I may mix the metaphors, often absorbed in a rising tide. More recently, in many industries it has become the practice to make industry-wide wage settlements which involve the imposition on the wage-drift of an explicit general increase.

The general level of industrial activity and output is a factor to which no account has been paid, and it is not necessarily the same thing as "demand"—particularly in a period when employers have at some times been hoarding under-employed labour and at other times releasing it.

Shortages of labour have, in any event, often been concentrated in particular skilled occupations of especial significance in certain industries. General "demand" is significant but local "demand" which is sometimes of even greater importance is often as much a matter of occupation and of geography as of industry. "Demand" does not always reflect underlying realities—it is dependent on the expectations and beliefs of employers which sometimes may be deceived. "Unfilled vacancies" as recorded in the *Ministry of Labour Gazette* can in some circumstances be an almost meaningless and valueless statistic. For one thing alone employers in times of shortage sometimes ask for 20 men of a particular category in the expectation of getting three. "Registered unemployed" is generally a figure which more closely reflects reality.

There have been various institutional changes affecting the post-war situation. To some extent both the disappearance of the Conditions of Employment and National Arbitration Order in 1951 and the reinstitution and subsequent rescission of the Notification of Vacancies Order have introduced discontinuities into some of the series, as have changes

in subsidy and taxation policy also. A current change which may have considerable significance in this matter is the revocation of the Industrial Dispute Order. So-called compulsory arbitration at the instance of one only of the parties has, I believe, had considerable effect both directly and indirectly on establishing patterns of wage increases which have been followed in diverse industries where current economic circumstances have varied widely.

On two points of industrial classification: Order V does not to any appreciable extent fall within the engineering industries, nor do some sections of the other orders, notably shipbuilding: it is, however, true that shipbuilding and engineering time-rate wage settlements generally tend to march together, but not other factors. Order XVII includes both building and civil engineering, the former having in its labour force a large proportion of craftsmen and the latter consisting predominantly of labourers.

While I think that the present study is a very interesting and suggestive one, a much closer analysis of particular fairly homogeneous industries using earnings as the subject of analysis would be rewarding.

Professor PHELPS BROWN: I was particularly interested in the claim in para. 1.7 that as there proved to be little correlation between the index of demand and that of price changes it was possible to obtain significant estimates of their separate effects. I should like to consider that against the setting in which I have thought of these data as being generated, namely a setting dominated by the current pressure of demand, and, arising from that, those expectations on the part of employers (mentioned by the last speaker) which guide their decisions whether they are taking a bigger risk in accepting a rise in cost through higher wages or in incurring a strike.

We have been living through years in which generally employers have had reason to believe—I suppose mainly because of the state of their order books—that a rise in their labour costs can be passed on in prices without detriment to their turnover. In such circumstances trade unions have only to knock at the door to be admitted. I heard of one research officer of a union who, when a claim was pending, made a number of enquiries and obtained data which showed that the official index of the cost of living under-stated the actual rise since 1939; after the claim had been negotiated and a satisfactory rise obtained he asked the general secretary if his material had been useful. The general secretary did recollect using it but added: "But what matters is whether the rise is there. If it is, you will get it whatever your arguments, and if it isn't you won't, however good they are. But of course you have to have some arguments, it's only polite." The question is whether in such a setting the different elements which may enter into the arguments are capable of analytic treatment.

I think the analysis set before us this evening has confirmed the observations of that general secretary in two respects. Firstly, the main conclusion has been the predominant influence of the pressure of demand. Secondly, when the conditions of different industries were examined separately, it was the level of general and not of local demand that was found to be significant, with the one notable exception of the textile industry. Where a progressive rise in money wages is generated in such circumstances, can we hope to find any general relations which will connect the extent of that rise in successive years with changes in measurable variables?

As we see the wage round of each year taking shape, it seems rather to depend on a number of historical circumstances—the effect of a keynote bargain struck early on; the attitude of the government; the influence of a particular personality such as the chairman of a Court or Wages Board. There seems little doubt that the settlement in engineering this autumn was different from what it might otherwise have been by reason of the previous report on the wages of dockers. Among these various elements is the rise in the cost of living, but I doubt whether we can estimate from the record its separate influence, in a period when the pressure of demand has been so high throughout that the rise will have been compensated in any case. I think the authors would agree with me that we cannot infer from such a period how far cost of living rises would be compensated by wage rises if they were the only ground for a wage rise. It is possible that in the coefficient

which the authors have calculated we have only the observation that when pressure of demand is generating a persistent rise in money wages, this rise goes on rather faster when the cost of living has been rising faster.

Mr. Glassborow: I too have been closely associated with some wage negotiations on the employers' side, and in the past few years we have always had put to us four separate arguments: (1) a wage increase is justified in any case by price changes, (2) other wages have risen in the interval since the last increase, (3) there are shortages of staff due to the relatively low wages paid in the industry and (4) there has been an increase in productivity.

The first two are usually alternatives—whichever suits the unions best—but the shortage of staff and increases in productivity are always regarded as additional and the unions add a percentage to their claims for each.

In the causal influences which the authors have introduced in their calculations, which of these do they take account of? Price changes, yes; shortage of staff, to some extent; but not productivity. I wondered why that, and also comparison with other wage increases, which is quite vital in any individual case, were left out. I do not know how the latter could be treated. There must be great difficulties, but in my own experience this has a very considerable influence on the final settlement.

I should also like to take up one of Professor Champernowne's points; that the wage rate index is an inadequate measure of changes in wages in the period under discussion. This point is valid in 1946 and 1947 if, as I understand it is, this is an index of weekly wage rates.

In both 1946 and 1947 in a large number of industries there were reductions in the normal weekly hours of labour without a change in the wage rate paid for the week. There was an increase of some 9 per cent. in wages in consequence of a reduction from 48 to 44 hours a week. I do not think the weekly wage index takes account of this, and there are other ways in which the weekly wage rate index is inadequate for the present purpose.

I should like to make the plea that the Ministry of Labour might come here and explain exactly what they do in producing the index of weekly wage rates. It is often not possible in the short interval between receiving a wage claim and having to produce the preliminary answer to it to find out the facts about the wage index which are relevant to the points made.

Dr. J. Wise: I found this paper interesting and informative and was impressed by the stability of the results obtained. The authors have broken down their data in various ways for analysis (by quarters, industries, etc.) and have found a surprisingly constant pattern, which suggests that the results are sensible and fairly reliable.

However, it is important to remember that what is being measured is in fact an approximation to a complicated single relationship in a feedback system, and that the paper deals with the post-war period which is rather exceptional in a number of ways. I refer particularly to the persistency of the low percentages of unemployment and of the high annual rates of price- and wage-increase. This leads to two obvious difficulties.

Firstly, if we wish to see (for purposes of economic policy application) how far it is possible to control the price-wage inflation by increasing unemployment, we must extrapolate outside the observed range, and thereupon immediately enter the highly non-linear part of the relationship. I think that this is the most important conclusion that can be drawn from Professor Phillips's *Economica* paper, and I am somewhat surprised that he did not mention it himself. The point, in the form made by the authors, needs to be put more strongly.

Secondly, the persistency of the employment levels and price- and wage-trends must inevitably have influenced the expectations of employers and trade unionists regarding market conditions—to some extent through direct extrapolation from the recent past—and this would certainly have affected the outcome of wage negotiations. The need to use four-quarter moving averages, and the average lead of about six months in the employment

variable (partly due to "lagged feedback", and partly to the "expectations effect"), tends to blend the determinants to some extent. It would only be possible to disentangle and separate the short-run effects of expectations and of lagged feedback from the long-run relationship, by analysing time series relating to longer periods for which there is much less persistency in the trends in the explanatory variables.

The following contribution was received in writing after the meeting:

Mr. R. J. BALL: The authors of the foregoing paper have done us a singular service in attempting to put some substance on the bones of current discussions of the problem of inflation.

I propose to divide my comments into two parts, the first confined to the essentially statistical aspects of the paper, the second to its economic implications.

Since the wage rate equation is a single equation in an interdependent economic system, the familiar problem of least squares bias comes to the fore. In general throughout the paper, the authors at all stages emphasize the "goodness of fit" or percentage of variation explained rather than whether their estimated parameters are biased or unbiased. It is true that they do consider the particular possibility of bias in least squares estimates due to the presence of autocorrelation in residuals. But we can distinguish between more than one kind of bias, between small sample bias, bias due to autocorrelation in the disturbance terms and bias due to the fact that the relation being estimated is a single equation within an interdependent system regardless of the presence or absence of autocorrelation. Consider for example, the relationship,

$$y_t = \alpha(x_t + x_t - 1) + u_t \tag{1}$$

where u_t is a random disturbance. Suppose that the u_t are autocorrelated and satisfy a first order autogressive scheme such as

$$u_t = \rho u_{t-1} + v_t \tag{2}$$

where v_t is a random disturbance with finite variance and for which $E(v_t) = 0$ and $E(v_t\, v_{t-\theta}) = 0$.

Then it follows that (1) can be written,

$$y_t = \alpha(x_t + x_{t-1}) + (\rho y_{t-1} - \rho\alpha x_{t-1} - \rho\alpha x_{t-2} + v_t), \tag{3}$$

from which it further follows that the least squares estimate of α will be biased since $(x_t + x_{t-1})$ and u_t will not be independent. In many cases, such as that under consideration by the authors, the problems created by the existence of autocorrelation between the disturbance terms, can be overcome if the appropriate transformation of variables is known (as it is in the text by the argument of (3.9)). However, this does not eliminate the bias resulting from interdependence within a system of equations. An attempt to avoid this kind of bias might have been made by use of the method of limited information maximum likelihood applied to a single equation. Certainly this would have altered the magnitude of the computing problems, but alternatively recourse might have been had to the method of "two stage least squares" developed by Theil, which is a consistent estimator where the additional computational burden would not have been heavy. Little difference will arise between the estimates obtained from least squares, and those obtained by these other methods when the overall correlation is high, but some of the parameter estimates presented are accompanied by relatively low overall correlations, so that somewhat more satisfactory results might have resulted from the use of one of the above alternative estimating procedures. It appears unlikely that much difference would result with regard to the estimates based on the 1950 IV–1956 IV sample period. In the paper referred to in the text, Ball and Klein applied both limited information maximum likelihood and least squares estimating procedures to a linear equation, whose economic logic substantially parallels that followed by Dicks-Mireaux and Dow. Based on a sample of quarterly data, 1948 I–1956 IV, the equation related changes in wage rates, to a distributed lag in prices, a moving average of unemployment, and a constructed variable, taking the value

0 up to 1951 IV and I thereafter, to take into account the changed attitude of trades unions toward pushing for higher money wages, under a Conservative as opposed to a Labour Government. It may be noted that this constructed variable bears some resemblance to the measure of "pushfulness" adopted by Dicks-Mireaux and Dow, and appeared to play an important role in the relationship estimated. The result was that there was little appreciable difference between the estimates obtained by the two estimating procedures. Nevertheless it would seem important that the same result should occur when alternative estimating procedures are applied to the equations as formulated in the text. If it does, then together with the apparent absence of multicollinearity between the distributed lag in prices and the moving average of demand, the parameter estimates may be considered fairly firm. In the particular after application of the autogressive transformation to the equation based on data, 1950 IV–1956 IV, the estimated parameters stand well with respect to their standard errors.

With regard to the problem of identification, nothing can be said until further relationships have been specified. This is implicit in the discussion about the relevant lags, and about the role of import prices in determining the price level. In the first place it ought to be earnings and not wage rates which enter explicitly into the determination of the price level, together with import prices and other factors that are thought relevant. Thus a link is required between changes in earnings and changes in wage rates. It follows that the discussion in (5.8) is somewhat obscure. In general one cannot simplify the situation to an equation in wage rates depending on price changes and excess demand, and a price equation depending on changes in wage *rates* and changes in import prices. If one could, then formally the presence of import prices in the price equation would be sufficient to identify the wage rate equation. To ensure identification of the price equation, further relationships would be required, since it is arguable that demand will not identify the price equation very well, since one may always be on the verge of including demand explicitly in the equation itself (*cf.* further remarks below on this point). Nevertheless the authors are on good ground, with regard to the role of import prices, and once the additional relationships are specified it is highly likely that the wage rate equations will be identified in their present form. This leads back to the fact that the wage rate equation is a single structural equation in an interdependent system, so that advantage might have been taken of the additional identifying information in the method of estimation.

Dicks-Mireaux and Dow point out that "for the values fitted the results are little different from what would have been obtained with a linear form of equation." This remark serves to point up a fundamental problem of which the authors are fully aware. It is arguable that *a priori* a non-linear relation between wage rate changes, price changes and excess demand is more plausible than a simple linear one, to take into account in some measure, the essential asymmetry involved in wage rate changes. Wage rates in general go up more easily than they come down. The sample of data available from the post-war period does not however enable this hypothesis to be tested, being unrepresentative in so far as it does not include a period of falling wage rates and prices. Hence as the authors emphasize, with regard to their results, "it is almost certainly not valid to project them into the zone of under-full-employment."

This fact is of considerable importance when one comes to consider the economic implications of the results, in particular in the context of policy. The relative importance of the demand coefficient is striking. An analogous result was obtained by Ball and Klein with respect to the influence of unemployment on wage rate changes. However, the jump from this to the conclusion that only a small reduction in demand is required to stabilize the money wage level is a big one to make, and it is disquieting that at least one element of the financial press should have made this jump with respect to the recently published findings of Professor Phillips in this field. From a statistical point of view, we are not able to say what will be the reaction of money wage changes to a fall in demand, although we may all have our different intuitions as to the possible outcome.

This question apart, it is possible to accept the result that the level of excess demand has played a key role in determining money wage changes over the post-war period without

accepting the prescriptions of the so-called demand inflationists for policy. I would be prepared to argue that the statistical evidence presented by Dicks-Mireaux and Dow is sufficient to discredit the view that responsibility for changes in money wage rates over the post-war period can be ascribed to militant trades unions and feedback effects of increased import prices alone. This does not however imply that the current remedy for inflation is to reverse the engine, for the asymmetry already referred to may be such that the tracks have been stealthily taken up behind it. Certainly if demand is reduced sufficiently there will probably be a breaking point somewhere; but where is not known. In consequence the result may be rather like curing the patient of a headache by cutting off his head.

It is suggested that the relation in time between the dependent and the independent variable throws light on the question of the point at which demand "impinges". Present excess demand is depicted as working in the labour market, on changes in money wage rates, increases in which are subsequently translated into increases in prices. This assumption may well be valid, but does not rule out quite generally the direct influence of demand either lagged or unlagged on the price level. If the price equation is postulated as a strict "mark-up" over cost, then it is only through the labour market that demand can work. It is of course possible to postulate an alternative price equation into which demand is introduced as an explicit variable together with changes in costs. This is equivalent to saying that excess demand for goods and services has some direct influence on the price level in so far as entrepreneurs may exploit favourable conditions by raising prices and simultaneously attempting to meet some of the excess demand by employing additional labour.

Dicks-Mireaux and Dow have made their point in a convincing fashion. "What we think is the important result of our analysis is the conclusion that the level of demand much in excess of the (still high) minimum encountered in the period, had a major effect on the rate of wage change." This seems undoubtedly true. They further argue, "This certainly leaves economic policy where it was, as much political as economic". Not quite. The policy makers have been provided by Dicks-Mireaux and Dow with further evidence on which to base their judgements.

The authors subsequently replied in writing as follows:

It is convenient to deal first with the question of autocorrelation raised by Professor Phillips. The question concerns autocorrelation arising out of our use of moving averages as an explanatory variable. The discussion shows our treatment to have been inadequate. First, as Professor Phillips points out, the Durbin-Watson test relates only to first order autocorrelation. Second, as was pointed out to us before the meeting by Professor Champernowne, our transformation $(1 - \frac{3}{4}E^{-1})$ is likely to remove from the residuals autocorrelation only of the first order. A better treatment seems possible. In the four component equations of equation (2) we have error terms (ϵ) which reappear in equation (2) as:

$$\begin{aligned} \nu_t &= \epsilon_{t-3} + \epsilon_{t-2} + \epsilon_{t-1} + \epsilon_t \\ &= (1 + E^{-1} + E^{-2} + E^{-3})\epsilon_t \\ &= \left(\frac{1 - E^{-4}}{1 - E^{-1}}\right)\epsilon_t, \end{aligned}$$

or

$$(1 - E^{-1})\nu_t = (1 - E^{-4})\epsilon_t.$$

This suggests that a transformation taking first differences of the variables would remove autocorrelation of orders up to and including the third, provided that the original error terms (ϵ) are serially independent. Our original equation (2) was re-estimated for the period 1950 IV to 1956 IV using this new transformation, and gave estimates of the coefficient for price-change, $\alpha = 0{\cdot}45$ $(0{\cdot}14)$ and for demand $\beta = 3{\cdot}86$ $(1{\cdot}15)$, which are similar to the previous results (see Table 5).

We must next consider the question raised in one form or another by most speakers whether, by trying to explain wage-change as a joint function of demand and of price-change (price-change being itself a lagged function of wage-change), we obtained biased estimates of the parameters. If there is feed-back some bias will result, and the residuals will also be autocorrelated. Such autocorrelation is much more difficult to allow for with our methods. Since the Durbin-Watson test applies only to first order autocorrelation it cannot be demonstrated generally that it does not exist; our attempted demonstration in 5·6 that feed-back is not present is therefore faulty. Further it cannot be demonstrated that the standard significance tests apply.

The validity of our results remains therefore not definitely established. Belief that our estimates of the parameters are unbiased must rest on general arguments (a) that the feed-back interval is probably long (our paras 5·4–5·5) or (b) that most of the variation in the rate of price-change was in this period not due to variation in the rate of change of labour costs (our para 5·7). These arguments seem to have been accepted by most speakers.

Mr. Ball criticises us for making least squares estimates of a single equation in what is an interdependent economic system. If the above arguments are valid the criticism would not apply in this case; and we note that the results he obtained with Dr. Klein appear to bear this out. We can also claim that our method enabled us to experiment with variant forms of the general hypothesis being tested; and that the precise *form* of the equation is one of the important things to investigate.

Professor Champernowne asked us to consider the suggestion that our price coefficient might refer merely to a short-run effect, and that the long-run effect might be more nearly unity. In essence our explanation of the wage-change (over twelve months) was in terms of the contemporaneous price-change. Professor Champernowne related the wage-change to the price-change over the three years starting one year previous and ending one year after the year of wage-change to be considered, and found a coefficient of about 0·8. We are in sympathy with Professor Champernowne's object, but we do not think his correlation is a test of it. It so happens that the rate of wage-change has varied in *short* bursts (see our Fig. 2) so that a short moving average reproduces the same cycle with reduced amplitude. This being so it is to be expected that the coefficient would come out higher. The correlation cannot tell us what would have happened if the rate of price-change had gone on accelerating markedly for several years together. With the same general object in view, we ourselves hoped to find a larger price coefficient by relating wage-change over 24 months to price-changes over the same period: in fact it was little different (para 3.21).

Professor Phelps-Brown suggests that employers felt able to pass on cost increases because throughout the period examined they expected the general rise in prices to continue, and that this was an important cause of the rise in wages and prices. We would agree that this probably was the case. If it were possible to make a parallel analysis of a period when expectations were different we would expect this to be reflected in different estimates of the parameters, or the constant term: and if they were a fairly *constant* influence in our period, their omission may not have distorted the former. Nevertheless our account may well be incomplete, and the estimates of the parameters we did make should probably be understood as valid only with the rider "expectations being what they were". We do not see any *special* reason to doubt the general applicability of our estimated price coefficient, as Professor Phelps-Brown seems to in his concluding remarks; analysis of other experience has yielded similar estimates (see our 5.12–5.17).

Mr. Burton has queried the reliability of the figures of unfilled vacancies which we employed together with those of unemployment in our index of demand. In the earlier study referred to in the paper we came to the conclusion that notifications of unfilled vacancies were in fact a surprisingly reliable index, notwithstanding the various factors which, we agree with him, might have been expected to prevent this being so. We agree entirely that wage-drift, neither covered nor intended to be covered in our study, cannot be overlooked in any complete account of wage and price formation.

Mr. Glassborow suggests that productivity-change should have been employed as an additional explanatory variable. In the period studied this tends to be correlated with

our index of demand, so that one can if one chooses think of our demand effect as including a productivity effect. Doubtless there is more to be said on this subject.

As a result of the ballot taken during the meeting the candidates named below were elected Fellows of the Society:

Munir Ahmad
John K. Chung
Dorothy Enid Cole
Lavender Violet Dollar
John Henry Emerson
Peter Hunt
Frederick John Johnson
Terence I. Jones
Gordon Graham Kalton
Ratan Lal
Henry Ballantyne Lawson
Richard Wildon Lofthouse
Peter Macdonald
Michael John Mackie
Richard Murray Paine
David Atheling Peel
Terence Aubrey Prichard
Peter Sprent
Peter Charles Webb

Corporate representative

Kenneth John Brooks *representing* International Latex Corporation

Production and Productivity Movements in the United Kingdom Since 1900*

By K. S. Lomax

University of Manchester

[Read before the Royal Statistical Society, January 21st, 1959, the President, Sir Harry Campion, C.B., C.B.E., in the Chair]

1. Introduction

It has been known for some time that existing annual index-numbers of industrial output for the United Kingdom before the second world war can now be improved upon in the light of information which has become available subsequent to their publication. The data on which they were based consisted only of quickly available current material and appear rather meagre, certainly, in relation to the knowledge which can now be assembled. No use was made, for instance, of the comprehensive output information provided in Censuses of Production and Import Duties Act enquiries. The technical methods of construction, too, were far from ideal. Weighting, in particular, was usually based on a single year. Further, most of the existing indices cover only limited periods. One attempt only (Hoffman, 1955) has been made to construct from original sources a continuous index over a long period and this was not subdivided by industrial groups except in a very broad way. A total index, both including and excluding Building, and indices for consumer goods and producer goods separately, only were presented. More recently (Ridley, 1955; O.E.E.C., 1955) attempts have been made by linking together, after adjustment, existing indicators for different periods, to construct a continuous total index of industrial output annually from 1900. While these give a general picture of the rising trend in total production over this period which is broadly satisfactory, it will appear that the values in individual years and the fluctuations exhibited do not inspire the same confidence. Furthermore the Ridley index is not subdivided industrially at all and the O.E.E.C. group figures are most unconvincing. In fact, the O.E.E.C. index-numbers exclude building, but values are given for seven main industrial groups.

Continuous annual indices covering limited periods are due to Rowe (1924) for 1907–23; London and Cambridge Economic Service (1939) for 1920–38; and the *Board of Trade Journal* and the *Statistical Abstract of the U.K.* for 1924, 1927–38. These all being constructed in the manner of a currently published index depend solely on quickly-available data, and each is based on a weighting system relating merely to a single year at the beginning of the period for which the index is calculated. The raw material of these indices must be regarded as inadequate and a weighting system on the above lines can be justified only on the grounds of expediency. It is not surprising that the results appear to be less than satisfactory.

* An expanded version of the paper delivered to the joint conference of the Royal and Manchester Statistical Societies held in Manchester, September 1958. A summary of the section relating to the calculation of new index-numbers of production, 1900–38 was published in the *London and Cambridge Economic Bulletin*, in the *Times Review of Industry*, June 1958. *The Times* and the Editor of the *London and Cambridge Bulletin* have given permission for reproduction of parts of that article.

Constructing now, in the 1950's, indices of output annually from 1900 to 1938, permits the use of all the statistical material relating to the period which is ever likely to become available. Optimal methods of construction, too, which are inapplicable in a currently published index, for example, can be called into play. Various weighting systems can be incorporated. Index-numbers can be computed for different industrial groupings. The whole exercise can be carried out in a manner best facilitating an economic survey of the period.

This article presents the results of such a completely new calculation of index-numbers of the volume of industrial production for the United Kingdom, annually, from 1900 to 1938. These can be linked through the 1935–48 comparisons of Brown (1954) and the interim index of industrial production with the official post-war index to provide a continuous series (except for the war years), 1900–57. Production movements in this period can be compared with changes in employment and hours of work to show trends in productivity.

2. PRODUCTION INDEX-NUMBERS

The method adopted in measuring production trends has been to calculate first, for each industry, index-numbers for Census of Production or Import Duties Act Enquiry years; 1907, 1912, 1924, 1930, 1933, 1934, 1935 and 1937, based on the figures provided in the published reports on these enumerations. These data are the most comprehensive of all statistics relating to production in this country. Yet there are still formidable problems to be solved in calculating index-numbers of production from such sources. Many items are specified in value only and not in quantitative terms. Corrections have to be made for changes in geographical coverage and for the contributions of small firms which are usually excluded from census returns. For weighting purposes, an estimate has to be made of net output of the principal products of a trade, a figure which is not given in census reports. With some difficulties we can do very little. For example, the quality of articles under the same description, no doubt, changes appreciably over a period of time. These deficiencies are always present in an index-number, however, and are not peculiar to the approach adopted here. The comprehensive nature of census material certainly would suggest that index-numbers on this basis must form the key figures in the final compilations.

The index-numbers calculated for 1907, 1912, 1924, 1930, 1933, 1934, 1935 and 1937 from the census tabulations were next used, then, as pivotal points with respect to which indices for other years were calculated by interpolation and extrapolation. The basis of these operations consisted of all annual data, published and unpublished, which it proved possible to assemble.* In many cases this interpolation and extrapolation was difficult because of the lack of agreement between the movements shown by the annual series and the Census of Production figures. It was, of course, difficult to reconcile these differences in every case and on one or two occasions an appraisal of the discrepancies suggested rejection of the census figures and reliance exclusively on annual series as indicators, even for census years.

The continuous annual index-numbers 1900–38 on a Standard Industrial Classification

* The collection of material, an enormous task, was most efficiently carried out by my two research assistants, Miss C. McGlade and Mrs. N. Connelly. This assistance together with part of the heavy computing burden was financed from Conditional Aid Funds.

basis could now be linked through to the official indicators of production movements since the second world war. The first stage of this linking process was to use the calculations of Brown (1954) which compared the levels of output in 1935 and 1948 by using the Censuses of Production for those two years, a method consistent with the general approach here to production index-number construction. Then the years 1946–48 were completed by using the official interim index of production for this period.

The various stages in the computation of production movements since 1900 must now be described in greater detail.

2.1. *The Census Year Index-numbers*

Index-numbers of production for census years based on census material have, of course, been constructed before; notably by Tolles and Douglas (1930), Devons (1939), Maddison (1955) and, indeed, in some of the census reports themselves. However, here the calculations were carried out completely afresh from the original census tabulations for several reasons. Firstly, the published results in the above cases were by no means sufficiently detailed for our purposes and sometimes not sufficiently wide in scope; nor were the industrial classifications adopted identical with those desirable in our approach. Then again, as we have already seen, it is desirable to incorporate various corrections to the census data which were not always carried out in these earlier calculations mentioned. Finally, it was intended to treat the weighting problem more completely.

In calculating the census index-numbers the attempt was made to include all census and I.D.A.E. data whether specified both in quantity and value or in value only. When quantity figures are given for a commodity a direct comparison of the volume of production is feasible. In the latter case, however, when only the value of output is entered, a value relative can be worked out but must then be deflated by an indicator of the prices or average value change in order to arrive at a quantity comparison. In many cases, therefore, prices or average-value index-numbers were computed or extracted from a variety of sources and used as deflators to give the estimated quantity changes. One of the sources for average-value indicators was the census reports themselves in that frequently the quantity change for an item specified in value only was estimated on the assumption that the average-value change for the item was similar to that for an allied product which was entered in both quantity and value. In other cases, in the absence of a suitable readily available deflator, the required average-value index was constructed from data relating to changes in the prices of raw materials and of labour. Through these and similar means, then, it was possible to include all the census output data in the calculations and in this sense the new index-numbers can be said to have, effectively, 100 per cent. coverage in census years. It was thought to be preferable to make some estimate of a quantity change for each item rather then to leave it out altogether. If any item specified in value only had been omitted it would have been implicitly assumed that the change in the volume of production for the item in question was the same as for the rest of the industry. There appears to be little reason why this should be so.

Next we come to the corrections necessary to ensure that the index-numbers are comparable throughout with regard to geographical coverage. Southern Ireland became independent in 1923 so that while the 1907 Census related to Great Britain and the whole of Ireland, the 1924 and subsequent censuses covered only Great Britain and Northern Ireland. This lack of comparability affected only the 1907–24 comparisons. The problem

was dealt with in this way. It was assumed that the 1907–24 change in the volume of production for Great Britain and Northern Ireland was the same as for Great Britain and all Ireland. The 1924 census results were then converted where necessary to include the Southern Ireland contribution. In the great majority of industries no correction was necessary since production in Southern Ireland was either *nil* or negligible. Where a correction was necessary it was made on the basis of the known production in Southern Ireland in 1926, when a census was taken, and the change in the volume of output from 1924 to 1926 in Great Britain and Northern Ireland which was assumed to apply to Southern Ireland. The 1924–26 change for Great Britain and Northern Ireland was computed from the annual indicators assembled for the purpose of interpolation between census points (see Section 2.2 below).

A further set of corrections was necessary in order to put the index-numbers as far as possible on a Standard Industrial Classification. This was feasible only to a limited extent although certainly the necessary re-grouping of industries could be achieved. A complete re-grouping of principal products between trades was not only very difficult in itself: it would have prevented any estimate being made of the net output of the principal products of a trade, a concept which was required for weighting purposes.

It is appropriate to describe the method of correcting for the contributions of small firms in some detail since this affords the most convenient opportunity for explaining the construction of an index-number of production from census material. Firstly, such an index will be calculated on a principal products basis and weighting between industries should be proportional to the estimated net output of the principal products of the trade. Within an industry weighting has to be related to gross value since net output cannot be allocated among the different commodities making up the principal products of a trade. The detailed output tables in reports of censuses since 1924 provide figures of production of principal products in establishments employing at least 10 persons.

An index of production will normally be calculated as a weighted arithmetic average of production relatives. If, then, i is an industry index, for year x on year y as base, calculated from census output material, we shall have

$$i = \frac{\Sigma(p_y q_y)\, q_x/q_y}{\Sigma(p_y q_y)}, \tag{1}$$

where the q's are quantitative output figures, the p's are (gross output) prices, and summation extends over the individual principal products of the trade.

This can be written:

$$i = \frac{\text{Gross output, principal products, large firms, year } x, \text{ year } y \text{ prices}}{\text{Gross output, principal products, large firms, year } y, \text{ year } y \text{ prices}}, \tag{2}$$

where the term "large firms" implies the establishments employing at least 10 persons, which are covered by the census.

Formulation (2) shows that even when quantitative output figures are not available it may be feasible to revalue, at base-year prices, output in value terms and calculate the index by the alternative, but formally equivalent, method (2). This was generally the approach for Engineering and Shipbuilding trades, for Building, and for some branches of the Clothing industry. Here output is difficult to measure in a significant quantitative form.

If now it is assumed that

$$\frac{\text{Gross output, principal products, all firms, year } x\text{, year } y \text{ prices}}{\text{Gross output, principal products, large firms, year } x\text{, year } y \text{ prices}}$$

$$= \frac{\text{Gross output of trade, all firms, year } x\text{, year } y \text{ prices}}{\text{Gross output of trade, large firms, year } x\text{, year } y \text{ prices}}$$

$$= \frac{\text{Gross output of trade, all firms, year } x\text{, year } x \text{ prices}}{\text{Gross output of trade, large firms, year } x\text{, year } x \text{ prices}}$$

and

$$\frac{\text{Gross output, principal products, all firms, year } y\text{, year } y \text{ prices}}{\text{Gross output, principal products, large firms, year } y\text{, year } y \text{ prices}}$$

$$= \frac{\text{Gross output of trade, all firms, year } y\text{, year } y \text{ prices}}{\text{Gross output of trade, large firms, year } y\text{, year } y \text{ prices}},$$

then the corrected index for an individual industry, say i', will be:

$$i' = \frac{\text{Gross output, principal products, all firms, year } x\text{, year } y \text{ prices}}{\text{Gross output, principal products, all firms, year } y\text{, year } y \text{ prices}}$$

$$= i \times \frac{\dfrac{\text{Gross output of trade, all firms, year } x\text{, year } x \text{ prices}}{\text{Gross output of trade, large firms, year } x\text{, year } x \text{ prices}}}{\dfrac{\text{Gross output of trade, all firms, year } y\text{, year } y \text{ prices}}{\text{Gross output of trade, large firms, year } y\text{, year } y \text{ prices}}}.$$

The denominators in the ratios on the right-hand side were known. The contributions of small firms, enabling the numerators to be computed, were estimated by assuming that gross output per head in small (1–10 employees) establishments was the same as for the nearest size group, generally those employing 11–24 persons. The latter figure could then be multiplied by the estimate of employment in small firms published in census reports. There was just one exception to this procedure. It was not, in fact, necessary for the year 1924 since the gross output of small firms, as well as the employment, was actually published in the 1930 census report for each industry in 1924.

The logical basis of group or total index numbers in the present approach would be to average the corrected industry indices i', weighting by net output of principal products for all firms in the base year. That is, if I is a group index for year x on year y as base:

$$I = \frac{\Sigma\,(\text{Net output, principal products, all firms, year } y\text{, year } y \text{ prices}) \times i'}{\Sigma\,(\text{Net output, principal products, all firms, year } y\text{, year } y \text{ prices})},$$

summation extending over all trades included in the group. These net outputs relating to the principal products, however, are not known and cannot directly be calculated.

They can be estimated, if we assume

$$\frac{\text{Net output, principal products, all firms, year } y\text{, year } y \text{ prices}}{\text{Net output of trade, all firms, year } y\text{, year } y \text{ prices}}$$
$$= \frac{\text{Gross output, principal products, all firms, year } y\text{, year } y \text{ prices}}{\text{Gross output of the trade, all firms, year } y\text{, year } y \text{ prices}}$$
$$= \frac{\text{Gross output, principal products, large firms, year } y\text{, year } y \text{ prices}}{\text{Gross output of the trade, large firms, year } y\text{, year } y \text{ prices}}$$

by an earlier assumption.

Hence:

$$\text{Net output, principal products, all firms, year } y\text{, year } y \text{ prices}$$
$$\doteq (\text{Net output of trade, all firms, year } y\text{, year } y \text{ prices}) \times \frac{\text{Gross output, principal products, large firms, year } y\text{, year } y \text{ prices}}{\text{Gross output of the trade, large firms, year } y\text{, year } y \text{ prices}}$$

and therefore

$$I = \frac{\Sigma\,(\text{Net output, trade, all firms, year } y)\,\dfrac{(\text{Gross output, principal products, large firms, year } y)}{(\text{Gross output, trade, large firms, year } y)} \times i'}{\Sigma\,(\text{Net output, trade, all firms, year } y)\,\dfrac{(\text{Gross output, principal products, large firms, year } y)}{(\text{Gross output, trade, large firms, year } y)}}$$

Everything in this formula was known except the net output of the trade for all firms in the base year. It was thus necessary to compute the net output of the trade in year y for small firms. This was estimated, generally, from employment in small firms by making the assumption that net output per head in establishments with 1–10 employees was the same as in those employing 11–24 persons. Again there was one slight exception to this procedure. In the case of a small number of industries, net output per head figures for small firms in 1924 were published in the 1930 census report. In contrast to the corresponding gross output statistics this was by no means true of all trades. The above calculations had to be carried out each time a different base year y was used. In the period 1920–38, for example, it will be seen that index-numbers with 1924, 1930, and 1935 weighting were used, so that three separate and distinct sets of computations, with $y = 1924$, 1930 and 1935, had to be completed.

A most important matter in a production index is, in fact, the year chosen as base. This affects not only the weighting system but also the procedure in calculating production relatives. The statement that a particular year is base implies that the production relatives must be computed with the base-year figure in the denominator and also that the weighting system relates to this particular base year. Once these two implications have been taken care of, simple scaling can then be used to bring the index for any desired year equal to 100. It is appropriate to consider at this stage the procedure adopted with respect to

choice of base year since the matter first arises in connection with the calculation of pivots from census data. The remarks here apply, however, equally well to the annual index-numbers described and discussed below.

Suppose we are comparing output in year x with the level in a later year, y. If we take year x as base (the Laspeyres index) we will normally obtain results different from those when year y is base (Paasche). In theory, looking back at the period (x, y) there is nothing to choose between the two or, indeed, between either and the results with any other base. In measuring the change in the volume of production from year x to year y probably the best compromise is to take the geometric mean of the results with year x and year y as bases and use the Fisher "Ideal" index formula. This has been the principle adopted in the calculation of the indices here presented but with an important modification. It was felt that the period 1900–38 was too long to be treated as one unit for weighting purposes. Thus for 1900–24, index-numbers were calculated as geometric means of indices with 1907 and 1924 as base-years. For 1920–38, the geometric means of index-numbers with 1924, 1930, and 1935 weighting were computed. Non-census years can never be used as bases in production index-number construction since it is only in a census year that we have net output figures which form the most desirable basis of the weighting system in such an index. Within an industry, of course, as we have seen, we must needs use gross output weighting since net output figures are not and conceptually cannot be divided up as between the different products. Between industries, however, the ideal weighting system would be based on the net output of the principal products of the trade. This is a figure which can be estimated only from information given in census reports.

A single set of indices was calculated for the overlapping period 1920–24, by taking geometric means, with moving weights, of the two sets; the one based on 1907 and 1924, the other on 1924, 1930, and 1935. The whole series of index-numbers was finally presented with 1924 = 100 but, again, it must be stressed that this is not to imply in any sense that 1924 was the base-year.

Before considering the calculation of annual index-numbers it should be mentioned that while the corrections for the contributions of small firms, for changes in geographical coverage and to bring the industrial classification into line, were made throughout the whole range of industry, additional specific corrections were sometimes made for known or apparent deficiencies of the census data in particular industries.

2.2. *Annual Index-numbers of Production*, 1900–38

Annual index-numbers for 1900–38 were next obtained by interpolation and extrapolation from the pivotal points provided by the calculations for census and I.D.A.E. years. The basis of these processes consisted of all the information relating to the period which could be assembled. Where possible, continuous, consistent, annual statistical series were obtained but account was sometimes taken of short-term production comparisons. The main sources of the data collected were as follows:

(i) All the information used in the Hoffmann (1955), Rowe (1924), London and Cambridge (1939), and Board of Trade index-numbers. The Board of Trade kindly made available, with the permission of the firms and trade associations concerned, all the unpublished figures incorporated into the pre-war official index.

(ii) Official committee and Royal Commission reports, Statistical Abstracts, etc.

TABLE 1

Index-numbers of Industrial Production:

Year	Total All Industries	Manufacturing: Total Manufacturing Industry	Building Materials	Chemicals	Metal Manufacture: Total	Ferrous	Non-Ferrous	Engineering, Shipbuilding and Electrical Goods: Total	Mech. Eng.	Elec. Eng.	Ship-Bldg.	Vehicles	Precision Instruments etc.
1900	73·9	71·3	—	60·6	76·1	77·7	66·8	79·5	87	28·7	112·2	28·2	—
1901	73·6	71·2	—	60·3	72·7	73·2	67·8	81·8	85	30·3	130·1	28·3	—
1902	76·4	73·0	—	61·6	78·8	79·4	75·0	79·6	87	31·8	108·3	28·4	—
1903	76·4	72·9	—	67·4	79·3	80·9	69·5	79·2	87	34·0	102·9	30·8	—
1904	75·7	73·0	—	70·7	81·6	81·9	78·9	78·8	88	35·4	96·2	30·9	—
1905	78·3	77·2	—	72·1	87·3	91·0	67·2	90·4	97	38·4	128·0	32·2	—
1906	80·2	78·6	—	74·0	88·6	92·4	67·8	90·7	93	41·9	139·6	32·5	—
1907	81·6	79·9	80·7	79·0	87·9	91·3	69·1	84·3	85	44·3	126·9	35·1	83·9
1908	77·1	75·3	—	79·0	84·3	84·9	79·8	72·9	82	43·7	71·0	31·1	—
1909	78·9	77·6	—	78·2	88·7	92·7	66·8	78·1	87	45·5	82·1	29·9	—
1910	80·6	80·1	—	81·6	94·9	99·9	81·1	84·2	92	51·4	93·1	30·6	—
1911	83·0	83·9	—	84·0	94·3	96·2	83·4	94·0	90	55·7	152·4	33·9	—
1912	84·2	86·3	77·4	84·8	91·4	94·1	76·4	89·9	85	59·7	141·3	36·0	—
1913	90·5	92·2	—	90·0	101·3	105·5	78·3	104·2	103	64·6	156·5	43·3	—
1914	84·8	85·9	—	86·6	94·7	94·7	93·9	95·5	92	68·1	138·5	—	—
1915	86·4	90·4	—	88·3	95·9	92·3	114·0	94·4	99	74·8	100·4	—	—
1916	81·8	84·6	—	91·7	93·1	91·7	99·8	101·4	100	80·0	132·2	—	—
1917	76·4	78·4	—	92·3	96·1	90·1	126·3	110·9	112	85·7	139·2	—	—
1918	73·8	76·1	—	93·3	98·0	92·6	125·9	118·5	115	91·4	166·8	—	—
1919	81·3	85·0	—	94·1	85·0	87·5	72·1	100·3	92	94·4	148·2	42·9	—
1920	90·3	93·6	87·2	101·3	93·5	97·1	74·5	104·9	95·4	94·0	154·9	70·7	98·2
1921	73·5	72·8	84·4	72·9	45·5	45·4	45·6	78·2	62·7	80·4	125·0	62·7	89·0
1922	85·0	84·8	73·3	86·3	65·6	67·1	57·4	73·4	76·2	63·2	88·2	71·5	92·6
1923	90·0	90·8	83·8	94·1	92·6	95·2	73·4	87·3	95·6	83·0	73·6	85·9	99·3
1924	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
1925	103·9	103·1	113·7	96·6	92·7	89·4	111·2	108·1	114·0	113·2	70·1	107·6	104·6
1926	98·4	99·8	116·1	89·0	64·2	57·0	103·7	98·2	99·7	103·6	52·7	113·1	108·5
1927	113·4	110·4	128·7	101·9	108·5	106·7	118·2	110·4	110·9	112·7	105·7	125·1	117·1
1928	110·2	110·0	118·9	106·8	101·3	96·0	129·8	111·4	116·9	117·5	93·3	123·7	123·7
1929	115·8	114·5	127·9	112·2	106·4	104·3	117·8	115·9	120·9	120·2	109·9	133·4	129·0
1930	110·8	109·6	122·1	106·6	95·9	93·0	111·6	107·9	107·2	121·6	95·8	127·8	123·2
1931	103·7	102·2	114·9	102·3	74·6	70·5	97·5	87·6	87·6	116·9	36·2	109·3	110·9
1932	103·2	102·7	110·7	109·1	76·4	74·7	85·8	79·5	77·4	126·1	11·3	106·4	110·9
1933	110·1	109·4	126·2	114·2	89·5	86·9	103·4	80·1	79·0	127·0	13·9	118·8	117·1
1934	121·1	120·4	146·6	123·3	110·0	106·3	129·9	99·5	92·7	155·9	48·2	139·8	123·9
1935	130·3	131·3	157·8	133·9	121·4	116·4	148·7	117·2	110·4	181·0	59·1	166·6	136·0
1936	142·0	143·5	171·7	139·6	136·1	130·9	164·3	132·9	119·8	202·9	91·0	199·4	145·1
1937	150·5	152·2	179·7	148·7	152·9	146·9	185·8	146·0	128·0	219·0	117·8	221·2	154·3
1938	146·4	147·8	168·5	141·0	127·0	118·3	174·6	145·2	130·2	215·5	96·4	224·7	158·3
1946	149·4	153·6	168·7	209·8	157·5	138·6	264·7	179·2	—	—	—	244·9	175·2
1947	158·7	164·3	194·3	213·3	168·6	145·0	304·2	202·9	—	—	—	255·8	203·9
1948	171·7	178·4	213·0	232·9	187·1	167·4	297·7	220·1	—	—	—	279·9	208·1
1949	182·5	190·5	223·2	253·6	191·0	174·8	279·8	233·7	—	—	—	309·8	221·6
1950	195·1	206·6	237·3	289·3	201·5	182·8	304·5	256·2	—	—	—	339·2	229·7
1951	201·2	215·2	247·9	313·9	212·7	189·2	345·3	279·7	—	—	—	348·2	234·9
1952	195·9	206·2	245·8	310·5	217·8	195·2	344·1	279·3	—	—	—	347·6	208·3
1953	207·9	219·8	256·0	356·1	213·5	198·2	295·3	284·4	—	—	—	385·4	225·4
1954	222·5	237·6	263·7	392·4	230·1	208·6	349·2	308·4	—	—	—	434·7	256·6
1955	234·5	253·1	274·1	415·7	249·2	224·1	388·8	340·5	—	—	—	480·9	262·8
1956	234·4	249·9	271·1	431·1	251·1	229·5	369·4	331·5	—	—	—	449·0	249·3
1957	237·6	254·4	264·3	448·1	255·2	235·5	361·4	342·9	—	—	—	461·8	254·5

United Kingdom, 1900–57 (1924 = 100)

Industry: Metal Goods n.e.s.	Textiles	Leather	Clothing	Food, Drink and Tobacco: Total	Food	Drink	Tobacco	Timber	Paper and Printing	Other Manufacturers	Mining and Quarrying	Building and Contracting	Gas, Water and Electricity Supply
—	88·7	96·6	104·7	76·0	47·7	128·3	52·5	89·1	49·0	—	86·0	108·0	39·1
—	91·0	92·3	104·2	76·0	48·8	126·9	51·2	87·1	46·1	—	83·4	108·5	41·6
—	92·0	84·2	106·1	78·2	52·8	126·6	52·9	94·2	50·6	—	86·7	119·0	43·7
—	86·4	76·2	105·9	79·7	56·5	124·8	54·1	90·3	52·0	—	87·8	115·4	46·1
—	88·7	76·9	102·1	80·3	59·1	121·9	55·3	82·7	53·6	—	88·8	105·7	44·3
—	99·1	71·5	104·1	80·3	60·2	120·0	56·4	87·6	54·1	—	90·2	95·2	48·3
—	101·8	77·5	107·0	81·2	61·3	120·7	57·2	91·0	55·3	—	95·4	94·1	51·6
84·1	108·5	79·8	108·3	82·9	63·9	120·7	59·7	84·4	60·7	67·3	100·9	87·4	55·2
—	96·5	84·2	105·5	81·5	62·4	118·4	61·1	84·9	63·7	—	98·2	75·0	57·1
—	102·4	82·6	104·9	82·4	65·1	117·2	60·4	84·9	62·8	—	98·8	74·9	58·5
—	99·0	91·1	109·3	85·8	70·3	118·0	63·1	81·6	68·5	—	98·8	70·8	60·1
—	111·0	82·5	111·5	88·6	71·7	122·7	66·3	78·2	64·9	—	101·2	59·9	63·6
—	119·2	93·0	114·3	92·0	77·0	123·9	67·6	79·1	71·4	—	96·6	57·9	67·1
—	121·7	89·3	116·8	94·1	78·4	127·2	69·9	85·9	74·6	—	106·3	64·0	70·6
—	102·6	100·5	114·6	92·5	78·0	121·7	74·4	84·8	75·5	—	98·7	58·0	75·6
—	137·7	129·7	105·3	91·4	80·8	112·0	80·2	83·6	70·5	—	93·5	45·0	77·3
—	122·9	101·9	92·5	89·7	82·7	105·2	77·1	60·4	49·9	—	93·8	35·2	80·6
—	110·0	121·5	82·9	76·2	75·9	75·7	79·6	35·3	30·7	—	91·1	24·5	85·3
—	94·4	130·5	70·9	72·7	69·7	74·5	81·7	41·8	33·4	—	84·3	20·8	89·3
—	122·9	121·4	78·7	91·7	78·2	106·2	110·2	57·2	67·7	—	84·9	29·8	92·8
115·1	107·8	75·7	80·4	101·2	89·3	117·3	106·9	71·9	89·9	—	86·1	66·2	90·5
71·4	71·6	63·5	86·0	93·4	85·7	103·5	97·9	66·8	53·3	—	61·7	91·7	82·8
81·3	102·8	79·7	94·9	93·3	93·2	92·1	97·2	79·6	84·1	—	91·7	75·9	84·9
95·4	86·7	91·6	94·2	97·5	97·4	97·7	97·4	91·1	94·5	—	102·1	63·1	92·0
100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
106·3	99·4	99·0	103·3	102·7	103·5	100·8	105·0	107·2	102·2	—	92·2	127·7	105·3
107·9	95·6	95·6	104·7	103·2	106·2	97·1	108·7	112·3	105·8	—	51·1	148·5	109·5
113·8	101·0	104·0	111·9	106·7	110·4	99·3	113·6	125·4	110·6	—	95·9	163·4	117·3
117·2	95·0	107·3	112·9	109·8	115·0	98·7	121·8	129·6	110·5	—	91·2	131·6	123·6
124·1	96·0	94·3	114·1	112·7	117·9	99·9	130·2	142·6	116·7	—	99·1	142·2	131·0
119·3	83·9	103·4	113·7	113·8	120·5	98·5	131·8	143·5	117·4	129·2	94·2	131·3	133·4
107·4	88·4	102·3	112·7	110·7	123·3	89·2	123·0	132·5	116·8	—	84·9	124·1	136·9
109·0	95·8	103·1	112·8	110·5	131·2	79·1	118·7	120·9	122·2	—	80·6	117·0	140·4
113·9	104·1	107·2	119·3	114·1	133·3	84·4	123·8	138·5	125·4	150·5	80·7	136·0	148·7
135·7	107·3	108·2	120·8	121·0	140·1	91·5	130·1	148·3	129·5	165·7	86·6	153·4	159·8
150·1	111·6	117·2	123·8	129·3	151·1	96·7	136·9	163·4	132·5	177·3	87·5	157·0	174·4
167·2	121·7	127·6	128·6	135·0	157·2	101·2	144·4	180·0	139·5	—	90·5	171·9	192·5
182·5	122·5	125·9	131·0	142·7	163·7	109·3	155·7	184·7	142·9	200·3	95·4	179·2	208·7
180·5	111·3	110·0	129·5	146·7	168·3	112·7	159·3	178·7	141·7	—	90·7	171·8	216·3
208·9	86·5	102·1	85·2	162·8	182·8	124·2	216·2	133·0	132·9	208·7	70·6	131·9	289·8
222·6	93·5	116·2	90·9	162·1	183·5	125·1	194·1	126·0	141·5	256·5	72·7	137·6	300·6
232·1	108·3	166·5	95·3	168·9	196·3	126·9	184·5	139·7	147·7	306·4	76·9	149·9	319·5
232·8	117·3	117·5	103·9	175·5	207·7	124·4	182·7	157·2	170·3	314·7	79·4	156·9	341·2
242·5	128·2	119·8	109·9	180·7	219·9	122·1	184·7	169·9	196·7	357·9	80·1	157·1	371·6
262·5	128·4	113·0	105·8	177·5	213·0	118·4	191·0	178·1	207·2	376·3	82·9	150·9	395·5
257·4	104·2	100·0	100·2	182·4	223·0	119·4	188·4	160·8	174·9	341·0	83·8	155·4	407·4
239·5	123·4	110·3	108·9	192·5	241·6	118·8	193·5	173·6	196·3	371·4	83·5	166·7	427·8
257·9	127·1	112·1	108·8	196·3	246·6	121·1	201·1	203·0	229·1	426·8	84·4	173·0	462·6
289·2	125·0	111·8	112·7	198·3	242·4	131·0	204·1	204·8	249·3	469·1	83·4	174·3	488·2
285·5	124·3	106·1	114·9	204·0	251·3	134·0	204·1	195·0	246·4	453·8	84·0	184·5	511·5
269·9	123·9	106·8	114·1	206·9	251·3	142·4	209·8	200·3	250·4	486·6	84·1	184·5	519·5

(iii) Working party reports, Anglo-American Productivity Council reports and other published industry studies.

(iv) Published information in books, periodicals, academic and trade journals, reports of trade associations, etc.

(v) In some cases, in the absence of output data it became necessary to estimate movements between census years on the basis of raw material input. Here the important article by Saunders (1952) was one useful source of material. Trade returns were also fruitful when an input was almost wholly imported. When raw material input was used as an indicator it was necessary to correct for changes in output per unit of input. This was done by calculating the output-input relation in census years and assuming smooth movement in the ratio between these points.

(vi) In a small number of, generally, relatively unimportant industries, when neither output nor raw material input figures were available, recourse was made to Ministry of Labour employment data. These often had to be adjusted for changes in scope and definition. Corrections also were always necessary for changes in labour productivity. Again, these were affected by calculating output-employment relations in census years and assuming smooth changes in the ratio between census points.

Lastly, but by no means least;

(vii) A considerable amount of unpublished information provided by private individuals, firms, trade associations, development councils, and the like. Most of these must remain anonymous since the data were obtained under the assurance of confidentiality. The industry groups in which this unpublished information was most important were Building Materials, Chemicals, and Engineering.

A rough count indicates that approximately 250 separate annual series were used in the construction of the final index-numbers. A comprehensive list of the individual sources of these data for interpolation between census points is given in the Appendix. The assembly of this considerable quantity of information permitted the computation of annual index-numbers, 1900–38, for the 80 or so most important industries within the broad fields of Mining and Quarrying, Manufacturing, Gas, Water, and Electricity Supply, and Building and Contracting which is the normal coverage of an index of industrial output. Results of the above calculations are included in Table 1 for the corresponding 17 Orders of the Standard Industrial Classification, together with some convenient subdivision. Metal Manufacture is broken down into ferrous and non-ferrous metals. Separate series are provided for Mechanical Engineering, Electrical Engineering, and Shipbuilding. The Food, Drink, and Tobacco group is divided into its main constituent parts. In Table 1 the series are continued through to 1957 using the official index-numbers, from 1946 on, in a manner described below.

2.3. *The Quality of the Index-numbers*, 1900–38

The index-numbers, naturally, vary a good deal in quality. Greater confidence can be placed in the values since 1920 compared with earlier years. There is more than one reason for this. Much more information is available about output since the first world war compared with pre-1920. Several of the most reliable production series for the inter-war period do not extend beyond 1914. The records of private firms rarely go back as

far as 1900 and many trade associations did not exist in pre-1914 days. Secondly, for all industries we have pivotal points at 1924, 1930, and 1935, and for many at 1933, 1934, and 1937 as well, based on census material and, clearly, very reliable. Before 1924 we have only the two pivots at 1907 and 1912. The war years, 1914–18, have to be regarded with some reserve. For many groups such as Mining, Metal Manufacture, Textiles, Food, Drink, and Tobacco, and the Utilities, etc., there is little reason to doubt the values, but with groups where there was a considerable switch to war production the figures must be used with caution. Not too much caution is necessary, however, because some of these industries happen to be just those for which output changes were estimated from raw material or labour input and omission of war material from output statistics would not affect the results. With regard to comparisons between industry groups, in general, it must be admitted that some difficulty has been experienced with the Engineering, Building and Clothing Orders, largely because of obstacles to satisfactory measurement with both the census material and the annual information. In each of these groups, census output data are presented, mainly, in value terms only, so that even the calculation of the pivotal values involved the rather hazardous business of computing suitable deflators to allow for price changes. This proved not too troublesome for Building and for Clothing since fairly reliable price index-numbers exist for the output of these industries but it was by no means an easy exercise for Engineering. Something can be got from movements in the average values of U.K. exports but in foreign trade statistics, just as with the census, there is difficulty in specifying many engineering products in meaningful quantitative terms. Again, some indication can be obtained from movements in iron and steel prices and in engineering wage-rates. With the annual figures several formidable problems arose. In Building, for example, most of the annual information related to "plans passed", a concept which is rather tenuously connected with actual output. Then, with Shipbuilding the long period of production leads to difficulty in output measurement. Certainly no single one of the series: tonnage started, tonnage under construction, or tonnage completed, would be completely satisfactory. Again, Mechanical and Electrical Engineering form one of the sections in which published material of any kind was particularly sparse. It was necessary here to rely to a considerable extent on figures supplied, confidentially, by private firms. Naturally, there was much less scope for this in relation to the period before the first world war.

It will be noticed from Table 1 that where there is no information at all or where the data are rather weak no values are entered. This applies only for certain years to a relatively small number of the less important groups.

2.4. *Comparisons with Earlier Calculations*

There now arises the question how the new index-numbers for 1900–38 compare with previously used indicators. The only annual indices running back continuously to 1900 are those due to Hoffmann (1955), Ridley (1955), and O.E.E.C. (1955). Beveridge (1944) provides an index of industrial activity but the figures presented give merely fluctuations about the trend. Hoffman calculates an original total index, including and excluding Building, an index for consumer goods, one for producer goods, and tabulates the individual series of which the index-numbers are composed. The total index including Building is presented in the form of a double ten-year moving average. Ridley links together, after

adjustment, various already available indices covering parts of the period from 1900, to produce a continuous total index. This is not subdivided by industry at all. The O.E.E.C. index covering Manufacturing, Mining, and the utilities but excluding Building is subdivided into broad industrial groups. A full description of the calculation of these O.E.E.C. indices is not given but a list of sources appended suggests that the method adopted was

TABLE 2

Total Industrial Production (1924 = 100)

Year	*Including Building*		*Excluding Building*		
	Ridley Index	*New Index*	*Hoffmann*	*O.E.E.C.*	*New Index*
1900	70·7	73·9	85·0	—	71·1
1901	70·7	73·6	83·6	71·4	70·8
1902	70·7	76·4	85·1	72·9	72·9
1903	70·7	76·4	85·1	72·9	73·2
1904	70·7	75·7	84·7	71·4	73·3
1905	75·7	78·3	90·4	77·1	76·9
1906	78·9	80·2	93·8	78·6	79·1
1907	81·3	81·6	95·7	80·0	81·2
1908	75·0	77·1	90·5	74·3	78·8
1909	78·8	78·9	92·0	80·0	79·2
1910	80·4	80·6	95·4	81·4	81·3
1911	86·2	83·0	98·8	85·7	84·9
1912	89·1	84·2	100·2	90·0	86·4
1913	94·2	90·5	110·3	97·1	92·6
1914	—	84·8	103·1	—	87·0
1915	—	86·4	104·4	—	89·9
1916	—	81·8	98·7	—	85·6
1917	—	76·4	93·9	—	80·7
1918	88·3	73·8	89·1	—	78·2
1919	100·0	81·3	97·6	—	85·5
1920	102·3	90·3	100·2	97·1	92·4
1921	71·7	73·5	62·7	65·7	72·0
1922	86·8	85·0	85·7	85·7	85·8
1923	89·5	90·0	93·2	97·1	92·5
1924	100·0	100·0	100·0	100·0	100·0
1925	100·7	103·9	98·6	98·6	101·7
1926	88·8	98·4	83·5	84·3	93·9
1927	112·0	113·4	110·4	107·1	108·9
1928	109·2	110·2	108·5	105·7	108·2
1929	117·3	115·8	116·5	112·9	113·3
1930	108·4	110·8	109·5	105·7	109·0
1931	99·0	103·7	99·9	97·1	101·9
1932	99·7	103·2	100·4	100·0	102·0
1933	108·7	110·1	107·2	107·1	107·7
1934	123·0	121·1	121·3	118·6	118·0
1935	130·8	130·3	125·8	128·6	127·9
1936	143·4	142·0	135·6	141·4	139·1
1937	153·1	150·5	144·4	152·9	147·7
1938	146·2	146·4	129·0	142·9	144·1

merely to link together existing indices covering different parts of the period in the Ridley manner.

The new total index, broadly, in general trend agrees fairly well with Ridley and O.E.E.C. throughout the period. There is apparent serious disagreement with the level of the Hoffmann index in the first half of the period but fair agreement thereafter. The comparisons are shown in Table 2. For comparability with O.E.E.C. Building had to be excluded from the new index and, since the Hoffmann index including Building is given

in the form of a moving average, the basis of comparison, here, too, is taken to be the total index excluding Building.

The disagreement with the Hoffmann index seems to hinge almost entirely on the immediate post-war period. In fact the contrast in post-war behaviour of the two indices is most marked. The growth in output from 1900 up to 1913 as measured by these different indicators, is almost identical and the fall from 1913 to 1918 little different. Yet the increase from 1919 to 1924 is 17% for the new index and only 2% for Hoffmann. Thus the well known and often-quoted criticism of the Hoffman index that it seriously under-estimates the growth of output up to 1924 in fact should refer predominantly to the period 1919–24. Outside this period the trends exhibited by Hoffmann are in fair agreement with the new calculations. This can be seen if the Hoffmann index is "adjusted" by scaling the annual values 1900–19 by a constant factor. Table 3 illustrates this and also shows the broad agreement in trend of the various long-run indices.

TABLE 3

Annual Average

Total Industrial Production (1924 = 100)

	Including Building		*Excluding Building*		
Period	*Ridley Index*	*New Index*	*Hoffmann "Corrected"*	*O.E.E.C.*	*New Index*
1900–06 . . .	72·6	76·4	74·4	74·1	73·9
1907–13 . . .	83·6	82·3	83·6	84·1	83·5
1914–19 . . .	—	80·7	83·8	—	84·5
1920–24 . . .	90·1	87·8	88·4	89·1	88·5
1925–31 . . .	105·1	108·0	103·8	101·6	105·3
1932–38 . . .	129·3	129·1	123·4	127·4	126·6

Indeed it appears to be true that the discrepancy with Hoffmann can be located even more finely. It is noted from Table 2 that with 1924 = 100 the 1922 and 1923 values for the two indices are almost identical. Hence it is the change over the period 1919–21 which is in such marked contrast.

While the broad sweep of the new index agrees with that of the Ridley and O.E.E.C. indices and, apart from the serious contrast over 1919–21, with Hoffmann, too, in certain individual years quite substantial differences are observed, which is not unexpected. The timing, though not the amplitude of the fluctuations, is similar. Generally, the new index tends to show rather less severe falls in output in years of recession. There was probably inadequate representation of new and rapidly developing products in the older indices and a tendency to rely too heavily on traditional declining products which suffered severely in times of bad trade. This can, perhaps, be seen more clearly from comparisons with annual indices (Rowe, 1924; Lond. and Camb. Econ. Ser., 1939; the *Board of Trade Journal* and the *Statistical Abstract*) covering shorter periods which are usually subdivided by industry group. The annual London and Cambridge Economic Service index 1920–38, for example, omitted altogether the rapidly growing Gas, Water and Electricity Supply, Building Materials, and Timber products groups. The advancing Chemicals group was also clearly seriously deficient.

TABLE 4

Index-numbers of Production (1924 = 100)

Chemicals and Allied Trades

Year	*London and Cambridge*	*New Index*
1920 . . .	94·7	101·3
1921 . . .	64·0	72·9
1922 . . .	83·4	86·3
1923 . . .	93·5	94·1
1924 . . .	100·0	100·0
1925 . . .	93·3	96·6
1926 . . .	77·4	89·0
1927 . . .	96·9	101·9
1928 . . .	104·2	106·8
1929 . . .	109·8	112·2
1930 . . .	102·8	106·6
1931 . . .	90·8	102·3
1932 . . .	99·6	109·1
1933 . . .	97·5	114·2
1934 . . .	100·2	123·3
1935 . . .	106·3	133·9
1936 . . .	110·4	139·6
1937 . . .	121·9	148·7
1938 . . .	113·3	141·0

It is true, on the other hand, that the London and Cambridge index omitted Clothing which was not a rapidly expanding group and considerably over-estimated the increase in Metal products and in Building which related only to houses built and ignored other types of construction.

TABLE 5

Index-numbers of Industrial Production

	Excluding Building (1924 = 100)		*Including Building* (1930 = 100)	
Year	*Board of Trade*	*New Index*	*Board of Trade*	*New Index*
1924 . . .	100·0	100·0	—	—
1927 . . .	106·8	108·9	—	—
1928 . . .	105·5	108·2	—	—
1929 . . .	111·8	113·3	—	—
1930 . . .	103·2	109·0	100·0	100·0
1931 . . .	93·7	101·9	—	—
1932 . . .	93·3	102·0	—	—
1933 . . .	98·6	107·7	—	—
1934 . . .	110·8	118·0	106·1	109·3
1935 . . .	—	—	113·5	117·6
1936 . . .	—	—	124·4	128·1
1937 . . .	—	—	132·8	135·7
1938 . . .	—	—	124·3	132·1

Comparison with the Board of Trade index brings out fairly clearly the tendencies of a currently published, quickly available index to under-estimate the rate of growth in output trend and to over-estimate the falls in recession years.

Index-numbers for Census of Production years only, based on the census data, have, of course, been calculated before, notably by Tolles and Douglas (1930) and Devons (1939). These figures were adjusted and corrected in various ways and presented in a revised form by Maddison (1955). Indices for broad industrial groups and in total are

available, although Maddison confined his attention to manufacturing industry. In principle these calculations for census years should agree with the new index tolerably well and in total this indeed proves to be the case. Comparison with the Maddison index is fairly typical and, certainly, most appropriate:

TABLE 6

Index-numbers of Industrial Production (1924 = 100)

Manufacturing Industry

Year	*Maddison*	*New Index*
1907	78·9	79·9
1924	100·0	100·0
1930	107·0	109·6
1935	133·2	131·3

When individual industry groups in the new index are compared with the earlier calculations for census years some differences are revealed. There could be several reasons for these. The methods differed in a number of respects. In the first place, the weighting problem has been much more comprehensively treated in the new index. Maddison, for example, following Devons, used merely 1930 weighting for the 1924, 1930, 1935 indices. Secondly, a more elaborate small firm correction was made compared with that used by Maddison. Then Maddison and apparently Tolles and Douglas made no adjustment for the secession of Southern Ireland in 1923. Finally but possibly most important of all was the different treatment accorded to items specified in value only in the census and for which no quantitative data exist. Tolles and Douglas were not consistent in their treatment of this problem but certainly substantial parts of output were not represented in the resulting index. The Devons method, while consistent and systematic, meant again that necessarily by no means all the output recorded in the census reports was in the calculation of the index-numbers. Our method was to include all output even though some temerity had occasionally to be shown in compiling suitable deflators to correct value figures for price changes.

2.5. *Annual Index-numbers*, 1946–57

The index-numbers, 1900–38, were computed as far as possible on the Standard Industrial Classification so that it was possible to link up with the official post-war index of production. The link was provided by the calculations of Brown (1954) in which the level of output in 1948 was compared with 1935 on the basis of census data. Brown presented results with both 1935 and 1948 as base years, so that it was possible to compute for each industry a 1948–35 linking factor as a geometric mean of the indices on the two bases. This was consistent with the method used for 1900–38. Unfortunately, Brown omitted Building from his calculations so that for this group the comparisons with 1935 given in the pamphlet on the Interim Index (Central Statistical Office, 1949) had to be used and spliced on to the Brown figures. It was thus possible to construct continuous series, apart from the 1939–45 war years, from 1900 up to 1957. The results are presented in Table 1.

2.6. *Production: The Final Picture*

The main outlines of the picture shown by the new statistics are, of course, familiar. Fig. 1 portrays the movements in the total index from 1900 to 1957. The graph is drawn on a ratio scale.

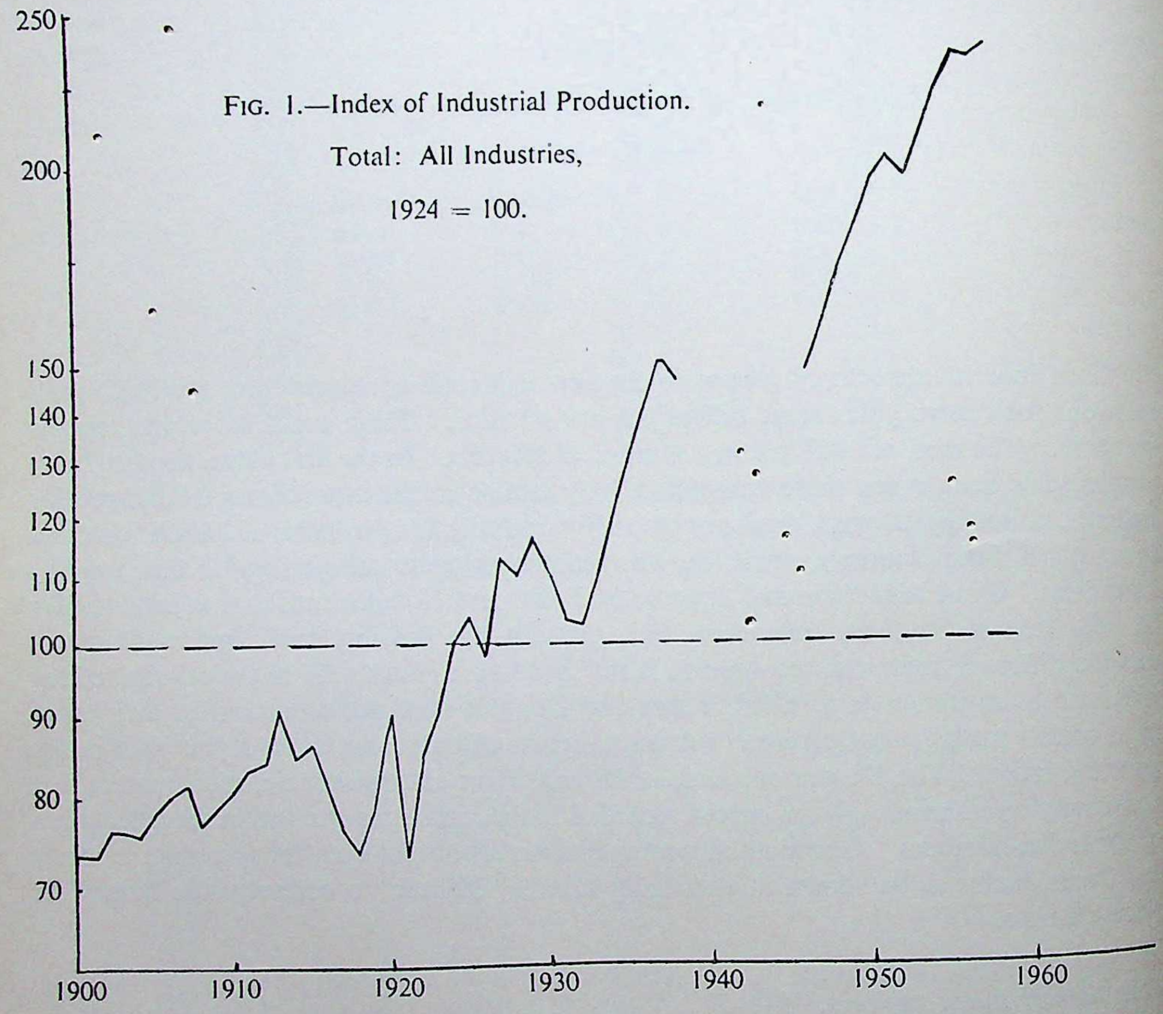

FIG. 1.—Index of Industrial Production. Total: All Industries, 1924 = 100.

The trend exhibited by the index of total industrial production corresponds to an accelerating rate of growth throughout the period. From 1900 up to the outbreak of the first world war total industrial production increased at a cumulative rate of only 1·6 per cent. per annum. In the inter-war period, the average rate of cumulative advance was 3·1 per cent. per annum. Since the last war, total output has increased cumulatively at 4·6 per cent. per annum. The average rate of cumulative growth for manufacturing industry in the three periods has been 2·0 per cent., 2·9 per cent., and 5·1 per cent., per annum respectively. The most striking feature among the groups is the contrast between the rapid growth of the Vehicles, Utilities, Chemicals, and Metal-using groups and the relative stagnation of Textiles, Leather, Clothing and Mining. The level of output in Electrical Engineering increased more than 24 times between 1900 and 1957, in Vehicles 17 times, in Gas, Water and Electricity Supply, more than 13 times, and in Chemicals

nearly $7\frac{1}{2}$ times. The 1957 volume of production in Mining, on the other hand, was actually below that of 1900, in Leather and Clothing only about 10 per cent. above, and in Textiles 39 per cent. above the 1900 level. It can be argued that this relative switch from the older traditional industries to the newer developing trades should have been even more rapid for Britain's needs as a country strongly affected by trends in foreign trade. With the industrialization of overseas territories world demand for machinery was increasing rapidly whereas we were meeting increasing competition, at home and abroad, and shrinking markets for Textiles and Coal. The rather modest rate of growth in Mechanical Engineering up to 1938 would appear to support the view that we were falling behind our competitors in this respect.

It is, indeed, illuminating to compare rates of growth in industrial output in different countries over the three principal subperiods since 1900. In these calculations slight variations occur in the periods taken in order to secure comparable trends. It was endeavoured always to measure the rate of increase between points at the same phase of the trade cycle; for example, from peak to peak and not from trough to peak. Figures for manufacturing industry are taken to ease comparability.

These rates of growth were calculated from figures published in the two main sources, (League of Nations (1945), United Nations (1948–)), of pre-war and post-war national production index-numbers. They are, in consequence, subject to the qualification that

TABLE 7

Average Rates of Increase in Manufacturing Output per cent. per annum

	1900–13	1920–37	1948–56
United Kingdom	2·0	2·9	5·1 (1955)
United States	5·2	2·5	4·2
Canada	8·1	3·4	3·6
Belgium	4·1	4·1	4·2
France	3·5	3·4	6·9
Netherlands	—	3·7	7·6
Sweden	3·6	5·0 (1938)	2·3
Germany	3·9	4·2 (1938)	—
New Zealand	7·6	3·4 (1938)	5·4 (1955)
Japan	6·4	6·6 (1938)	—

the quality of these index-numbers for other countries has not been examined in any detail. Even so, it seems fairly clear that the rate of expansion in the United Kingdom, generally, compared unfavourably with countries overseas before the first world war, but there has been progressive improvement in the comparative position since then. In the most recent period, since 1948, the rate of growth in the United Kingdom has been by no means depressing relative to experience abroad. Figures for Western Germany and Japan since the war have not been entered because these countries, starting from a very low level of output, have experienced enormous increases quite out of line with the rest of the table.

3. PRODUCTIVITY MOVEMENTS IN THE UNITED KINGDOM*

The measurement of productivity movements poses many difficult questions. It is

* The calculations in this part of the paper bear obvious similarities to some of the work of Maddison (1955). While none of Maddison's figures has been used, he must be recognized as entitled to claim priority in the field.

commonly the practice to compare production movements with changes in some measure of labour input. This leaves out of account other inputs or factors of production such as capital, raw material, management, etc. which will also have been changing. Most of these other variables are, however, generally difficult to measure and sometimes even conceptually vague and elusive. Further, it is useful to know what changes have taken place in output per unit of labour input and, perhaps, to explain these partly in terms of changes in the other factors. Another problem is whether to use total employment or merely number of operatives as a measure of labour. It is the number of operatives which is more sensitive to changes of trade. It is the operatives who are more liable to be dismissed when trade is bad. But an operative basis ignores the changing proportion of operatives as capital investment and more mechanization takes place. As a process becomes more automatic, more administrative and supervisory personnel are needed and relatively fewer operatives. Then again, should hours of work be brought into account? This certainly seems desirable if productivity comparisons are to be made over a long period during which there have been big changes in hours worked. Yet it could be the case that an operative works shorter hours but works harder and produces just as much as when hours of work were longer. There are, then, possible objections to almost any method of comparison.

In relating production movements to changes in labour input over a fifty-year period—the problem here—it seemed most appropriate, all things considered, to work on an output per operative-hour basis. No attempt was made to allow for changes in length of holidays or in the age and sex composition of the labour force.

An immediate difficulty was that figures of actual hours worked were only available for certain years. It is only since 1940, in fact, that the regular Ministry of Labour earnings and hours enquiries have been carried out. Before that, enquiries were held only in 1906, 1924, 1928, 1931, 1935 and 1938. The years 1928, 1931 and 1938 were, however, times of recession and it might have been misleading to include in a comparison of this kind, years at different phases of the trade cycle. Again, the numbers of "operatives" employed in the different industries are given only in the employment section of a census of production. Ministry of Labour employment figures, normally, do not distinguish "operatives". Indeed, annual labour statistics before the early 1920's are rather primitive. Provided the (September/October) 1906 hours of labour could be taken to apply to 1907 there were, then, strong reasons for confining the pre-war productivity comparisons to the census years 1907, 1924 and 1935. It is tempting to argue that the use of census employment data ensured comparability between output and operative-hours but, in fact, an output index calculated from census material must be on a "principal products" basis whereas census employment figures always relate to the trade. Strict comparability, for all industries, is really quite impossible to achieve.

Measurement of productivity changes since the war raised a number of considerations. Hours of labour statistics have existed continuously throughout the period and we have had an annual census of production in which numbers of operatives are tabulated for each industry. The 1948 Census applied, however, to Great Britain only and not to the United Kingdom. All later censuses repaired this omission. Finally, output has been more or less stationary since 1955 and during this time there has clearly been idle capacity in the economy. All this suggested that the most sensible and appropriate post-war comparison would be 1949–55.

TABLE 8

Average Rate of Productivity Increase in the United Kingdom per cent per annum*

	1907–24	1924–35	1935–49	1949–55	1907–55
Total: All industries	1·6	2·2	1·3	3·2	1·9
Total: Manufacturing	2·0	2·0	2·0	3·1	2·1
Building materials	1·9	2·1	2·3	2·5	2·1
Chemicals	1·3	2·3	2·7	6·2	2·5
Metal manufacture	1·1	1·7	1·2	3·3	1·5
Engineering, etc.	1·3	1·3	1·0	3·3	1·4
Vehicles	5·1	3·7	4·0	3·9	4·3
Metal goods	2·4	1·7	1·3	1·8	1·8
Precision instruments, etc.	0·6	3·3	2·2	1·2	1·8
Textiles	0·7	2·3	2·0	1·4	1·5
Leather	2·1	0·7	1·1	−0·3	1·2
Clothing	1·8	1·5	0·3	2·0	1·3
Food, Drink, Tobacco	1·2	1·0	1·6	0·5	1·2
Timber trades	2·4	1·0	0·1	5·3	1·8
Paper and Printing	3·2	1·0	2·5	4·2	2·6
Other manufacturing	1·3	3·6	2·1	4·3	2·4
Building and Contracting	1·7	1·2	−3·6	3·6	0·2
Gas, Water and Electricity	1·9	2·8	3·7	4·5	2·9
Mining and Quarrying	−0·8	2·6	−0·4	−0·1	0·2

* Output per operative-hour.

The output index-numbers were thus compared with changes in operative-hours over the periods 1907–24, 1924–35, 1935–49, 1949–55. The results in the form of average rates of increase in output per operative-hour are presented in Table 8.

This is a picture of fairly steady progress but at rather modest rates of productivity increase. The rates of change have risen quite markedly in the most recent post-war period. The well known phenomenon of productivity advance being closely associated with production growth is again apparent. The period of most rapid growth of productivity has also been that during which output has most increased. Productivity rarely seems to rise when the economy is stagnant or in recession. Performance in the individual groups has mostly been as expected although there is much scope for analysing and explaining individual industry movements in a detail which would be out of place here. The new developing industries and those in which there have been important technological changes show up in the most favourable light. The consistently high rate of productivity growth in Vehicles and also, to a rather lesser extent, in Paper and Printing are examples. The rapidly accelerating movement in Chemicals and, again, in the Utilities group are striking cases of productivity in newer industries following the production growth pattern.

It is difficult to derive comparable productivity figures for many other countries. Canada and the United States are two cases where comparison is feasible on an output per operative-hour basis for manufacturing industry.

TABLE 9

Average Rates of Productivity Increase per cent per annum: Manufacturing Industry

	1907–24	1924–35	1935–49	1949–55, 56
United Kingdom	2·0	2·0	2·0	3·1
United States	3·2	3·3	1·5	3·7
Canada	—	—	1·6	2·2

The figures for the United States and Canada have been calculated from data provided in the *U.N. Statistical Year Book*, the *Federal Reserve Bulletin*, the *Statistical Abstract of the United States* and the *Historical Statistics of the United States*.

It is seen that the United Kingdom performance relative to the United States and Canada has been, perhaps, not as depressing as is the general impression. A word of warning is necessary, however, again concerning the quality of the index-numbers used for other countries. It is known, for example, that United States production index-numbers contain an appreciable labour element. Output changes in a number of industries are measured by changes in man-hours worked. This would tend to depress any measure of productivity change. Such a serious deficiency is not present to any significant extent in the United Kingdom figures. The employment content of the new index-numbers calculated is quite negligible, and, in any event, is corrected for changes in productivity.

APPENDIX: SOURCES OF DATA FOR INTERPOLATION BETWEEN CENSUS POINTS

Order II: Mining and Quarrying

Coal mines:
Annual reports of the Secretary for Mines.
Ministry of Fuel and Power Annual Digest.
Statistical Abstract.

Metalliferous Mines and Quarries:

Iron ore and ironstone Lead ore Tin ore Zinc ore Tungsten ore etc.	Annual reports of the Secretary for Mines. Statistical Abstracts of the U.K.

Salt:
Annual reports of the Secretary for Mines.
Imperial Institute.
Statistical Abstracts of the U.K.

Slate:
Annual reports of the Secretary for Mines.
Statistical Abstracts of the U.K.

Non-Metalliferous Mines and Quarries:

Igneous rocks Clays and shales Gravel and sand Sandstone Limestone, etc.	Annual reports of the Secretary for Mines. Statistical Abstracts of the U.K.

Order III: Building Materials

Bricks and Fireclay: Sample of private firms.
China and Earthenware: Home consumption of ball and china clay.
Cement: Cement Maker's Federation.
Glass: Private firms.
Building Materials: Building plans passed.
Manufactured Fuel:
Ministry of Fuel and Power.
Statistical Digest.

Order IV: Chemicals and Allied Trades

Chemicals, Dyestuffs, and Drugs:

Heavy chemicals: I.C.I. production index.
Dyes and dyestuffs: Reports of Dyestuffs Development Committee and *Board of Trade Journal.*
Coal tar products: National Benzole Company.
Sulphuric acid: National Sulphuric Acid Association Ltd. and net imports of sulphur.
Ammonia products: Trade Association and Annual reports on Alkali etc. works by the Chief Inspector.
Sulphate of copper: British Sulphate of Copper Association.
Oxygen: British Oxygen Company Ltd.
Acetylene: British Oxygen Company Ltd.
Sodium phosphates: Information from private firms.
Chloroform: Private firms.
Sulphuric ether: Private firms.
Cellulose acetate: Private firms.
Acetone: Private firms.
Various chemicals: Minerals for use in Chemicals and Allied Industries; Annual reports of Secretary for Mines.

Fertilizers:

Fertilizer Manufacturers Association.
Gelatine and Glue Federation.
Retained imports of phosphate of lime and rock phosphates.

Seed Crushing: Retained imports of oil-seeds.

Soap, Candles, etc.:

Soap: Information from private firms and retained imports of soap-making materials.
Candles: Estimate by expert in trade.

Coke and by-products:

Coke: O.E.E.C. Industrial Statistics, Secretary for Mines Reports, and Ministry of Fuel and Power Digest.
Tar: Alkali Act reports.
Pitch: Alkali Act reports.

Petroleum Refining: Board of Trade Journal.
Industrial Explosives: Board of Trade.
Starch and Polishes: Private firms.
Matches: Monopolies Commission Report.
Paint: Interpolation between census points on employment basis.

Order V: Metal Manufacture

Iron and Steel; Blast Furnaces:

National Federation of Iron and Steel Manufacturers.
British Iron and Steel Federation.

Iron and Steel; Smelting and Rolling:

N.F.I.S.M.
B.I.S.F.

Tinplate:

E. H. Brooks *Chronology of the Tinplate works of G.B.*
N.F.I.S.M. and B.I.S.F.

Foundries: Foundry pig iron production adjusted for foundry and forge imports and exports.
Tubes: Estimates from private firms.

Non-ferrous Metals:

Imperial Institute; the mineral industry of the U.K.
International Tin Study group statistical Year Book.
Metallgesellschaft Statistische.
Economist Annual Commercial Histories.
American Bureau of Metal Statistics.
G. A. Roush: *The Mineral Industry.*
The Mond Nickel Co. Ltd.

Non-ferrous Metals (cont.):
Aluminium Development Association.
W. Lewis: *The Light Metals Industry*.
General: Committee on Industry and Trade; Survey of Metal Industries.

Order VI: Engineering, Shipbuilding, and Electrical Goods
Mechanical Engineering:
1900–19. Interpolation effected mainly on the basis of iron and steel consumption figures (Saunders, 1952) and employment data.
1920–38. Individual series for following items compiled:

Locomotives:
Railway returns.
Railway Gazette
Railway Handbook.
Railway Yearbook.

Estimates from private firms:
Gas and oil engines
Steam engines and turbines
Boilers and boiler-house plant
Cranes, excavators, and material handling equipment
Furnace plant
Pumps and pumping machinery
Pressure gauges
Bearings
Industrial valves
Printing machinery
Machine tools
Textile machinery
Refrigerating machinery
Marine engines and equipment
Grain milling machinery
Paper-making machinery
Air heating, filtering, and ventilating plant
Sugar-making machinery
Sugar confectionery and jam-making machinery
Machinery for brewing industry
Agricultural machinery
Air and gas compressors and exhausters
Water meters
Gas meters and gas street lighting appliances
Office machinery and accessories
Coin-operated vending machines
Metal windows

Electrical Engineering:
1900–19. Interpolation effected mainly on the basis of employment data.
1920–38. Individual series for following items compiled:
Switchgear and control gear: Sample of private firms.
Generators, motors, transformers, etc: Sample of private firms.
Electrical instruments and meters: Sample of private firms.
Electric lighting equipment: Sample of private firms.
Electric lamps: Electric Lamp Manufacturers' Association.
Batteries and accumulators: Private firms.
Radio valves: British Radio Valve Manufacturers Association.
Wireless sets: Estimated from licensing statistics and foreign trade.
Domestic electrical appliances: Sample of private firms.
Wires and cables: estimated from consumption of copper.

Shipbuilding:
Interpolation on basis of composite index compiled from:
Tonnage launched, Tonnage commenced, Tonnage under construction: Lloyds Register.

Telegraph and Telephone Apparatus:
Interpolation on basis of information from private firms.
General: Committee on Industry and Trade; Survey of Metal Industries.

Order VII: Vehicles, etc.

Motor Vehicles and Cycle Trades—

Motor Vehicles:
Private cars and taxis, Commercial vehicles: Annual reports of the Society of Motor Manufacturers and Traders.
Motor Cycles, *Cycles*: Figures from the British Cycle and Motor Cycle Manufacturers and Traders' Union Ltd.
Aircraft: Miscellaneous figures from wide variety of publications relating to development of air transport.

Railway Rolling Stock and Equipment:
Interpolation on basis of figures from:
Railway returns.
P. Redfern: "Net investment in fixed assets in U.K.", *J. R. Statist. Soc.* A, 1955.
A. K. Cairncross: *Home and Foreign Investment.*

Order VIII: Metal Goods n.e.s.

Interpolation on basis of information from private firms, trade associations and employment data.

Order IX: Precision Instruments, etc.

Interpolation on basis of employment data.

Order X: Textiles

Cotton Spinning and Doubling:
Cotton yarn produced:
G. T. Jones: *Increasing Returns* for early years.
Joint Commitee of Cotton Trade Organizations.
J. Jewkes and G. N. Daniels: *Lancashire Cotton Industry* (Manchester Statistical Society: 1927).

Cotton Weaving:
Cotton piece goods produced:
Joint Committee of Cotton Trade Organizations.
Cotton Board.
Cotton yarn consumed:
G. T. Jones: *Increasing Returns.*
Direct estimates from production, foreign trade, and "other use" statistics.

Wool:
Wool consumption figures:
Saunders, (1952).
Hoffmann (1955).
Machinery activity index: Woollen and Worsted Trades' Federation.

Silk and Art Silk:
Raw silk consumption:
Trade returns.
Rayon and Silk Directory.
C. T. Saunders.
Rayon production:
Rayon and Silk Directory.
Customs and Excise Returns.
Balfour Committee on Ind. Trade.
Rayon consumption:
Rayon and Silk Directory.
C. T. Saunders.

Jute:
Net imports of raw jute.
Numbers of bales cut up: Association of Jute Spinners and Manufacturers.

Linen and Hemp:
Consumption of flax and hemp: C. T. Saunders.
Yarn production in the flax spinning trade: Flax Spinners' Association.

Textile Finishing:
Statistics from private firms.
Employment data.
Output of cotton, wool, silk, etc., piece goods.

Hosiery: Interpolation on basis of employment figures.
Other Textiles: Employment data.
General: Committee on Industry and Trade; Survey of Textile Industries.

Order XI: Leather, Leather Goods, etc.

Fellmongery: Estimates from private firms.
Leather production: *Consumption of Hides:* W. G. Hoffmann. Data from United Tanners' Federation.
Leather Goods: *Consumption of Leather:* W. G. Hoffmann.

Order XII: Clothing Group

Clothing:
Home cloth supplies ± foreign trade.
Estimated consumption function for clothing ± foreign trade.
Employment data.

Boots and Shoes:
Private firms.
Federation of Boot and Shoe Manufacturers.
National Institute of Economic and Social Research.
Employment data.

Hats and Caps:
British Felt Hat Manufacturers' Federation.
Employment figures.

Gloves:
Private firms.
"Safeguarding of Industries" reports.

Order XIII: Food, Drink and Tobacco

Food:

Grain Milling:
Home consumption of wheat:
Statistical Abstract of U.K. and trade returns.
Broomhall's *Corn Trade News.*
R. Stone *Consumers' Expenditure.*
Wheat Commission Reports.
London and Cambridge Economic Service.

Bread, Cakes and Biscuits:
Flour consumed in bread etc. making
Wheat Commission Reports.
A. R. Prest: *Consumers' Expenditure.*
R. Stone: *Consumers' Expenditure.*
London and Cambridge Economic Service.

Sugar and Glucose:
Statistical Abstract of the U.K.
Economist Annual Commercial History and Review.
Report on Sugar Beet Ind. (Min. of Agriculture).
Sugar Industry Committee Report (Cmd. 4871).
Report of Sugar Commission.

Sugar and Glucose (cont.):
Trade accounts for imports of unrefined sugar.
London and Cambridge Economic Service.
Cocoa, Chocolate and Sugar Confectionery:
Estimates from private firms.
National Institute of Economic and Social Research.
Cocoa, Chocolate and Confectionery Alliance.
Bacon and Ham:
Ministry of Agriculture (Bacon Marketing Scheme).
Imperial Economic Committee.
Interpolation on basis of pig-meat available.
Lard: Estimates from private firms.
Margarine: Estimates from private firms.
Dairy Produce
Butter, Cheese, Condensed milk, Cream, Milk powder: Stone: *Consumers' Expenditure.* National Institute of Economic and Social Research.
Ice Cream: Stone: *Consumers' Expenditure.*
Fish curing: Stone: *Consumers' Expenditure.*
Cattle, Dog and Poultry Foods:
Ojala: *Agriculture and Economic Progress.*
Ministry of Agriculture.
Drink:
Brewing and Malting
Beer:
Statistical Abstract of the U.K.
Customs and Excise Reports.
Brewers' Year Book.
Malt:
Quantities used in Brewing and Distilling.
Statistical Abstract Customs and Excise.
± Foreign Trade (Trade Returns).
Spirits Distilled:
Annual Statistical Abstracts.
Monthly Trade and Navigation Accts.
Customs and Excise reports.
Spirit Rectifying: Customs and Excise reports.
Aerated Waters, Cider, etc.
National Institute of Economic and Social Research.
Statistical Abstract of the U.K.
R. Stone: *Consumers' Expenditure.*
Tobacco:
Quantity of unmanufactured tobacco entered for home consumption (Trade Accounts).
Figures given in Tobacco chapter (anonymous) in *Sources and Nature of the Statistics of the U.K.* Vol. I. Edited by M. G. Kendall.

Order XIV: Timber Group
Saw milling etc: Interpolation on basis of consumption of softwoods (Saunders, 1952) and employment data.
Furniture:
Information from private firms.
Retained imports of hard wood.

Order XV: Paper and Printing
Paper: Retained imports of paper-making materials; trade accounts.
Wallpaper: Figures from private firms.
Printing, etc.: Interpolation based on employment.
Newspapers: Circulation figures. Consumption of newsprint.
Cardboard Boxes etc.: Interpolation based on employment figures.

Order XVII: Building and Contracting

Building:

Building plans passed: *Ministry of Labour Gazette* and private source.
Numbers of new buildings assessed to income tax; Statistical Abstracts.
Houses built: Ministry of Health.

Local Authorities: Interpolation on basis of local authority loan expenditure.

Order XVIII: Gas, Water, and Electricity Supply

Electricity: Garcke's *Manual of Electricity Undertakings.* Electricity Commission reports.
Gas: Board of Trade annual returns relating to authorized gas undertakings.

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Discussion on Mr. Lomax's Paper

Miss PHYLLIS DEANE: As an eager consumer of the kind of statistics presented in to-day's paper I am particularly glad to take this opportunity of proposing a vote of thanks. Mr. Lomax has constructed a new industrial index of production from the results of eight censuses or partial censuses and some 250 annual output indicators, and the result is the most elaborate and reliable index of industrial production yet produced for this country for such a long period. Those who have had to battle with the task of reconciling or interpreting the conflicting indices of British industrial progress so far available for this period will welcome Mr. Lomax's index and admire his achievements.

My own comments on the paper are from the point of view of a student of long-term growth, and my interest in the index is much more in its broad trends than in its year to year variations.

My first query relates to the treatment of Ireland. I assume that what we have here is an index which omits Southern Ireland from the whole period 1900–1957 and I take it also that the correction for Southern Ireland was so small that in fact it scarcely affected the result. Perhaps Mr. Lomax would confirm these assumptions and say whether there are any industrial groups or major industries for which the correction for Southern Ireland was not either nil or negligible.

My second point relates to the weighting system. It would be helpful to have in addition to the rather detailed and useful list of sources given in the Appendix an indication of the weights attached to the principal industries or industrial orders. This would assist those who would like to re-compile the indices in different ways, or possibly to compare them in more detail with other production indices.

Thirdly, I wonder if Mr. Lomax would elaborate on his reasons for making his productivity estimates on an output per operative hour basis instead of an output per employee hour. Was it largely because he lacked information on hours of work of salaried staffs or because he thought it gave a more interesting theoretical result to measure productivity in terms of operative hours?

I come to a point of interpretation. Most would-be users of this index will find section 2.4 and the annual comparisons in Table 2 particularly helpful. In this connection I should like to take issue with Mr. Lomax on his interpretation of the difference between his index and the Hoffman index. This seems to me a particularly important comparison because the Hoffman index covers a much longer period and will generally be used to carry the index back into the nineteenth century. I find it difficult to agree with Mr. Lomax "that the broad sweep of the new index agrees . . . apart from the serious contrast over 1919–1921, with Hoffman." It seems also to be misleading to say that " . . . the well known and often-quoted criticism of the Hoffman index that it seriously under-estimates the growth of output up to 1924 in fact should refer exclusively to the period 1919–1924."

This is perhaps a disagreement in point of view, in perspective even, rather than in substance, but as I see it the most striking feature of the comparison between the Hoffman index and the new index is that the Hoffman values fluctuate at a totally different level in the first two decades of the period. This is not true of the comparisons with the Ridley and the O.E.E.C. indices. The Ridley index varies widely in the post-war years when conditions were abnormal anyway, but it is at much the same sort of level as the new index for the first two decades. It is this prolonged higher level of the first two decades which accounts for the low rate of growth exhibited by the Hoffman index. If we omit the abnormal war- and post-war years altogether and compare the averages for the decades 1904–1913 and 1924–1933 we find that the Lomax index implies an increase of nearly 29 per cent. in industrial output compared with nearly 9 per cent. suggested by the Hoffman index. This is a substantial difference.

A difference of perspective, a longer-term point of view, would also lead me to qualify a little the optimism of the final picture suggested by Mr. Lomax. This shows an accelerating rate of growth from 1·6 per cent. in the period before World War I to 3·1 per cent. in the inter-war period, and then again to 4·6 per cent. after World War II. This I think needs to be considered in relation to the very severe setbacks of the two world wars. If a long-term measure is made, for example, from the decade 1900–1909 to the end of the period covered by this index, 1948–1957, there appears to have been an average rate of growth of about 2·1 per cent. per annum. This is roughly the same as the rate of growth which Hoffman finds for the period 1850/59–1900/09, that is, the previous half century.

There is more than one way of looking at any statistical series, however, and different users will derive varying implications according to their different methods. To have reduced such a formidable mass of statistical material to the order of this new index is, in my view, a substantial achievement and I am glad to propose a hearty vote of thanks to Mr. Lomax.

Mr. T. M. Ridley: Although criticising indices of production has been a fairly popular pastime for quite a while this is the first opportunity I have had of indulging in it. Being concerned in the computation of these indices, it has always seemed, from my side of the fence, that their critics are inveterate perfectionists. I suppose, therefore, that on this occasion I should point out some of the ways in which the paper before us falls short of the ideal.

At the end of section 2 Mr. Lomax states that the years 1946 to 1948 were linked by

means of the official interim index of production. This index shows a rise of 21 per cent. against the 15 per cent. rise shown in Table 1. The official index based on 1948 was projected back to 1946 and I suspect that it was this index that was used for the link.

Coming to section 2.1, those interested in the statistical theory of index numbers will be disappointed that what I call the *vulgar* formula—that is, net output weights applied to gross output quantities—has been used throughout and that no attempt has been made to connect the pivot years by means of the formula associated with the name of Dr. Geary. The remarks in the opening paragraph of the Introduction about the deficiencies of existing annual index numbers lead one to suppose that only ideal technical methods would have been used, or at least tried out.

My chief criticism of the index concerns the comparisons with other calculations given in section 2.4. No doubt an index of production depending on a few basic raw material output statistics with no allowances for the building up and running down of stocks will, in some cases, be unduly sensitive to short-term fluctuations. On the other hand it is also true that Mr. Lomax's partial dependence on employment statistics and input material with smoothly graduated correction factors (section 2.2, paras. v and vii) makes for positive insensitivity in the inter-pivot years. He himself makes a significant admission on the close association between productivity and production movements towards the end of the paper.

Moreover, indices derived from the output statistics of a sample of private firms will also be unduly insensitive unless the sample includes some of those firms which go out of production in a depression and some of those which "mushroom" in a boom. If the sample is confined to well established businesses reminiscent of Marshall's "representative firm" there is a useful reminder by Mr. D. C. Hague in the *Economic Journal* (December, 1958) that in Marshall's view, when output rises the extra production is likely to come not from the representative firms but from those just struggling into business. Whether or not Mr. Lomax's samples are properly balanced we do not know. For some obscure reason we are told (section 2.2, para. vii) that most of the firms and individuals providing information must remain anonymous and that the production records of one or two generations ago must be regarded as confidential! This makes a proper appraisal of the figures difficult, but I am glad to hear that it will not necessarily prevent anyone taking up from where Mr. Lomax and his colleagues have left off.

Shortage of time rules out an adequate discussion of the figures given in Table 1 but I should be grateful if Mr. Lomax would satisfy my curiosity about the reasons for the very low figure for mechanical engineering in 1919. The figure looks all the more surprising against the record figure in the adjacent column for electrical engineering in the same year.*

I prefer to leave the discussion of the section of productivity movements to other speakers, but I would like to remove the impression given that hours worked by operatives are available for certain pre-war and post-war years on a comparable basis. In the first place the hours and earnings enquiries were not conducted on a comparable basis; there were several changes of definition in the pre-war years. Secondly, the figures are not available for years but only for certain individual weeks, the weeks being selected for their optimum rather than their average characteristics; any annual figures derived from them are, in fact, derived from a biased sample of 2, 4 or in one case 8 per cent. only. One consequence is that changes in the length of holidays, believed to be substantial since the end of the war, have had to be explicitly ignored.

In his concluding paragraph Mr. Lomax states that it is known that the United States production index numbers contain an appreciable labour element. This seems to conflict with the claim of the Federal Reserve Board that in their annual index of production man-hour figures account for only 4 per cent. of the weighting (*Federal Reserve Bulletin*, December 1953, p. 1,254).

While having certain reservations about using the figures in Table 1 for the calculation of productivity indices, I feel that Mr. Lomax has made a valuable contribution to the

*This entry has now been corrected—Editor.

economic history of a period which, for speed of technical development, clearly surpasses the industrial revolution which started 200 years ago. Indeed, it is not impossible that future historians may judge the period leading up to the death of the steam locomotive to be more significant, from the point of view of industrial development, than the period leading up to its birth, and in this event the results of the research embodied in the paper before us would become a valuable legacy.

I have much pleasure in seconding the vote of thanks.

The vote of thanks was put to the meeting and carried unanimously.

Mr. MAIZELS: Mr. Lomax's new series will fill an important gap in our knowledge of production trends over the past half century. His results are particularly useful because they are based on the Standard Industrial Classification.

Mr. Lomax has been rather modest in presenting his paper. I am sure that he has much useful working material which could well be published. This is particularly true of the series for the constituent items within each industry group. It would also be useful to have, as Miss Deane has suggested, the weighting systems used at the census dates. It would help in judging the reliability of the annual series if Mr. Lomax would say how well his original unadjusted series fitted the movement as recorded at the censuses.

I now turn to the detailed techniques which Mr. Lomax has used to construct his index numbers. First, Mr. Lomax's procedure is to calculate indices of the volume of gross output for individual trades, and to weight these by the net output of each trade. This procedure assumes that the technical input-output ratio for each industry remains constant over time, and that the gross output volume measures the volume of work done. We know that there is a general tendency for more work to be done per unit of materials, so that in fact this method is bound to result in indices with a downward bias. Mr. Ridley suggested that the Geary method might be tried. I think this method, which involves re-valuing materials consumption as well as output, could indeed be used to a limited extent for the period 1933–37. The Import Duties Act, 1932, allowed the Board of Trade to collect information about the quantity and cost of materials purchased and used, and the 1933 Inquiry was the first at which some detailed particulars were collected. The list of materials was extended progressively, until in 1937 the list was quite considerable. It would, nevertheless, be necessary to make assumptions about the price movement of materials not recorded by quantity, though these assumptions might in fact be more reasonable than those underlying Mr. Lomax's method.

Second, Mr. Lomax's indices are defined as relating to the output of "principal products", and not to the whole output (whether principal products or not) of an industry or industry group. The distinction between the two concepts is an important one, and I find rather lacking in the paper a discussion of the advantages or disadvantages of each. Though in principle one should derive the same index number for total industrial production on either the "principal product" basis or the "trade" basis, this is not the case for individual industries or even industry groups. The advantage of using the "principal product" basis is that comparisons can be made with commodity statistics, such as those of foreign trade: but no such comparisons are made by Mr. Lomax. The advantage of using the "trade" basis, on the other hand, is that the production indices would then be comparable in scope with other trade aggregates, and in particular with the employment figures. Mr. Lomax is conscious of this difficulty, but he argues that "strict comparability . . . is really quite impossible to achieve". In fact, his procedure may have introduced some significant errors into his productivity calculations.

Third, in allowing for the output of small firms (p. 189) Mr. Lomax makes two main assumptions: (*a*) that the price movement of the output of small firms from one census to the next is the same as for large firms in each industry; and (*b*) that the ratio of "principal products" to the output of the trade is the same for small firms as for large.

The first assumption seems a reasonable one in general. But the second may be mis-

leading and, in any case, seems unnecessary. The small firms tend to specialize much more than the large firms, so that the ratio of their characteristic products to their total output would be higher. A better assumption would therefore be that this ratio remained the same up to 1937 as it was in 1924, for which year full details of the small firms' output were published.

Fourth, Mr. Lomax (p. 190) estimates the net output content of principal products by making the further assumption that the net output content of principal products of the large firms is equal to the net output content of their non-characteristic products. This assumption again is unnecessary and could be misleading. For many trades a better estimate can be derived from the ratio of net to gross output for specialist groups within each industry.

Fifth, and more important than the points made earlier, is that Mr. Lomax nowhere mentions the problem of duplication. Unless changes in the degree of duplication are allowed for, the indices of production may be seriously distorted. One example relates to the movement in the volume of building work between 1924 and 1930. The census report for 1930 gives a volume index of 129 (with 1924 = 100), while Mr. Lomax gives an index of 131 for the same trade. However, the census index relates only to the output of the large firms and there was a very large expansion in the output of the small firms in this period. The relevant employment figures are as follows:

	1924 (*Thousands*)	1930 (*Thousands*)	*Increase* (*Per cent.*)
Large firms	419	454	+8
Small firms	136	163	+20
Total	555	617	+11

Since the small firms worked very largely on sub-contract work for the large firms, it follows that a large part of the volume increase of 29 per cent. was in fact produced in the small firms, though recorded as part of the output of the large firms. Taking the increase in total employment and allowing for some increase in productivity it still looks doubtful whether the total volume of production increased by more than 20 per cent., after eliminating the duplication in the recorded output figures. There are numerous other examples one could give. Duplication is extremely important in the textile industries and in the motor car trade and calculations which do not attempt to adjust for this are inevitably suspect.

My final technical point relates to Mr. Lomax's use of cross-weighting in calculating the movement in production from one census year to the next. In theory, as Mr. Lomax says, when several base years are included in a calculation of this sort there is nothing to choose between them. However, in practice there may be something special about one of the base years which precludes its use in a weighting system. I believe that this was true of the year 1930, which Mr. Lomax uses as one of the weighting links. The recession in that year caused prices to fall and there was a fall both in physical output and in new orders. Though stocks may have increased in some sectors, such increases would have been involuntary and would have been offset, or more than offset, by reductions in stocks of materials, finished goods and work in progress elsewhere. Businesses would, in any case, have written down their stock values at the end of their accounting years to the lower prices then ruling. These changes, affecting different industries and products in very different ways, almost certainly resulted in an abnormal relationship of the recorded unit values of the main items of output. This explains why 1930-based volume indices are invariably significantly different from those based on 1924 or 1935. It would, I think, have improved his series had Mr. Lomax omitted the 1930 results from his weighting system.

Coming now to the series presented in Table 1, I was rather struck by the differences between the movement from 1937 to 1938 shown by Mr. Lomax's index for Total Manu-

facturing, and the corresponding changes shown by other authorities. The relevant figures are:

		Per cent.
1.	London and Cambridge Economic Service	−10
2.	O.E.E.C.	−6
3.	Board of Trade	−6*
4.	Mr. Ridley	−4½*
5.	Mr. Lomax	−3
6.	Prof. Stone	−1
7.	Prof. Bowley	0

* Including building.

I must confess that I am not entirely convinced that manufacturing output fell by only 3 per cent. between 1937 and 1938. One general indicator is the level of employment in manufacturing industry, which also fell by 3 per cent. in this period. We know that since the war a fall in employment has generally been accompanied by a more than proportionate fall in output; but it is not at all certain which way output per head moved in 1938. The other general indicator is the volume of exports of manufactured goods, the official index falling by 12 per cent. in 1938. Assuming exports represented one-fifth of total production of manufactures in 1937, a fall of only 3 per cent. in the total implies no significant change in output for the home market. The evidence suggests, however, that there was a recession in home demand, as well as in exports, in 1938 so that, on this basis, the fall in production shown by Mr. Lomax looks on the low side.

It may be that the Board of Trade can throw some further light on the extent of the recession in industry in 1938. An Import Duties Act Inquiry was made for 1938, with a more restricted industry coverage than in 1937, though the 1938 Inquiry was interrupted by the outbreak of war.

Mr. Lomax has shown fairly conclusively that the pre-war annual production indices failed to reflect accurately the movement shown by the full Census detail. This disparity is always likely to be present in the construction of a monthly or annual index, so that the point applies equally to the present official index published by the Central Statistical Office. The next important task in this field therefore seems to be the calculation of volume indices for the post-war period based on the full census details. This could in principle be done both on a gross output and on a net output basis. In view of the magnitude of the computations, this will presumably have to be undertaken by the official statisticians.

Miss Maton: I should like to follow up the remarks about weighting. My impression is that although this index has been presented as a consistent index covering a long period, one could alter it by using a different pattern of weighting. The one used is not necessarily the best one for all the year to year movements over the period. We know that the effect of using different bases for weighting can be very significant. Mr. Brown calculated the increase in output between 1935 and 1948 using 1935 weights at 40 per cent., but using 1948 weights at 28 per cent. I would like to ask Mr. Lomax if he would give us the various sets of industry weights he has used to construct the series for the two periods, 1907–1924 and 1920–1938. It would be very informative to see what are the differences in the weighting patterns. It would also be most useful if we could know what the index numbers for each industry, and for the total of all industries, would be for each of the years 1907, 1924, 1930 and 1935, using each of these years in turn for weighting; I think that would help us to form a better judgment of the value of these indices.

My second point concerns the use of census material to derive the changes between the pivotal census years. I doubt whether that material is really adequate to give us a precise measure of what I think we mean by the change in production. This can be illustrated by the figures available for the engineering industry. The 1907 census gives no quantity figures. The 1924 and 1930 censuses give quantity information in terms of the weight of fairly broad categories of plant and machinery. All we know is that so many tons of, say,

weighing machinery were produced in 1930 and so many in 1924; this is scarcely an adequate basis for a precise measure of the change in the volume of production. For 1935 and the post-war full censuses we have obtained output information in more detail. Thus the changes in production between the successive pivotal years covered by the new series must inevitably rely on information which is far from precise but which becomes gradually more precise as the years go by. I would suggest to Mr. Lomax for his further consideration that this gradual increase in the detail available could have the spurious effect of accelerating the rate of industrial expansion during the period 1900–1935, as measured by the new series.

My last point relates to the comparisons between particular industries. The information available from industries about their current output is very different in type. A factor is thus introduced which is quite unrelated to the changes in production one is trying to compare, and must to a large extent invalidate comparisons over a long run of years between the performance of different industries.

Mr. R. Robson: Perhaps one of the most striking comparisons made in this paper is that between the new index and the pre-war Board of Trade index of industrial production, the former showing between 1924 and 1934 about twice the rate of growth shown by the latter. It would be interesting to know how far this difference is due to the different weighting of the various industries and how far to differences in the rate of growth attributed to the individual industries. In so far as it is due to the latter the question arises to what extent the present method includes a purely statistical increase arising from the greater detail now shown in census returns than was shown in earlier years. This factor could be important if we are comparing rates of growth in different countries. We have to be sure in such a comparison that the index of production of country A is not only weighted on the same basis as that of country B but also that within each industry it relates to the same range of principal products, otherwise we may get a difference which reflects in part the excellence or otherwise of the statistical material in the two countries. One wonders whether it might not be better to rely on value figures and to make more use of suitable deflators which would show different changes in quality.

It would also be interesting to know how much of the increase in productivity shown is due to the relative growth of high productivity industries and how much to increases in productivity within each industry. If in fact we take an arithmetic average as a measurement of the latter then it suggests that almost all the increase is within industry, which I find rather surprising. For productivity comparisons it is surely necessary to take account of changes in quality: for example, the output per head of yarn is roughly inversely proportional to its fineness, and there are somewhat similar relations in other reduction processes, e.g. steel rolling. One can imagine the effect on a productivity index which does not take account of fineness if there is a substantial increase, for example, in 1,500 denier tyre yarn as compared with 15 denier hosiery yarn. One wonders, therefore, whether there is much point in making productivity comparisons unless quality is allowed for or unless one makes a detailed industry-by-industry study.

Mr. Gribbin: Mr. Lomax's new index is a magnificent effort and he has given us a series which will end many arguments about the facts and start many arguments about what caused the changes he has shown.

Many speakers have suggested how Mr. Lomax could have improved his index had the information been available but I should like to make a few remarks about how the present index of industrial production could be improved from information which is now available. One of the things which strikes a person entering business is the speed of change. It is important that statisticians who use or prepare the index of industrial production should realize this. I suspect that they often fail to do so, and this is unfortunate because one gets the impression that we are presented with a rather precise measure of industrial activity. This has been particularly important in the economic discussions of the last few years.

What brought this to my mind was the table on page 4 of the *Official Handbook of the Index of Industrial Production* which gives the percentage of industry accounted for by the index. In many cases it is claimed that this is as high as 95 per cent. or even 100 per cent. This is quite a misleading conception of the index. The same impression is obtained from a recent article in *Economic Trends* which made a comparison between the new index of industrial production and the old index. The view was put forward that because the new index gave the same total as the old, we could be satisfied with its performance. But this is not so. I have made a calculation about one industry which I know in detail: the soap industry. The indicator used by the Board of Trade for this industry is the output of soap. Soap is not as important as it used to be, its place having been taken by the new synthetic detergents. If we use the Board of Trade indicator of this particular industry, the output in 1957 was 1 per cent. higher than in 1948. However, if we make a calculation which includes the new synthetic detergents, the output of the industry in 1957 was more than double that of 1948. This is confirmed by the figures of personal consumption of soap and detergents given in the Blue Book on National Income. I suspect that examples of this kind might be found in many industries. A comparison of similar sets of indicators with different weights is not a good test of the adequacy of the index. A much better test would be whether we can work from one detailed census year to another and obtain the correct change in output at constant prices. I suggest that for many industries a better index of the changes in output would be the value of output corrected for changes in the price of the output of the industry.

A further point I should like to make is that it seems unfortunate that the Board of Trade appears to be taking a detailed survey less frequently than it used to. I suspect that this is because of pressure from people in industry who are not willing to fill in the detailed forms for the census. Two possible changes here might enable us to take a more detailed census more often: firstly we should try to help people in industry by showing them how the census information can be used to their own advantage. I do not think enough is being done in this direction. Secondly, we could explore the possibility of taking more of the detailed censuses by means of samples.

Mr. G. R. White: The industry with which I am associated, namely, leather, happens to be one of those described as stagnating. I do not think we stagnate quite as much as Mr. Lomax's index seems to suggest. One important reason why the index for leather is so depressed in his index is another aspect of the problem of duplication mentioned by Mr. Maizels.

Over the 55 years or so covered by the index there have been very great changes in the leather industry. There is now a large production of leather produced by chrome tanning, whereas in the earlier stages leather used for a similar purpose was almost entirely vegetable tanned. In the latter process it is usual for one firm to take the hides and tan them to a rough tanned stage and then pass them to another firm. In the chrome process the firm starts with the raw hide and finishes it as dressed leather.

I will illustrate what this may mean. Under the vegetable tanning process we start with raw material at a value of, say, 70 and produce rough tanned leather which sells at 100. The man who buys it puts in more work and ends up with the finished leather which sells at 120. In the census of production that leather would appear twice, once at the value of 100 and again at the value of 120, a gross output of 220. In the chrome tanning process we start with the raw hide at 70 and end with the finished leather at, say, 110. Thus for one unit of finished leather there is a value figure of 220 on one process and 110 on the other. In the period 1930–1948 one of the most important sections of the industry, using the chrome tanning process, increased its output by over 700 per cent.

I feel that this variation in the extent of duplication at different dates inevitably depresses the ultimate index.

Mr. Browning: I would like to take up the point which has been raised with regard to the index of production. Production of detergents has for some time been represented

in the index. New and better information is now being collected and will, it is hoped, be incorporated in the index for soap and detergents in the not too distant future.

A suggestion was also made that one might get a better measure of the volume of production if one took a value series adjusted for price changes. This may well be the case for certain periods and for certain items of output—for example, commodities reasonably homogeneous in their nature and therefore ones about which one can get price information which is reasonably reliable. I must emphasize, however, especially in view of what Mr. Lomax said in section 2.3 of his paper, that it is often very difficult to compile a reliable price index. Fashions and qualities change and it is difficult to disentangle price movements from other factors affecting quotations. It is specially difficult in industries which produce custom-built items—complicated pieces of plant and equipment, power stations and the like. Even where a price index is in itself reasonably reliable, it is normally base-weighted, and care must be taken in using it to deflate a current-weighted value series.

I should like to ask what is to be subsumed under the heading of quality changes. Mr. Lomax has calculated index numbers of industrial production covering a period of nearly 60 years. How has he dealt with those industries which have grown up during the period, such as aircraft, plastic materials and man-made fibres? It is not clear from the paper how satisfactorily the development of industries such as those is reflected in the index numbers in Table 1.

The following contribution was received in writing after the meeting:

Mr. H. Ward: Anyone visiting factories regularly and noting the changes over the last forty years cannot but be impressed by the fairly universal improvement of quality of the products. After the war industry did wonders with poor raw materials. Since then the quality of raw materials has improved. Mr. Lomax ignores quality as though of necessity, but after hearing the discussion one might wonder whether some assessment on the extra productivity due to improved quality is any more difficult to estimate than productivity or production itself. The study of quality is a delightful subject. Criteria must be different for each product. Quality often depends on the initiative of relatively few people. It would be possible to form an estimate of the effort required to produce better and better goods. One might, for example, ask "how many hours would it have taken five years ago to produce the product that is being produced today?" In many industries there are higher rejects than before the war. Production figures are only for goods sold. Present day products may be better because rejects are high. Most buyers these days put on tighter specifications. There are many more ways of speedily and accurately measuring dimensions, weights, surface finish, strength, temperatures and performance than existed before the war. Mass production often demands production to close dimensional limits. Such limits make the next process of machining far easier. I should like to feel that the Royal Statistical Society would induce studies of quality. The study of how to obtain statistics of quality would be useful. Would it be too much to say that the average of all manufactured products, if proper allowance was made for improved quality, would show that productivity has increased by, say, 5 per cent. more over the last six years?

Mr. Lomax: I should like to thank the speakers for the stimulating contributions to the discussion.

There are two points I might mention briefly. One is about the weighting system. The great difficulty there is that the different sets of weights can be published but in using them it is a matter first of working out index-numbers with each set and then taking the geometric mean of the results. If one carries out this process for any year with the individual industry indices the answer will still not be the same as my overall figure because of the final adjustment to bring everything into line with the pivotal points. There was thus the danger that the use of the weighting system might be misinterpreted. Again I had space considerations in mind in not including the full weighting system.

The other point concerns the confidential figures given by firms. When this work was started one had to give the assurance that any information provided would be regarded as

confidential. We thought that this was the right approach and having given that assurance we could not ignore it. It would be difficult to say "This is what we have done, can we publish the figures you gave us?"

Mr. LOMAX subsequently replied in writing as follows:

Several contributers to the discussion mentioned the desirability of publishing further detail. It is intended to publish a good deal more elsewhere, but it would have unduly lengthened the paper to include much more here. I have been awarded a Simon Senior Research Fellowship of the University of Manchester for the purpose of writing a book on the subject of the paper. In this, many more individual industry series will be published and also full schedules of the weighting systems used. These latter will, in fact, serve more than the one purpose of giving the weighting basis on which the index-numbers have been calculated. They will indicate the changing pattern of industrial structure in the United Kingdom since 1900.

On the technical basis of the paper I agree with most of the speakers, particularly Mr. Maizels, that alternative, possibly more satisfactory, assumptions could have been made. When the work was started it was, indeed, intended to look at all the most obvious alternatives, including the Geary method suggested by Mr. Ridley. However, these calculations take such an enormous amount of time that quite early on it was decided to adopt what appeared to be the most generally acceptable compromise. This is not the type of computation which can easily be programmed for an electronic computer. Again it was held to be desirable as far as possible to adopt methods applicable fairly consistently over the whole range of industries and throughout the entire period. The Geary method, for example, is most attractive theoretically but over the period of these calculations only the 1937/35/34/33 comparisons could possibly have been computed on these lines. Even then the schedules of materials used were rather slender and the calculations could only have applied, of course, to industries included in Import Duties Act enquiries.

With regard to some of the suggestions made, it would, I fear, have been possible to do little. Mr. Maizels mentioned duplication. The degree of duplication within individual industries, which is what is relevant here, cannot be allowed for since generally we do not know what the inter-firm transactions are within an industry. The most important cases are, I suppose, cotton spinning and doubling where the doublers buy the single yarn from the spinners, and engineering, building, and, possibly, clothing where sub-contracting by the larger units to smaller firms in the industry is common practice. The extent of duplication in the case of cotton could, perhaps, be assessed, but not in the other industries. The effect of duplication becomes more serious when there have been important changes in process as in the case quoted by Mr. White. Mr. Robson and Mr. Ward pointed out that changes of quality can have a serious effect on the reliability of productivity measurement but, here again, we are limited in the extent to which allowances can be made by availability of data and the detail of the Census classifications. Miss Maton, too, questioned the adequacy of much of the Census material and I would agree with her, but what is the alternative?

I regret that some of the procedures adopted were not sufficiently justified in the paper. A principal products basis was used in preference to a trade break-down, the matter raised by Mr. Maizels, for three main reasons. The Census output figures were all on a principal products basis; consistency between the annual series and the Census data was ensured; and, finally, comparability with currently published index-numbers and linkage with the post-war official index were eased by this approach.

Now, to one or two detailed matters:

Southern Ireland.—Corrections generally were almost negligible. The only industries where they were appreciable were in the Food and Drink group.

Productivity.—The main reason for working with output per operative-hour was that it is the number of operatives (and not total employment) which is sensitive to changes in trade. Generally a wide range of output would be possible with a fixed supervisory employ-

ment element. I agree, however, that this argument is much stronger for short-term comparisons that it is over a long period.

Comparison with Hoffmann.—I am afraid I must hold to my view here. A graph of the two indices brings out clearly, I believe, what I was trying to say. The fact that Hoffmann appears to "fluctuate at a totally different level (in) the first two decades of the period" is simply due to scaling the two indices to 1924 = 100. If, for example, 1913 is put equal to 100 we get:

	Hoffmann	*New Index*
1900	77·1	76·8
1907	86·8	87·7
1913	100	100

which can hardly be described as "widely different levels".

The 1946–48 *link.*—It was, indeed, the official index of production projected back to 1946 which was used. I should not have included the term "interim", although, in fact, the figures for 1946 and 1947 were, of course, based on less complete information than later years in the official index.

Mechanical Engineering in 1919.—I regret that the figure here in the galley proofs was wrong. It has now been corrected in the published version.

New Industries.—By considering separately the sub-periods between successive benchmarks, the problem of splicing on new products and industries becomes, in practice, not too difficult, certainly less troublesome than if a direct comparison of 1900 with 1938 were made. This, inevitably, raises the question of the true meaning of an index number of production extending over a long period such as has been considered; a question not at all easy to answer.

As a result of the ballot taken during the meeting the candidates named below were elected Fellows of the Society:

Ronald Charles Carpenter
Hilary Harding
John Allan Hargreaves
Warren Alexander Hay
Douglas Wynne Holloway
Frederick Charles Piper
Albert Henry Russell
Anthony Barrett Truman
John Edward George Utting

Corporate representative

Peter Brian Mooney *representing* Kemsley Newspapers

Using an Electronic Computer in a Problem of Medical Diagnosis

By T. H. Hollingsworth

*Department of Human Ecology, University of Cambridge**

Summary

An electronic computer was used to find an optimum method of early diagnosis of suspected cases of primary carcinoma of the lung or bronchus. A combination of the signs and symptoms was obtained which proved to be 85 per cent. efficient for the 200 cases on which the analysis was based. The advantages and difficulties connected with the use of an electronic machine in this type of problem are discussed.

Introduction

The signs and symptoms which a patient presents during an illness and the history of his case are the pointers which lead to a diagnosis. It is well recognized that some of them are critical in making the diagnosis and that others only tend to confirm it. Thus one case may present a long history and many signs and symptoms; another, only a few signs, no symptoms, and no history; yet their diagnosis may be the same. A large part of medical training is concerned with making the right diagnosis of a case by correctly weighing each sign and symptom, and this training is continually being amplified by experience. When a new aid to making a diagnosis is introduced, difficulty arises over how much new information it gives. Situations also arise in which doctors cannot agree what the diagnosis should be. Statistical analysis is now well recognized to be a quick way of solving these problems although the really appropriate statistical method of discriminant analysis has very seldom been used.

In this paper the application of discriminant analysis to a diagnostic problem is described which illustrates clearly the difficulties that are likely to be encountered and the achievements of the method. The data were collected by Mr. C. Parish, Consultant Thoracic Surgeon, United Cambridge Hospitals, and Dr. D. Barron Cruickshank, Clinical Pathologist, Sims Woodhead Memorial Laboratory, Papworth Hospital, and were passed to the Department of Human Ecology for analysis. The problem was as follows.

In the course of clinical practice a chest surgeon and his team of workers studied a series of 200 consecutive cases of suspected lung cancer which had been referred to him for operation. All the information found at the initial clinical examination of the patient was recorded in a standard manner, and each case was observed at least until it was apparent whether a carcinoma was really present or not.

A special feature of this study was the inclusion of the result of the cytological examination of the bronchial aspirate by a pathologist. The criteria of malignancy are individual, and so the same pathologist made all the examinations. The aim was to assess the value of the cytological examination of the bronchial aspirate in the early diagnosis of the cases.

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The 200 cases were ultimately divided into two groups:

(a) Those conclusively proved to be cases of primary carcinoma of the lung or bronchus, of which there were 97 cases.

(b) The remaining 103 patients, who did not have a primary carcinoma of the lung or bronchus.

The problem was to discriminate between groups (a) and (b) in an optimum manner on the basis of the information available after the initial clinical examination.

METHOD

The method used was the standard linear discriminant function technique, based on least squares.

A random variable y is assumed to be equal to unity for each patient in group (a) and equal to zero for each patient in group (b). Then each y is equated with a linear function of a set of x_i's plus an error term, say $y = \Sigma\beta_i x_i + \epsilon$, where $E(\epsilon) = 0$. The x_i's represent the factors present or absent in each case, and we may take $x_i = 1$ when the i^{th} factor is present and $x_i = 0$ when it is absent. For example, a case of carcinoma in which only factors 1, 3 and 4 were present would be represented by the equation

$$y = \beta_1 + \beta_3 + \beta_4 + \epsilon = 1.$$

The coefficients β_i are estimated by least squares, and the F-test can be used for testing the significance of the β_i's. If a particular β_i does not differ significantly from zero there is no evidence that the corresponding factor is relevant to the discrimination, and the sign or symptom to which it relates may be ignored in making the diagnosis. The procedure is equivalent to a multiple regression of y on the x_i's. From this point of view, more complicated factors can be readily incorporated into the system: for example, we can regress linearly upon age. The least squares estimates b_i of β_i will give the discriminant function. In practice, $L_i = n_1 n_2 b_i/(n_1 + n_2)$ is easier to find, where n_1 and n_2 are the numbers in each part of the total population. Here $n_1 = 97$ and $n_2 = 103$, so $L_i = 49{\cdot}955\ b_i$.

It is convenient to correct y and each x_i for their respective means before beginning analysis.

FACTORS

There was a large number of factors available, each of which could contribute to the discrimination. It was necessary to decide first which factors ought to be included in the discriminant analysis and which factors might be ignored, and second, how the chosen factors should be incorporated into the system we have described.

Fourteen factors were chosen, and were included as follows:

1. *Sex.*—We took $x_1 = 1$ for a female patient, and $x_1 = 0$ for a male patient.

2. *Age.*—The ages were given in 10-year age-groups, and a linear effect was assumed: $x_2 = 2$ for ages 20 to 29, $x_2 = 3$ for ages 30 to 39, and so on.

3. *Significant weight loss.*—This and the next five factors were clinical symptoms, which were either present or absent. We took $x_i = 1$ for the presence of each factor $(i = 3, 4, \ . \ . \ . \ , 8)$, and $x_i = 0$ for its absence.

The five other symptoms included were:

4. *Haemoptysis.*
5. *Dyspnoea.*
6. *Cough.*
7. *Sputum.*
8. *Pain.*

The remaining symptoms noted were *wheeze* and *other*. *No symptoms* was also recorded separately. As entries under these headings were very uncommon there seemed little point in including them in the analysis.

The next two factors were:

9. *Signs of local extension.*

10. *Signs of distant spread.*—In 9 and 10 the actual signs were not considered relevant. Signs were thus either present or absent and the factors were treated in the same way as factors 3 to 8.

The next three factors were the results of special investigations. They were:

11. *Bronchoscopy.*—There were only two possible results, namely that positive evidence of a tumour was seen or was not seen, and it became a simple factor like 3 to 10. ($x_{11} = 1$ when evidence was visible, $x_{11} = 0$ otherwise.)

12. *Radiology.*—A radiological examination often gave us a site for the suspected tumour. Peripheral sites were subdivided into three groups: shadows under one inch in diameter; and shadows over one inch, which were further divided into solid and cavitated. Since only 15 per cent. of the cases showed peripheral sites, the numbers in these three groups were rather small, and so for simplicity the larger shadows (over one inch) were grouped together. The results of radiological examination were then summarized under one of the following five headings: a *hilar site* involved; a *peripheral site, shadow one inch or under in diameter*; a *peripheral site, shadow over one inch in diameter*; *nothing seen*; *atelectasis, etc.* Radiology as a factor therefore required four degrees of freedom.

13. *Cytology.*—The results of the pathologist's cytological examination of the bronchial aspirate were given in four groups: *positive*, *suspicious*, *metaplastic and inflammatory*, and *negative*. These four were regarded as being spaced at equal intervals on a linear scale. Cytology becomes a factor requiring only one degree of freedom by making a linear regression on this scale. The units were taken as *positive* = 3, *suspicious* = 2, *metaplastic and inflammatory* = 1, *negative* = 0. Placing cytology on a linear scale was a great advantage, and cannot have led to much loss of information, because only 11 per cent. of the cases had *suspicious*, *metaplastic* or *inflammatory* results, and so it hardly matters where we put such a small proportion, provided it is somewhere between the positives and the negatives.

The remaining factor was:

14. *Time from first symptom to cytology.*—This required only one degree of freedom, since we assumed a simple linear effect. Possibly something more elaborate should be sought, but there was no obvious relationship, although the times seemed to be slightly shorter in malignant cases.

ANALYSIS

Preliminary

The mean value for each factor was found for:

(a) Primary bronchial carcinomas (97 cases).

(b) The remainder (103 cases).

The results are given in Table 1.

Taking the factors singly, and doing simple χ^2 tests, age, visibility, site, and cytology appear to be very highly significant ($P < 0 \cdot 001$), haemoptysis and sputum highly significant ($P < 0 \cdot 01$), dyspnoea and cough significant ($P < 0 \cdot 05$), and sex, weight loss, pain, extension, distant spread, and time from first symptom not significant. However, the factors are correlated, and the significance of their contributions to the diagnosis may change when we do the full analysis.

TABLE 1

The Mean Value of Each Factor in the Two Groups*

Factor	*With Carcinoma*	*Without Carcinoma*	*Difference Between Means*
1. Sex	7·2% female	15·5% female	−8·3% female
2. Age	56·3 years	50·1 years	6·2 years
3. Weight loss	51·5%	42·7%	8·8%
4. Haemoptysis	63·9%	40·8%	23·1%
5. Dyspnoea	61·9%	44·7%	17·2%
6. Cough	91·8%	79·6%	12·1%
7. Sputum	85·6%	67·0%	18·6%
8. Pain	54·6%	47·6%	7·0%
9. Extension	8·2%	3·9%	4·4%
10 Distant spread	6·2%	1·0%	5·2%
11. Visible on bronchoscopy	54·6%	3·9%	50·8%
12. Radiology:			
Hilar	73·2%	25·2%	48·0%
Peripheral under 1 in.	1·0%	5·8%	−4·8%
Peripheral over 1 in.	10·3%	12·6%	−2·3%
Nothing seen	2·1%	13·6%	−11·5%
Atelectasis, etc.	13·4%	42·7%	−29·3%
13. Cytology	1·72 units	0·18 units	1·54 units
14. Time from first symptom	19·5 weeks	22·5 weeks	−3·0 weeks

* If a factor was either present or absent, its mean value is the proportion of cases in which it was present, and is shown as a percentage. The mean values of other factors are given in the appropriate units.

The Electronic Computer

The labour of calculation would have been very heavy without the aid of an electronic computer or some other high-speed computing device. In this investigation, the computing time was greatly reduced by using such an electronic computer, namely the EDSAC I at the Cambridge University Mathematical Laboratory. It was used to solve the linear equations which arise, and the solutions were the estimated coefficients of the linear discriminant function.

The contribution of each factor to the sum of squares could then be found at once, and hence the sum of squares for all the factors together.

Discrimination

The first run on the machine gave the estimates when all factors are assumed to matter. The analysis of variance is:

Source	Sum of Squares	Degrees of Freedom	Mean Square
Discrimination . . .	30·2013	17	1·7765
Residual	19·7537	182	0·1085
Total	49·955	199	—

This shows, as we should expect, that the discrimination is highly significant.

The major contributions (not, however, independent) to the discrimination sum of squares were:

Cytology . . .	10·1471	1 d.f.
Visibility . . .	7·2584	1 d.f.
Site	8·0886	4 d.f.

None of the others seemed important by comparison.

(2) We then discarded cytology, and solved the remaining equations. The discrimination sum of squares was 25·9976, with 16 degrees of freedom, and hence we deduce that 4·2038, with one degree of freedom, is the reduction in the sum of squares for cytology after allowing for the effects of all the other factors. This is statistically very highly significant. It means that cytology cannot be ignored without a considerable loss of information relevant to the diagnosis.

The actual results of the cytology were:

Cytology	Actual Condition: With Carcinoma	Actual Condition: Without Carcinoma	Total
Positive	49	1	50
Suspicious	10	4	14
Metaplastic, etc. . . .	0	8	8
Negative	38	90	128
Total	97	103	200

(3) It did not seem worth while doing this procedure again for visibility and site. We assumed that they would always be significant, and accordingly looked at the position when only cytology, visibility and site are taken into account.

The analysis of variance is:

Source	Sum of Squares	Degrees of Freedom	Mean Square
Discrimination using factors 11, 12, and 13 only .	27·3973	6	4·5662
Rest of possible discrimination	2·8041	11	0·2549
Residual	19·7537	182	0·1085
Total	49·955	199	—

The effect of the rest of the possible discrimination is just significant at 1 per cent. So we have found most of the available information by using only 6 of the 17 degrees of freedom, but the remaining 11 do provide us with something useful.

(4) The problem now is to find what simple combination of factors accounts for all but an insignificant part of the remaining sum of squares, 2·8041.

We can get some idea of the effects of the individual factors by a closer study of the components of the total sum of squares when all factors are included. After cytology, visibility and site, the most significant ones seem to be (in order):

Sputum, haemoptysis, age, dyspnoea, and sex.

This provisional order of importance will be altered later. However, sputum and cough were highly correlated (necessarily, all those who expectorated coughed), and the contribution to the "sum of squares" for cough was actually negative. This suggested that the sputum factor might not be very important after all.

The two most obvious factors are age and sex, since they are not subject to observer bias, and so we might as well use them if it will help at all.

Employing the same technique as before, we had three more runs on the EDSAC, allowing for:

(a) Factors 1, 11, 12, and 13 (i.e. sex plus factors already found relevant).

(b) Factors 2, 11, 12, and 13 (i.e. age plus factors already found relevant).

(c) Factors 1, 2, 11, 12, and 13 (i.e. sex and age plus factors already found relevant).

The interest lies in the splitting up of the 2·8041 into components for age, sex, and remainder.

The results were:

(a) Sex only	0·5475	1 d.f.
Remainder	2·2566	10 d.f.

Each is significant at 5 per cent.

(b) Age only	0·4556	1 d.f.
Remainder	2·3485	10 d.f.

Each is significant at 5 per cent.

(c) Age and Sex	0·8866	2 d.f.
Remainder	1·9174	9 d.f.

Each is significant at 5 per cent.

This is rather unsatisfactory in that both factors seem to matter but the remainder still contains more information.

(5) We now introduced factor 4, haemoptysis. Since sex appears to be of more significance than age (in contrast to our provisional order of importance), we had three more runs on the EDSAC, allowing now for:

(d) Factors 4, 11, 12, and 13.

(e) Factors 1, 4, 11, 12, and 13.

(f) Factors 1, 2, 4, 11, 12, and 13.

The combination 2, 4, 11, 12, and 13 seemed pointless, as sex ought to be brought in if age is also included.

The results were:

(d) Haemoptysis only	0·3349	1 d.f.
Remainder	2·4691	10 d.f.

Remainder is significant at 5 per cent.

(e) Sex and haemoptysis	0·9067	2 d.f.
Remainder	1·8974	9 d.f.

Each is significant at 5 per cent.

(f) Sex, age and haemoptysis	1·3088	3 d.f.
Remainder	1·4953	8 d.f.

The three factors together are significant at 1 per cent. The remainder is not significant.

This is much better, and a study of partial sums of squares leads to the conclusion that the order of importance of the three factors is sex, haemoptysis, and age. None of them, moreover, can be ignored, and since the remaining 8 degrees of freedom are not significant, we need go no further.

We have thus found 6 factors out of a possible 14 which discriminate between the two groups.

The regression coefficients which we will use for discriminating are the L_i's $\times 10^4$, where $b_i = 49 \cdot 955\ L_i$. See Table 2.

TABLE 2

Discriminant Coefficients for Each Significant Factor

Factor	$L_i \times 10^4$
1. Sex:	
Male	0
Female	−30·75
2. Age	8·67 (per decade)
4. Haemoptysis	18·92
11. Visible on bronchoscopy	61·50
12. Radiology:	
Hilar site	28·93
Peripheral site, shadow 1 in. or under . .	−26·73
Peripheral site, shadow over 1 in. . . .	−5·05
Nothing seen	−47·61
Atelectasis, etc.	−30·55
13. Cytology:	
Positive	81·56
Suspicious	54·37
Metaplastic, etc.	27·19
Negative	0

We assign a score to each case on this basis. The mean score for the carcinoma group is 150·09,* and for the others 35·06.† The obvious discriminant point is 92·57,‡ half-way between. Fig. 1 shows the distribution of scores in the two groups, with the broken line at 92·57. If we diagnose all those cases (86) with a score above 92·57 as with carcinoma and the remainder (114) as without carcinoma, the effect is as summarized

* On the original scoring basis of 1 for a carcinoma and 0 for no carcinoma, this mean score would be $49 \cdot 955 \times 10^{-4} \times 150 \cdot 09 = 0 \cdot 7498$.
† Similarly, this score would be 0·1751.
‡ 0·4625.

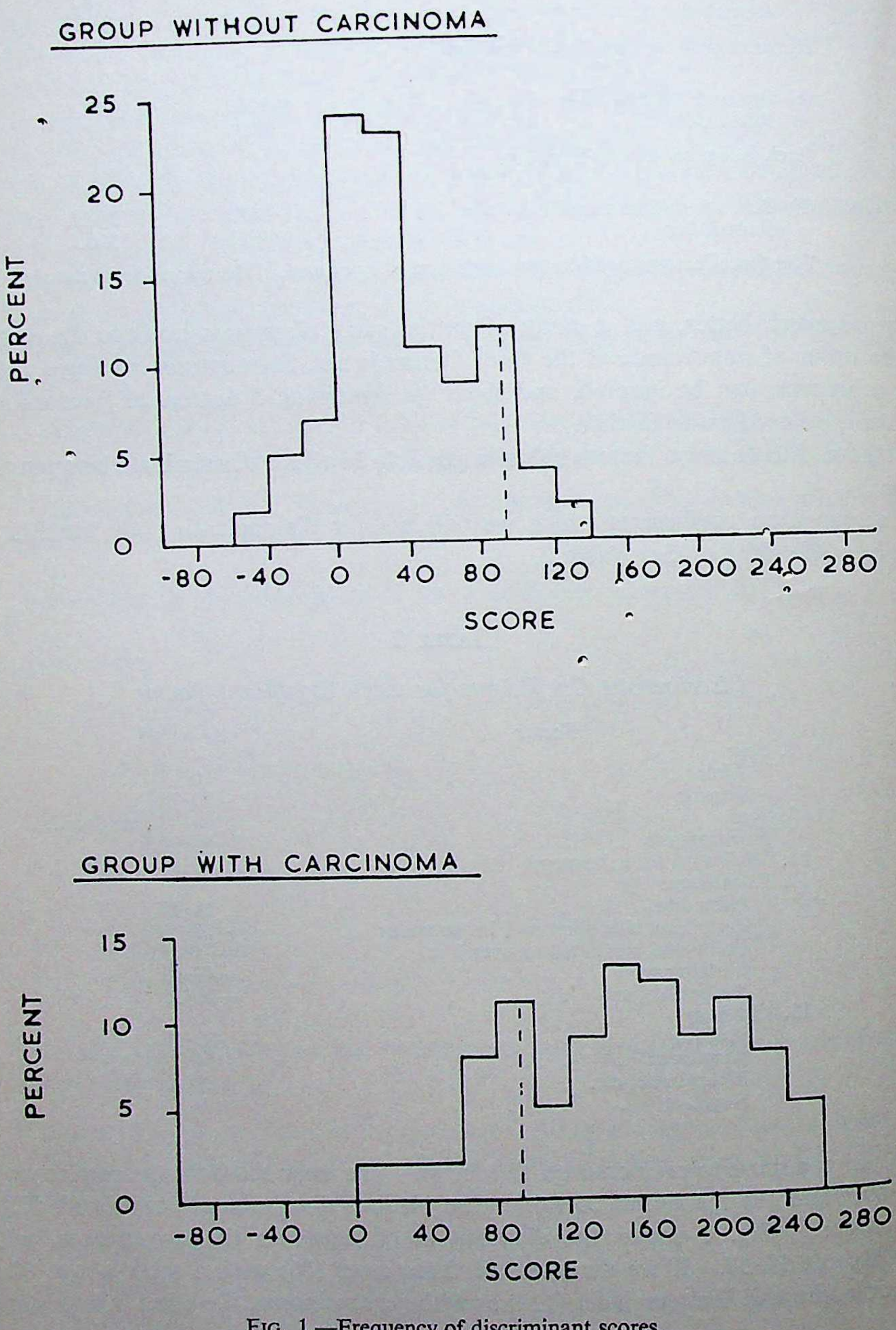

FIG. 1.—Frequency of discriminant scores.

below:

	Diagnosed With Carcinoma	Diagnosed Without Carcinoma	Total
Actually with carcinoma	77	20	97
Actually without carcinoma	9	94	103
Total	86	114	200

Since we are right 171 times out of 200, we might say that the discrimination is $85\frac{1}{2}$ per cent. efficient. We must expect, however, rather worse results from any other set of cases, since our discrimination is constructed to fit the given group.

It may be better to take the discriminant point rather lower. If, for example, we took it at 80·00, the table becomes:

	Diagnosed With Carcinoma	Diagnosed Without Carcinoma	Total
Actually with carcinoma	83	14	97
Actually without carcinoma	18	85	103
Total	101	99	200

In fact, if we take the discriminant point anywhere between 60 and 135, the "efficiency", as used above, is at least 83 per cent., with a maximum of $86\frac{1}{2}$ per cent.

The multiple correlation coefficient R for this discriminant function is 0·758, and for the complete set of factors, 0·773.

It will be seen from the diagram that the two distributions of scores not only have different means, but also somewhat different variances, and neither of them approximates very closely to a normal distribution. The distributions of the two sets of scores would be normal and have the same variance if the assumptions on which the analysis is based were satisfied. These are that the observations relating to the two groups of subjects are drawn from normally distributed populations, possibly with different means, but having the same variance matrices. Now many of the observations take the values 0 and 1 only, and hence variances and co-variances must depend on the means. Obviously the data do not fit the assumptions closely, and in the circumstances the resulting distributions of the scores seem satisfactory.

In view of the departures from assumption the significance tests may not be correct. Variance ratio tests have been shown to be robust in fairly general circumstances (Box and Andersen, 1955), but the effects of departures from assumption in such complicated cases as are here involved have yet to be studied. The fact that the numbers of subjects in the two groups are almost equal probably helps to keep the significance levels near 5 per cent., as in simpler cases where unequal variances are involved.

Errors in the significance tests might affect the number of factors it is found necessary to include but would not alter their order of importance. The first three factors could hardly be affected, but we cannot be certain whether all the remaining factors should be included or whether more should be sought.

After finding the first three important factors it might have been interesting to have followed the procedure suggested by Mannheim and Wilkins (1955), selecting those subjects for whom the scores left doubt as to interpretation, and recalculating the discriminant

function using these subjects only. This procedure, although intuitively appealing, is rather arbitrary and it is doubtful if the number of subjects in the present series warrants it.

DISCUSSION

The problem of systematic diagnosis of a medical condition is essentially that of discrimination, and the statistical technique of linear discrimination developed by Fisher (1936, 1938) and Rao (1952, Chapter 7) is applicable. Unfortunately, the number of factors which may *a priori* be relevant is often quite large, and the computational difficulties are considerable. The design is never orthogonal, and the factors are usually decidedly correlated, so there is no easy way of determining which factors are to be counted as relevant. The method commonly used is to include only those factors where the regression coefficient exceeds twice its standard error, when all factors are taken together. This corresponds to adopting the provisional order of the factors, and including only those where the apparent contribution to the sum of squares for factors is significant, compared with the residual. This approach is straightforward, but basically unsound. At the other extreme, there is the complete analysis; however, if n factors are considered, a complete analysis involves the solution of 1 set of n linear equations, n sets of $n-1$ linear equations, $\binom{n}{2}$ sets of $n-2$ linear equations, and so on. The time taken to solve a set of n linear equations is roughly proportional to n^2; if we call this time t_n, let $t_n \doteqdot an^2$. The time required to produce a complete analysis of n non-orthogonal factors is then

$$T_n = a\left\{n^2 + \binom{n}{1}(n-1)^2 + \binom{n}{2}(n-2)^2 + \ldots + \binom{n}{r}(n-r)^2 + \ldots + \binom{n}{n-1}\right\}$$

$$= a2^{n-1}(n+1)(n+2).$$

For the EDSAC I, $a \simeq 0 \cdot 02$ min., since a set of 19 linear equations was solved in about 8 minutes. We can now calculate T_n for various values of n.

$n = 5$ $\quad Tn \simeq 14$ minutes.

$n = 10$ $\quad Tn \simeq 19$ hours.

$n = 19$ $\quad Tn \simeq 6$ months.

Thus 10 is about the largest number of factors which can practicably be considered.

However, it is not necessary to solve every possible set of equations. If at any stage a factor or group of factors is found to be essential, at all later stages it must be included. A programme could be devised which would take account of this, and so reduce the amount of computation. At present, each solution of a set of equations has to be found separately, which increases the time spent on the analysis, but has the advantage that the statistician can be using his own judgment about which combination of factors to try next.

In the actual example there was probably no advantage in time by using an electronic machine instead of a desk machine. For a desk machine with a trained computer, $a \simeq 4$ min. A certain amount of time could be saved by using the Choleski (see Fox, 1950; Fox and Hayes, 1951) method of triangularization and choosing the order of the factors sensibly, and some 19 hours would have been required to perform the actual calculations that proved necessary. The EDSAC took about $1\frac{1}{4}$ hours to solve the 9 sets of equations

that were presented to it, but the extra time required to punch and check the tape was considerably greater than this, and probably exceeded 24 hours.

Thus although the electronic machine is some 200 times faster than a desk machine in this type of operation, the gain is only about 15 times when all the most time-saving techniques are used in desk computation. A very much more efficient programme is needed to make full use of the intrinsic high speed of an electronic computer. However, the data tapes for each set of equations could readily be formed once a correct tape was made for the complete set of 19, since all that needs to be done to eliminate the i^{th} factor is to replace the i^{th} equation by $x_i = 0$, which can be done easily. Without this advantage, there would certainly have been no saving of time unless a better programme had been available. If more factors had been involved, or the situation had been more complicated, more runs on the machine would have been necessary; the advantage of the machine would then have been clear even with the existing programme.

Electronic methods are no more suited to Mannheim and Wilkins' approach to the problem. Their method would require solving a preliminary discriminant analysis and then constructing different data tapes to find a second discriminant.

Conclusions

The cytological test of the bronchial aspirate proved indispensible to an optimum method of diagnosis. The other factors which appeared relevant were visibility, site, sex, haemoptysis, and age. Using the best combination of these six factors, we may expect to give the correct diagnosis in about 85 per cent. of all cases. If we wish, we can arrange that most of our mistakes will be diagnosing a carcinoma when there is none, rather than diagnosing no carcinoma when in fact the patient has one.

Acknowledgments

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Note on the Measurement and Prediction of Labour Turnover

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1. Introduction

In recent years there have been a number of statistical studies of labour turnover. The results of much of this work have been summarized and discussed by Silcock (1954). He observed that, in all investigations reported, groups of men with short service had a higher leaving rate than groups with long service. It follows from this fact that the usual figure of crude turnover* can be very misleading as a measure of stability, since its value depends on the length of service structure of the group to which it applies. Thus, even if other things were equal, a new group, or one which had recently been expanded, would be expected to have a higher turnover rate than one which was long established, because of its greater proportion of short service men. To avoid this difficulty, Lane and Andrew (1955), proposed that the expectation of service should be used as a measure of stability. Several contributors to the discussion suggested the use of a standardized leaving rate. The purpose of the present note is to give a method of expressing, in quantitative terms, this relationship between the length of service structure and the rate of leaving and to apply the results to various typical situations. The results have two main applications. In the first place they show when crude turnover rates can be used with safety and when their use may lead to serious error. The great simplicity of crude rates has led to their widespread use and it is necessary to know how far this use is justified. Secondly, the method will enable us to predict the effect of changes in factory size on the rate of leaving. This is of interest when planning future labour requirements.

2. Turnover as a Renewal Process

We begin by formulating the problem in its simplest form and subsequently extend the results to more complicated situations. Consider a group of men who start their period of employment at time $T = 0$ and suppose that the size of the group is kept constant by replacing each leaver. This is an example of a self-renewing aggregate to which the methods of renewal theory are immediately applicable. Let the frequency distribution of the length of service, t, at the date of leaving, for the original complement and all subsequent recruits, be $f(t)$. To avoid confusion with the length of service of men still on books, this will be referred to as "length of completed service". The implicit assumption that $f(t)$ is independent of time of joining is unlikely to hold over long periods; it will be sufficient for our purposes if it is approximately true for periods of about two years. If $L(T)$ is the rate of leaving at time T, then the renewal equation for the process is

$$L(T) = f(T) + \int_0^T f(t)\, L(T-t)\, dt. \tag{1}$$

* Crude turnover is defined as the number of leavers in the period divided by the average number of men employed during the period.

The turnover rate in the period T_1 to T_2 will then be given by

$$\int_{T_1}^{T_2} L(T)\, dT \times 100\%.$$

This result enables us to determine the relationship between age of group, T, and rate of leaving.

For the purposes of the present work it will be convenient to state briefly the main results concerning the solution of (1). The Laplace transform of $L(T)$ is found to be

$$f(s)/(1-f(s)), \tag{2}$$

where

$$f(s) = \int_0^\infty e^{-st} f(t)\, dt.$$

The basic theorem of renewal theory states that, under very general conditions,

$$\lim_{T\to\infty} L(T) = 1/\mu,$$

where $\mu =$ mean length of service.

If this limit held for all T, or was reached very rapidly, $L(T)$ would be independent of age of group and our problem would be immediately solved. This is, indeed, the case for the negative exponential distribution. However, as will be shown below, for distributions as skew as length of service distributions, the limit is only a good approximation for very large T and is greatly in error for small T.

It should be noticed that (1) can be solved numerically by treating time as a discrete variable so that

$$L(k) = f(k) + \sum_{j=1}^{k-1} f(j)\, L(k-j), \qquad k = 1, 2, 3 \;\ldots\;.$$

This method is useful for studying specific problems but lacks the generality of the analytical approach.

3. The Length of Completed Service Distribution

For the present purpose, the function $f(t)$ must satisfy two conditions. It must be capable of describing length of completed service distributions which occur in practice and must be such that (1) can be solved for $L(T)$.

Length of service distributions are usually extremely skew with a mode at or near zero. Two curves have been used in the literature to describe them. The first, due to Silcock, takes the form

$$f(t) = \frac{s}{\lambda}\left(\frac{\lambda}{\lambda+t}\right)^{s+1}, \qquad t \geqslant 0.$$

It was suggested on theoretical grounds and proved to be a good fit to most of the empirical distributions given in Silcock's paper.

The second is the lognormal distribution proposed by Lane and Andrew. They used it on the grounds that it was a good description of their data, but Aitcheson pointed out,

in the discussion of the paper, that there were theoretical reasons for expecting the log-normal to be appropriate.

Unfortunately it has not proved possible to solve the renewal equation for either of these distributions, so a third alternative is suggested. We shall use

$$f(t) = p\lambda_1 e^{-\lambda_1 t} + (1-p)\,\lambda_2 e^{-\lambda_2 t}, \qquad t \geqslant 0, \tag{3}$$

which may be described as a "mixed exponential" curve. This form is not believed to have any theoretical significance but has the advantages of fitting the data and yielding a simple solution to (1). A method of estimating the three parameters p, λ_1, λ_2 is discussed in the Appendix.

4. SOLUTION OF THE RENEWAL EQUATION FOR THE MIXED EXPONENTIAL DISTRIBUTION

The Laplace transform of (3) is easily shown to be

$$f(s) = p\left(\frac{\lambda_1}{\lambda_1+s}\right) + (1-p)\left(\frac{\lambda_2}{\lambda_2+s}\right).$$

Substituting in (2) and inverting, it is found that

$$L(T) = \frac{1}{\mu} + \left(p\lambda_1 + (1-p)\,\lambda_2 - \frac{1}{\mu}\right) e^{-(p\lambda_2+(1-p)\lambda_1)T}, \tag{4}$$

where $\mu = p/\lambda_1 + (1-p)/\lambda_2$. It will simplify subsequent formulae to write (4) as

$$L(T) = L + ae^{-bT}.$$

This is the fundamental result on which all the applications are based. The use of this formula is, however, limited by the following consideration. All of the empirical distributions to which the mixed exponential curve has been fitted were truncated at 18 months or 2 years. As the possible range of length of service is of the order of 40 years, there is no guarantee that this curve will provide an accurate description beyond the point of truncation. For this reason (4) should not be used for $T > 2$ years.

5. APPLICATION OF THE RESULTS

5.1. In order to demonstrate the application of the results, three typical length of service distributions have been chosen. Rather than select arbitrary values for the parameters we have used those obtained by fitting the curve to three empirical distributions. The distributions selected for this purpose have been taken from the papers by Silcock and by Lane and Andrew. They were chosen partly because of the large numbers of men involved and partly because they constitute a fairly representative sample of the distributions on record. They are given in Table 1 with the fitted frequencies. It will be seen that the agreement is sufficiently close to justify the use of the mixed exponential curve. The parameter estimates are given at the foot of the table.

TABLE 1

Length of Completed Service Distribution for Three Firms

Length of service	Glacier Metal Co. (1944–1947) Observed	Glacier Metal Co. (1944–1947) Fitted	J. Bibby & Sons Ltd. (Males) 1950 Observed	J. Bibby & Sons Ltd. (Males) 1950 Fitted	United Steel Cos. Branch A Observed	United Steel Cos. Branch A Fitted
Under 3 months	242	242·0	182	182·0	412	412·0
3 months	152	152·0	103	103·0	143	143·0
6 months	104	101·4	60	60·7	66	62·6
9 months	73	72·7	29	38·0	45	37·7
12 months	52	55·8	31	25·5	27	29·6
15 months	47	45·7	23	18·6	18	26·1
18 months	49	39·2	10	14·7	450	450·0
21 months and over	487	497·2	191	186·5		
Total entrants	1206	1206·0	629	629·0	1161	1161·0
$\hat{p}$	0·6513		0·4363		0·5377	
$\hat{\lambda}_1$	0·2684		0·2339		0·2187	
$\hat{\lambda}_2$	2·4228		2·5335		4·8946	

Examples of three different situations which arise will now be discussed. We shall consider the relationship between turnover and age of group for

(i) a group of constant size,
(ii) a group which is expanded by a fixed amount at a particular time,
(iii) a group which is expanding continuously.

Since the length of service structure is determined by the age and recruitment history of the group, these examples will enable us to draw general conclusions about the value of crude turnover as an index of stability.

5.2. *Turnover for a group of constant size*

The situation considered in this section corresponds to that of a new factory or department which starts off with its full complement of manpower. Using the result (4) we are able to calculate the expected turnover in successive periods from the start. This has been done by supposing, in turn, that all men have one of the length of completed service distributions discussed in the last section. Table 2 gives the turnover in successive quarters over a period of two years.

TABLE 2

Percentage Turnover in Successive Quarters for a New Group

	Glacier Metal Co.		*J. Bibby & Sons*		*United Steel Cos.*	
1st quarter	22·6	69·5	34·6	108·3	45·8	107·2
2nd quarter	18·2		28·6		28·0	
3rd quarter	15·3		24·2		19·0	
4th quarter	13·4		20·9		14·4	
5th quarter	12·1	44·6	18·5	65·4	12·1	43·6
6th quarter	11·3		16·8		11·0	
7th quarter	10·8		15·5		10·4	
8th quarter	10·4		14·6		10·1	

The table demonstrates clearly that the turnover is very dependent upon the age of the group. The quarterly rate has been reduced by at least one-half during the first eighteen months. For the United Steel Cos, the reduction is over three-quarters. This emphasizes the unsuitability of the crude turnover rate as a measure of stability when applied to new groups. It shows also that a declining rate does not necessarily indicate increasing stability but is rather an inevitable consequence of the ageing process.

It will be noted that the rate of change per quarter is small after the end of the first year. However, it cannot be assumed to be effectively constant after this point. The equilibrium turnover given by (4) would suggest that this was approximately so, but the uncertainty about the tail of the distribution beyond the truncation point leaves the question in doubt. For this reason the equilibrium figures, corresponding to $T=\infty$ have not been included in the table. The equilibrium state is of little practical interest because conditions are unlikely to remain constant for long enough for it to become a reality.

5.3. *Turnover for a group expanded at one step*

The results of §5.2 will now be extended to cover the case when the original group is expanded after its formation. Suppose that a group of N men is set up at $T=0$ and subsequently expanded to $N+N_1$ at $T=T_1$. Again it is assumed that all recruits have the same length of service distribution. The leaving rate will now be given by

$$\begin{cases} L(T), & 0 \leqslant T \leqslant T_1; \\ \dfrac{NL(T)+N_1L(T-T_1)}{N+N_1} & T \geqslant T_1. \end{cases}$$

Clearly the result can be extended to more than one increase.

Some specimen calculations have been made and are given in Table 3. The effect of the expansion depends on its size and when it takes place but the general tendency is to arrest the decline in turnover, sometimes causing a temporary increase.

TABLE 3

Percentage Turnover in Successive Quarters for a Group which has been Expanded (United Steel Cos. only)

	After 6 months		*After 1 year*	
	50% increase	*100% increase*	*50% increase*	*100% increase*
1st quarter	45·8	45·8	45·8	45·8
2nd quarter	28·0	28·0	28·0	28·0
3rd quarter	27·9	32·4	19·0	19·0
4th quarter	18·9	21·2	14·4	14·4
5th quarter	14·4	15·6	23·3	29·0
6th quarter	12·1	12·7	16·7	19·5
7th quarter	11·0	11·3	13·3	14·7
8th quarter	10·4	10·6	11·5	12·3

It is interesting to notice that by the eighth quarter the effect of the increases has almost disappeared. Further calculations using the other length of service distributions and different amounts and times of increase confirm this observation. We therefore draw the following empirical conclusion. Isolated increases, even if as great as 100 per cent., may

have a large immediate effect but their influence on turnover is negligible after about two years.

5.4. *Turnover for a group which is continuously expanding*

Instead of an increase at one time we may be faced with a group which is continuously expanding. Suppose that the rate of increase at time T is $N(T)$, $(N(T) \geqslant 0)$. The expected number of leavers in the period T_1 to T_2 will be

$$\int_{T_1}^{T_2} \left\{ \int_0^T N(t)\, L(T-t)\, dt \right\} dT, \tag{5}$$

and the average manpower during the same period

$$\int_{T_1}^{T_2} \left\{ \int_0^T N(t)\, dt \right\} dT/(T_2 - T_1).$$

For the purposes of illustration, we shall consider the case of a constant rate of increase in size, that is $N(T) = N$. Substituting for $L(T-t)$ in (5) the expression for the turnover rate for the period T_1 to T_2 becomes

$$L(T_2 - T_1) + \frac{2a}{b(T_1+T_2)} \left[(T_2 - T_1) - \left(\frac{e^{-bT_1} - e^{-bT_2}}{b} \right) \right]. \tag{6}$$

Quarterly turnover figures have been calculated using each of the three length of service distributions discussed above. The results are given in Table 4.

TABLE 4

Percentage Turnover in Successive Quarters for a Group Expanding at a Constant Rate

	Glacier Metal Co.	*J. Bibby & Sons*	*United Steel Cos.*
1st quarter . . .	23·5	35·8	49·6
2nd quarter . . .	21·3	32·9	40·5
3rd quarter . . .	19·5	30·2	33·5
4th quarter . . .	18·0	28·0	28·6
5th quarter . . .	16·8	26·2	25·2
6th quarter . . .	15·9	24·6	22·7
7th quarter . . .	15·1	23·3	20·9
8th quarter . . .	14·5	22·2	19·4

The figures in Table 4 differ in two ways from those for a group of constant size given in Table 2. In the first place, the quarterly figures are higher for the expanding group and secondly, the rate of decrease is slower. Thus, even when comparing groups of exactly the same age, the turnover would be misleading if one group was increasing in size.

6. SUMMARY AND CONCLUSIONS

In this note, a study has been made of the labour turnover process, with the principal object of assessing the value of the crude turnover rate as a measure of stability. The method used is based on a straightforward application of renewal theory. Its purpose is

to find how turnover rates are influenced by changes in the length of service structure of a group of men. If this influence can be shown to be large then the value of the crude turnover rate is seriously reduced. Although detailed conclusions can come only from an application of the method to individual cases, certain general conclusions can be drawn. These are of an empirical nature and may not hold if conditions are different from those considered in this note.

The crude turnover rate is influenced very much by the age and recruitment history of the group to which it applies. It cannot, therefore, be used as a valid method of comparing the stability of groups of men unless they are similar in these respects. In practice a serious error is likely to be made if any group is less than two years old or has had a major increase in size during that period. This emphasizes the need for using methods which are independent of length of service structure, such as that proposed by Lane and Andrew. On the question of predicting manpower requirements for a new or expanded factory we have the following conclusion. Not only must there be an adequate supply of manpower initially, but replacements will be required at a much higher rate than for a long established firm. If some knowledge of the length of service distribution of recruits is available than (4) can be used to predict recruitment needs.

APPENDIX

Fitting the Mixed Exponential Curve

The problem with which we are faced is that of fitting three-parameter curves to grouped frequency distributions, which are truncated at the upper tail. In most cases examined, about 40 per cent. of the total frequency lies beyond the point of truncation. The most straightforward method of estimation, in this case, is by equating percentage points. As the object is to obtain an accurate graduation, and not to estimate parameters in a theoretical model, this method has been employed in preference to maximum likelihood. Let β_i be the proportion of the total frequency lying beyond the point t_i; then the equations for λ_1, λ_2 and p are

$$pe^{-\lambda_1 t_i} + (1-p)\, e^{-\lambda_2 t_i} = \beta_i, \qquad i = 1, 2, 3.$$

For a given choice of t_1, t_2 and t_3, these equations can be solved by successive approximation. However, if the results given above are to be widely applicable, a more rapid method is desirable. The equations are therefore transformed as follows. Choose t_2 and t_3 to be multiples of t_1 so that

$$t_2 = 2t_1, \qquad t_3 = kt_1,$$

where k takes one of the integer values 3, 4, 5, ... Let

$$X = e^{-\lambda_1 t_1}, \qquad Y = e^{-\lambda_2 t_1};$$

then the equations become

$$\begin{aligned} p(X-Y) + Y &= \beta_1 \\ p(X^2 - Y^2) + Y^2 &= \beta_2 \\ p(X^k - Y^k) + Y^k &= \beta_k, \text{ say.} \end{aligned}$$

Writing $u = X + Y$, $v = XY$ these equations can be solved explicitly giving

$$\begin{aligned} p &= (\beta_1 - Y)/(X - Y), \\ v &= \beta_1 u - \beta_2. \end{aligned}$$

The expression for u depends upon k; the first four cases are given below.

$$\begin{aligned}
k = 3: &\quad u = (\beta_3 - \beta_1\beta_2)/(\beta_2 - \beta_1^2),\\
k = 4: &\quad u^2 = (\beta_4 - \beta_2^2)/(\beta_2 - \beta_1^2),\\
k = 5: &\quad u^3 - \beta_1 u^2 + 2\beta_2 u = (\beta_5 - \beta_1\beta_2^2)/(\beta_2 - \beta_1^2),\\
k = 6: &\quad u^4 - 2\beta_1 u^3 + 3\beta_2 u^2 = (\beta_6 - \beta_2^3)/(\beta_2 - \beta_1^2).
\end{aligned}$$

The last two equations can easily be solved by the Newton-Raphson method using the fact that the required root is in the neighbourhood of unity. The restriction of t_2 and t_3 to be integer multiples of t_1 does not limit the choice of percentage points unduly as both t_1 and k are at choice. The best choice of t_1 and k has been investigated empirically leading to the following scheme. Since $L(T)$ and $f(T)$ change most rapidly near $T = 0$ the graduation needs to be as accurate as possible in this region. Thus we have taken

$$\begin{aligned}
t_1 &= 3 \text{ months (the usual interval of grouping)},\\
t_2 &= 6 \text{ months}.
\end{aligned}$$

The tail is represented most accurately by taking the third percentage point near to the end of the available range at $t_3 = 18$ months ($k = 6$). In the case of the Glacier Metal Co. and United Steel Cos.' distributions, which are comparatively smooth, the parameter estimates are not very sensitive to changes in t_1 and k. For J. Bibby and Sons Ltd., where the tail frequencies are irregular, the estimates depend markedly on the particular choice of percentage points. Even in this case, however, the turnover rates calculated from (4) are not seriously affected by this variability.

References

Lane, K. F. & Andrew, J. E. (1955), "A method of labour turnover analysis", *J. R. Statist. Soc.*, A, **118**, 296–323.

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PROCEEDINGS OF A SPECIAL GENERAL MEETING OF THE ROYAL STATISTICAL SOCIETY held at the London School of Hygiene and Tropical Medicine on Thursday, December 18th, 1958, immediately on the conclusion of the Ordinary Meeting convened for the same day.

The Chair was taken by the President, Sir Harry Campion, C.B., C.B.E.

The Notice convening the meeting was, with the consent of the meeting, taken as read.

The President moved:—

"That the Bye-laws of this Society be amended as follows:—

1. By substituting for the first paragraph of Bye-law 5 the following paragraph:—

Every Fellow shall pay an annual subscription of three guineas except as hereinafter provided. Every Fellow who shall have attained the age of sixty years or paid at least twenty-five annual subscriptions shall pay an annual subscription of two guineas. Every Fellow, other than Fellows representing bodies or institutions associated with the Society under Bye-law 3, may on attaining the age of sixty years or at any subsequent time pay in lieu of all future annual subscriptions a composition fee of twenty-five guineas, less one guinea for each year of age in excess of sixty and less two guineas for every five annual subscriptions paid by him prior to such commutation. The minimum composition fee shall be ten guineas.

2. By deleting from the end of the second paragraph of Bye-law 5 the words 'apart from subscriptions in respect of the years 1940 to 1946 inclusive remitted by reason of conditions arising from the 1939–1945 war'.

3. By inserting after the word 'or' in the first sentence of the third paragraph of Bye-law 5 the words '(in case he shall have attained the age of sixty years and elected to pay such fee)'.

4. By deleting the whole of Bye-law 5 (a)."

The Honorary Treasurer, Mr. R. F. Fowler, seconded the motion.

Dr. W. J. Martin moved the following amendment:—

"To add at the end of the proposed first paragraph of Bye-law 5 the words 'except that for those Fellows, other than Fellows representing bodies or institutions associated with the Society under Bye-law 3, who have attained the age of seventy years and paid at least fifteen annual subscriptions the minimum composition fee shall be seven and a half guineas'."

Mr. A. T. Gore seconded the motion for the amendment.

The Amendment was put to the meeting and carried by a majority of those voting.

The Resolution with the Amendment was then put to the meeting and declared carried by the necessary majority.

The Proceedings then terminated.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*Planning of Experiments.* By D. R. Cox. New York, Wiley; London, Chapman and Hall; 1958. vii, 308 pp. 9″. 60*s.*

Dr. Cox has written a skilful and, to my mind, admirable, book on the principles of comparative experimentation. The book contains virtually no mathematics, and very little arithmetic. There are, for instance, no detailed instructions on the arithmetical steps necessary for analyses of variance, although some examples of the direct calculation of residuals, and of their sums of squares, are included, more for the purpose of explaining the nature of residual variation than as a method to be followed in practice. The book is therefore in no way a competitor of the large standard text-books on experimental design, and is more on the lines of Finney's *Experimental Design and its Statistical Basis*, although Cox's book is very much the longer and more detailed.

The first two chapters are introductory, and emphasize the assumptions underlying the designs and their analyses. Chapters 3 and 4 discuss very fully methods for reduction of error by designs like randomized blocks and latin squares, and by covariance adjustments. It is typical of the book that questions such as the relationship between different methods for reduction of error should be discussed more fully than in most of the more methodological text-books. There is also a good discussion on methods of replicating latin squares. Chapter 5 is concerned with the nature of, and reasons for, randomization. Chapters 6 and 7 are concerned with factorial designs, and contain a useful categorization of different types of factor. Chapter 8 is on the choice of sample size, which is perhaps the question most frequently posed to the consulting statistician, but is rarely discussed satisfactorily in books. This chapter contains the best available account of sequential methods in comparative experimentation. In Chapter 9, the author discusses the choice of units, treatments and observations. It would have been useful to have some guide here as to the soundness of using quantal responses in the standard experimental designs. The randomization model generally appealed to by the author assumes constant treatment effects, and this is in general not possible with quantal responses. In the same way, the randomization method of obtaining confidence limits breaks down. The remaining five chapters deal further with latin squares, and introduce incomplete designs, fractional replication,

confounding, cross-over designs, systematic designs for eliminating trend, problems of optimal allocation, search for optimal conditions, and assay-type experiments. These chapters contain short lists of the more useful experimental designs of various types. The description of different types of incomplete designs, and the discussion of the distinction between fractional replication and confounding, are particularly well done. There are some short tables of random digits and permutations.

One attractive feature of the book is the wealth of examples. In accordance with the author's general policy, numerical details are usually omitted, but the examples are carefully chosen, from a very wide variety of fields of research, to illustrate the principles or difficulties under discussion.

The book is intended primarily for the experimental worker. The basic ideas of experimental design can be explained without either mathematics or arithmetic, and this book should attract many scientists who are intimidated by the more orthodox text-books. On the other hand, some experimental workers and students, making their first contact with the subject, will find it easier to absorb the general principles if they can see them applied in detail on numerical examples. It would be profitable for such people to read Cox's book in conjunction with a standard book on experimental design, or on statistical methodology. Finally, the professional statistician will find much food for thought in the thorough discussion of basic ideas which is characteristic of this book.

P. ARMITAGE.

2.—*Principles of Statistical Techniques*: *a First Course from the Beginnings for Schools and Universities*. By P. G. Moore. Cambridge, University Press, 1958. viii, 239 pp. $8\frac{1}{2}''$. 22*s*. 6*d*.

In this introductory text-book Dr. P. G. Moore offers a broad, readable and patiently developed discussion of the basic principles and elementary techniques of statistics. A corrective of this kind is needed to supplement the elementary text-books now available, too many of which are collections of recipes rather than reasoned expositions of basic principles. A teacher experienced in applied statistics can make good use of text-books giving only concise formulations and exemplifications of statistical techniques because he can remedy their deficiencies. But as the teaching of statistics in schools depends mainly on teachers with little practical experience of the subject, Dr. Moore's book will be as valuable to teachers as to students.

The author rightly begins with a detailed description of the scope of statistics and the collection, tabulation and presentation of numerical data; he suggests many useful exercises on these rather neglected aspects of elementary work. He then introduces frequency distributions, averages, and measures of dispersion. A chapter on probability and sampling establishes the basic rules of probability and demonstrates (but does not prove) the sampling distribution of the mean. The binomial probability distribution is introduced and qualitative consideration of its form as the exponent increases in turn introduces the normal distribution. There follow two chapters on tests of significance which include tests on the difference of means of large samples and on the equality of proportions in two populations (by reference to the normal probability tables provided) and tests illustrating the use of the χ^2-distribution (with tables of significance levels of the χ^2-distribution also provided). The last three chapters discuss weighted means and standardized rates, time series and moving averages, and regression and correlation.

All this ground is covered without using calculus methods or, indeed, anything beyond Ordinary Level G.C.E. mathematics. It is evident that some important results have to be given without proof. Though the book therefore provides an excellent introduction for the non-mathematical reader, it will in places fail to satisfy those who, specializing in the physical sciences, could accept a more mathematical treatment. The addition of a concise mathematical appendix could help to give the book a wider usefulness.

Two other defects are apparent. Though there is a refreshingly original and carefully selected collection of exercises of all kinds (happily *none* from past examination papers) there are at present no answers to the numerical exercises—a fact which will discourage

the private reader. And though two statistical tables are given, they are buried in the text and, by an accident of pagination, are separated from the diagrams that explain to the student precisely what the tables give.

These defects could all be remedied in the future editions that this thoughtfully written and excellently produced text-book deserves to achieve. Unfortunately however, its price, though justified by what the book offers to the buyer, is high enough to restrict its adoption as a class-book, especially in schools where, after all, statistics is at best a minor subject.

B. C. BROOKES.

3.—*Statistics of Extremes*. By E. J. Gumbel. New York, Columbia University Press, 1958. London, Oxford University Press, 1959. xxi, 375 pp. 9″. 120*s*.

"A book should have either intelligibility or correctness. To combine the two is impossible." Thus the quotation at the head of the final summary in Professor Gumbel's book. The reviewer feels that this book errs on the side of correctness, but he writes as one not very familiar with the subject. The chief impediments to a ready understanding are an over-condensed literary style, and the mathematical notation. The author has obviously attempted to systematize the latter: that he has not been entirely successful is probably not his fault. One feels, however, that this is a case where a list of the (varying) meanings of the main symbols used might have proved helpful. On the stylistic side, the desire for conciseness leads to the use of many technical terms and abbreviations, some of which tend to convey a meaning other than that intended. Examples are the "probability" of x rather than its (cumulative) distribution function, and "initial median" for the median of the initial (i.e. parent) distribution. Fortunately the book possesses an adequate index. Another irritating feature arises because all that can be said about largest values can be said, with appropriate changes, about smallest values. But would it not be better left unsaid? Here is an example: "In consequence of (2) and (7), the logarithms of the average maxima (minima) of the smallest (largest) values decrease (increase) and the average maxima (minima) of the smallest (largest) values increase (decrease) as the logarithms of the numbers of extremes increase."

The book is intended for statisticians and statistically-minded scientists and engineers, and the author states that he has tried to write on an elementary level and to use graphical procedures in place of complicated calculations. The first objective has been achieved, in that an undue standard of rigour and generality has not been imposed; the book is definitely on statistics rather than on abstract probability theory. As for the graphs, there are about one hundred of these, nearly all illustrating theoretical relationships rather than practical data. Most of them use transformed scales on one or both axes, and in this case two or three different scales may appear against each axis. A variant is the drawing of more than one curve, to different scales, on the same diagram. Many will feel that the effort of unravelling the meaning of some of these is not worth while. Transformed scales are very useful for plotting experimental data in straight line form, but they hardly help one to get the "feel" of a theoretical relationship when it has been distorted into a straight line. It is unfortunate that the symbols on the diagrams are occasionally different from those in the text.

One of the most useful features of the book is a closely-printed 22-page bibliography of works in the pure and applied field; and selected references to further reading are given at the ends of sections. There are also "exercises" and "problems", the latter being more difficult questions, some of which could serve as research projects. Introductions and summaries are placed at strategic points throughout the book, and should prove helpful in using the book for reference purposes.

Chapter I is called "Aims and Tools". *General tools* reviews transformations of variates, symmetry, moments (ordinary, reciprocal and geometric), seminvariants (cumulants), convolution, the lognormal distribution and so on. Most of these topics could well be omitted: they are treated in texts on mathematical statistics, and the reader to whom they are unfamiliar is unlikely to make much progress anyway. The short section

on the gamma function is important in the sequel, but is marred by an ambiguous formula (3′), an incorrect one (unnumbered) and a notation for derivatives which contradicts convention. *Specific tools* introduces new concepts and treats, among other things, of probability papers and the associated "plotting problem". Given a set of ranked observations, $x_1, \ldots, x_n$, supposed to have been drawn from the type of distribution for which the probability paper has been constructed, what value P_i on the distorted probability scale is to be associated with x_i on the other scale? The author concludes that

$$P_i = i/(n+1)$$

is the most satisfactory solution. The remainder of the section is concerned with a modified least-squares method for fitting lines drawn on probability paper. The method is designed for ease of calculation rather than for optimal accuracy, and is typical of most methods of estimation considered in the book. The author states, "No distribution is stated without an explanation of how the parameters may be estimated, even at the risk that the methods used will not stand up to the present requirements of mathematically minded statisticians, who are welcome to find better estimates."

Chapter II deals first with some general theory of order statistics, and then with the distribution of "exceedances" (the number of observations in a second random sample which exceed a given order statistic of a first random sample). The statement (p. 51) that the sample median is the order statistic with minimum variance for any symmetrical unimodal distribution is wrong, and is all but disproved by an example given on the same page.

Chapter III treats of general formulae for, and properties of, extreme observations and their derivatives, which can be deduced without appeal to the form of the parent distribution. In the section on the use of the sample range as a tolerance limit, quite incorrect numerical values are obtained because the approximate formula used is only valid when $n\delta \ll 1$. A more serviceable formula is $1 - P = (1 + n\delta)\, e^{-n\delta}$ $(n \gg 1)$.

In Chapter IV general properties of extremes are discussed, and in particular the manner in which the location and scale of their sampling distributions change with increasing sample size. This behaviour varies widely according to the behaviour of the parent distribution in the appropriate tail, and three types of parent distribution are distinguished (exponential, Pareto-Cauchy, and limited) which are in turn subdivided. The theory is well illustrated by the use of well-known distributions (particularly the normal) as parents. The three types of initial distribution lead to three types of asymptotic distribution for the extreme, which are introduced in Chapter V. Fisher and Tippett's proof that these are the only "stable" forms of asymptotic distribution is given in slightly modified form. The properties of the first asymptotic distribution (stemming from the exponential type of parent) are then discussed in detail. Graph 5.3.2(3) appears to be incorrectly described on the previous page, and the curves for median and mode are not in fact comparable. (They do not approach one another for large *m*.)

In Chapter VI the first asymptotic distribution is itself taken as a parent distribution, and the properties of the maximum and minimum observations in a sample are considered. The author's general contention is that in the analysis of extreme observations such as maximum annual temperatures, the first asymptote has the same kind of theoretical position as has the normal distribution in the treatment of means. Like the latter it possesses two parameters determining location and scale, and methods of estimating these are discussed. The chapter concludes with numerical examples in which some of the foregoing theory is applied to floods, to meteorological data, and to breaking strengths ("weakest link" theory).

Chapter VII treats the second and third asymptotic distributions, which in their most general forms possess three parameters and thus have a greater variety of shape.

In the final chapter the asymptotic distributions of the mid-range and range are derived for samples from a symmetrical distribution of exponential type. In the second section the known logarithmic relation between the first and second/third asymptotes is used to

transform these results into the asymptotic distributions of the geometric range,

$$\sqrt{[(-x_1)(x_n)]},$$

and extremal quotient, $x_n/(-x_1)$, of samples from the Cauchy and limited types of parent. As far as the limited type of parent is concerned, these results seem highly academic, because they involve measuring each extreme from its own natural limit. For example, if the initial distribution is uniform between -1 and $+1$, the "geometric range" whose distribution is actually considered is not $\sqrt{[(-x_1)(x_n)]}$ but $\sqrt{[(1+x_1)(1-x_n)]}$.

Quite a few relatively minor slips and printer's errors have been noticed, but cannot be listed here.

Statisticians will be grateful to Professor Gumbel for writing a book which will be referred to for many years to come. The very high price is presumably due to the large number of graphs reproduced. G. S. JAMES.

4.—*Measurement and Statistics*. By Virginia L. Senders. New York [and London], Oxford University Press, 1958. xvi, 594 pp. $8\frac{1}{4}''$. 48*s*.

This book is written for students with no knowledge of mathematics who are taking their first course in statistics. It is intended primarily for psychologists. The approach of the author is unusual and refreshing and she manages to combine rigorous thinking with readability in a way which makes the book enjoyable. It is essentially a book to be read right through, rather than a reference book, although the author points out in the introduction that certain chapters can be omitted on a first reading.

She begins with an analysis of the nature of measurement, and considers four types of scale. First the "nominal scale" (objects classified according to some criterion); then the "ordinal scale" (objects ranked according to some criterion); next the "interval scale" (for which $x - y = (x + a) - (y + a)$ for any a) and lastly the "ratio scale", for which there is an absolute zero, and ratios of measurements are meaningful.

At least one chapter is devoted to each of these scales, and there is a discussion of the relevant descriptive statistics. In the chapter on the nominal scale Shannon's measure of information

$$H = -\Sigma p(i) \log_2 p(i)$$

is introduced as a measure of uncertainty in prediction. This does seem to have a more satisfactory interpretation than χ^2, and it appears to be a useful measure in psychological experiments on learning and reaction times. A related measure, T, is used for contingency tables, to show the extent to which knowledge of one variable reduces uncertainty in predicting the other. After this treatment of scales, which covers two thirds of the book, the author discusses probability and inference generally and their application to problems which arise with the various types of measurement. There are 20 tables at the end of the book, which include a table of

$$-p(i) \log_2 p(i),$$

diagrams for confidence limits for proportions and correlation coefficients and practically every other table one is ever likely to need.

The plan of the book is excellent and no concessions are made in rigour for the benefit of non-mathematical students. Amusing examples are scattered throughout the book, e.g., in introducing discrete and continuous variables we are asked to "Consider peas and pea soup". Nearly every section is followed by a set of carefully planned and often searching questions, the numerical ones being supplied with answers; the implications of the various tests of significance discussed are clearly stated. A short discussion on the difficulties arising in connection with the definition of terms might usefully have been included, although such difficulties are implied in the text and questions.

There are one or two errors which should be pointed out. What is called the "Standard error of estimate of y" from a regression line is in fact the standard deviation about the regression line; also an adjustment for degrees of freedom should be used here (p. 259).

The bivariate normal distribution with zero correlation does not in general have circular contours (p. 476), and there are a few other minor errors.

The first two thirds of the book can be appreciated by any conscientious student; the final third on statistical inference is rather concentrated and probably needs to be supplemented by lectures. Subject to this any student who works carefully through this book will be well equipped to formulate his research problems in such a way that the results can be treated by standard statistical techniques. Many teachers can probably also learn something about the presentation of material.

MONICA A. CREASY.

5.—*Sampling in Sweden: Contributions to the Methods and Theories of Sample Survey Practice.* By Tore Dalenius. Stockholm, Almqvist & Wiksell, 1957. viii, 247 pp. $8\frac{1}{2}''$. *kr*. 20.

The author is well known as a sampling statistician and this book, his Ph.D. thesis, presents an account of his contributions to the field. It is no disparagement of the book to describe it as a hotch-potch of dissimilar constituents. The author's work has been mainly in an official sampling organization and this has given rise to a diversity of experience whose range is well illustrated by the topics treated in the thesis.

The early part deals with basic theory and philosophy exemplified by Swedish experience during the past century. As in other countries, official acceptance was slow and suspicious. Nevertheless Swedes were well to the front in the use of random sampling, as is made clear by two remarkable examples quoted by the author which date back to 1910. The first refers to a housing survey in Gothenburg. The following description of the sampling design taken from Edin's (1912) paper may be quoted:

"The total number of occupied small apartments . . . was combined for all districts . . . Thereafter the addresses of all real estates containing such apartments were noted, together with their total, on small slips (cards). These slips were mixed in an urn and taken out one by one, whereupon the notes were transferred to the district lists, one for each investigator, until a total of small apartments was obtained slightly exceeding 1/5 of the total number of such apartments which existed in 1905. . . . The main object of the procedure here described has naturally been that no one could possibly have the slightest reason for saying that the sample was biased."

These are remarkable words when one considers the time at which they were written. Even more remarkable, however, is the second example described by Dalenius. This refers to the estimation of Sweden's forest capital in 1911. At discussions in 1909 it had been urged that the preliminary survey to be held in Värmland should be based on probability. The views of Professor I. Fredholm, professor of physics at Stockholm, were sought and his reply is worth quoting in full:

"(1) I consider that the probability calculus can be applied to the estimation of the forest capital in the manner suggested, and that the laws valid for the calculus mentioned, which have been successfully applied in many other fields, can be applied to equally great advantage to the problem with which the Commission is faced.

"(2) It should be possible, using the material available within a given district, to draw conclusions regarding the probable error, to which the survey can be subjected, and in this manner determine whether the survey percentage has been sufficient or not for the need.

"(3) It should be possible, using the proposed preliminary survey of the County of Värmland, to decide whether a strip survey with the low percentage proposed can give results sufficiently accurate for the purpose required.

Stockholm, January 11, 1910."

Swedish eminence in other branches of statistics is beyond question. These quotations, and the other material adduced by Dalenius, clearly establish the prominent place held by Sweden in the pioneering of sampling methods.

In later chapters Dalenius describes work with which he has himself been concerned. Of special interest here is the Swedish general purpose sample, designed to be used for

surveys of all kinds over an extended period on the American "master sample" principle. Unfortunately, although a great deal is said about how this sample was set up, very little is said about how useful it was found to be in practice. One infers that this is not the author's fault—evidently the sample was less widely used than had been hoped. (It will be recalled that in this country the Government Social Survey has explicitly rejected the "master sample" principle—see e.g. Gray and Corlett (1950), "Sampling for the Social Survey", *J. R. Statist. Soc.*, A, **113**, 150–199).

The last few chapters are taken up mainly with theoretical questions. The author's chief contributions in this direction are on the problem of optimum stratification; this can be stated as follows. Suppose that in advance of the survey we know for each individual in the population the value of a variable x, e.g. size, known to be correlated with the variable y under study. We wish to use this information to stratify the population into k strata. On what basis should we choose the values of x which define the boundaries between strata? Treatments of stratified sampling given in ordinary text-books ignore this problem completely; strata are conveniently regarded as given *a priori.* In real life they never are and one has to construct them oneself on the base of ill-founded conjectures. Dalenius has rendered a significant service by working out the basic theory on which rational choice of strata ought to be based. The account given in the thesis is based on three papers published in *Skand. AktuarTidskr.*, 1950–52.

The main result is easy to derive and easy to state but not alas, so easy to use in practice. Assuming $x \equiv y$ and optimum allocation of sample numbers among strata, the condition, for large populations, is

$$\frac{\sigma_h^2 + (x_h - \mu_h)^2}{\sigma_h} = \frac{\sigma_{h+1}^2 + (x_h - \mu_{h+1})^2}{\sigma_{h+1}} \qquad (h = 1, \quad . \; . \; . \quad , k-1), \tag{1}$$

where μ_h, σ_h are the mean and standard deviation of the h^{th} stratum, and x_h defines the boundary between the h^{th} and $(h+1)^{\text{th}}$ strata. This set of equations is, however, extremely cumbersome to handle.

It can be seen that when the population is rectangularly distributed (1) leads to strata of equal width, a result which is otherwise obvious. It is surprising that, in order to get a result which can be applied in practice Dalenius did not consider the simplest possible departure from rectangularity, a population whose density $f(x)$ has constant first derivative $f'(x)$, and hence work out a first-order correction to equal intervals.

Consider a population distributed on (0, 1), divided into two strata at x, with $f(0) = 1 - a$ and $f(1) = 1 + a$. Then

$$\sigma_1^2 + (x - \mu_1)^2 = x^2(2 - 2a + ax)/6(1 - a + ax)$$

and

$$\sigma_2^2 + (x - \mu_2)^2 = (1 - x)^2(2 + a + ax)/6(1 + ax).$$

To the first order $\sigma_1/\sigma_2 = x/(1 - x)$. Substituting in (1) we have to the first order, assuming a small, $x/(1 - x) = 1 + \frac{1}{2}a$.

The ratio of the mean densities in the two strata is $(1 - a + ax)/(1 + ax)$ which to the first order equals $1 - a$. Suppose now that we had another distribution with density mid-way between the rectangular and that of the above distribution, and suppose that this were stratified at the point x. For this distribution we would have a ratio of mean densities in the two strata equal to $1 - \frac{1}{2}a$, so the numbers in the two strata (stratum width $\times$ mean density) would be approximately equal. Thus if we construct this artificial distribution, and stratify it in such a way that equal numbers of individuals are included in the two strata, the stratum division so obtained will be nearly optimal for the original distribution.

This suggests the following procedure for achieving approximately optimum stratification for the general case of any population, finite or infinite, and for any given number of strata. Let x denote the stratification variable, and $F_1(x)$ denote the number of individuals $\leqslant x$. Let $F_2(x)$ denote the number of individuals calculated on the assumption

that the distribution is uniform between the smallest and largest values x_1 and x_N say, i.e. $F_2(x) = N(x - x_1)/(x_N - x_1)$ (ignoring an obvious end-correction!). Let

$$F(x) = \tfrac{1}{2}\{F_1(x) + F_2(x)\}.$$

Then $F(x)$ may be regarded as the cumulative frequency function of a distribution mid-way between the actual distribution and one which is uniform over the same range. Choose strata so that the artificial distribution gives an equal number of individuals in each stratum, i.e. $F(X_{h+1}) - F(X_h) = N/k$, where X_h is the lower boundary point for the hth stratum. Then the resulting strata should be approximately optimal for the actual distribution. Obvious modifications can be worked out for infinite distributions with infinite range. The method is crude but in problems of this kind the minimum is very flat and a first-order solution often takes one sufficiently close to the optimum. Approximate optimum allocation can be achieved by taking sampling fractions proportional to strata widths. An alternative and perhaps simpler procedure would be to find strata boundaries, first by taking equal numbers of individuals per stratum, secondly by taking equal strata widths and adopting the points mid-way between as an approximate solution.

The book ends with the treatment of a number of miscellaneous problems including choice of optimum numbers of strata where cost factors are involved, stratification with regard to a number of criteria, estimation for subregions and contributions to the study of non-response.

J. DURBIN.

6.—*Economic Arithmetic*. By Robin Marris. London, Macmillan, 1958. xvii, 349 pp. $8\frac{1}{2}''$. 28*s*.

This book is intended for second and third year economics honours students. The author aims to integrate their economic theory with statistical practice without going into detailed descriptions of statistical techniques, and only "O" level in Algebra in G.C.E. is required for nearly all of the book. The book is divided into three distinct parts.

The first part, entitled "Anatomy", discusses problems arising from the need for a general picture of the economic system. The first chapter is excellent in giving examples of official statistics with problems of their meaning and interpretation. The discussion is linked to a flow diagram of the use of labour in the United Kingdom. Unfortunately the following chapters do not maintain the level of interest. The second chapter deals with sources of economic statistics in the United Kingdom, and gives a short list of references. Although official publications are listed, no mention is made of *Economic Trends*, which, with its monthly articles on particular topics in official statistics, is an increasingly useful publication. Chapters three and four discuss the National Income and Expenditure Blue Book and make very dull reading. For those making a detailed study of the official publications these chapters will make most useful parallel reading, but they are unlikely to be very useful to others. The discussion of social accounting concepts is left to other writers and the principles of division of the economy into sectors and of accounts into various types are not discussed, the Blue Book divisions being accepted without comment. An example of the failure to distinguish between principle and practice comes in the discussion of gross national product. Among the "awkward" small items "which clearly require inclusion on the welfare criteria", "residual error" is listed with the other items such as "ownership of dwellings" and "net income from abroad". When the items are discussed in turn, residual error is described as a "technical adjustment" and the reader is referred for an explanation to the official *National Income Statistics: Sources and Methods*.

The second part of the book, "A Selection of Instruments" describes some statistical methods useful to economists. Elementary aids to inspection cover tabulation, rounding and graphs (very briefly), and ratios and percentages. Instead of carrying on with the usual order of averages and variation, the other two chapters deal with time series analysis and regression analysis. The discussion of the interpretation of time series is good but

the algebra will surely confuse. The positioning of time series values is very muddled and the statement (on page 123) that $\bar{X} (= (1/n) \sum_1^n X) = \frac{1}{2}n$ cannot be excused as a misprint since the result is used in the subsequent example. It is difficult to decide what the author is trying to accomplish when he goes (unnecessarily) to the trouble of using calculus to prove the least squares regression line formulae and gives an example of their calculation (without using an assumed mean) but refers the reader to the "standard texts" for the computation of moving averages. Similar criticisms may be made of the following chapter. Here the notation is confusing as β is used for both the regression coefficient and its estimate. When the coefficient is defined for X on Y it is written as $1/\beta$ so that $r = \sqrt{\beta_{XY}/\beta_{YX}}$—an unnecessary departure from usual practice. The construction of the example data is also confusing and unlikely to be of much help to the would-be model-builder. There are again some unfortunate slips: if the correlation is perfect the regression coefficient is the ratio of the two standard deviations, not variances (page 168), and a division by N is omitted from the second term in the numerator of formula (3) on page 161.

Part III, amounting to nearly half the book, deals entirely with economic index numbers. The first chapter discusses formulae referring to a simple quantity index example. It is followed by a long "Algebraical appendix" which is very hard going as the notation is cumbersome and the discussion is in general terms which are more difficult to appreciate than "prices" and "quantities". Its usefulness to a student new to the subject is also limited by the bias towards the author's own views which are not generally accepted. He likes to regard a Laspeyre quantity index, say, as the ratio of two weighted averages, $\Sigma q_1p_0/\Sigma p_0$ and $\Sigma q_0p_0/\Sigma p_0$, even though "little economic meaning can be attached" to the sum of the weights. In fact the value of the sum of the weights, Σp_0, depends entirely upon the quantity units which are priced, and in the case of the price index Σq_0 has no physical meaning since the quantities may be different in kind. Again, he does not like to use expenditure weights because they vary even if quantities purchased remain the same and only prices change. But, on the usual assumptions of demand analysis, such consumption behaviour is irrational and a "true index" (as defined in the subsequent chapter) for a consumer who maximizes his satisfactions is not the same as the Laspeyre and Paasche indices even when the two are equal. The other two chapters apply the formulae to the measurement of changes in real family income and in real national product. These are likely to be the most useful chapters in the book since the author gives a detailed account of the link between economic theory and the practical possibilities, a link which usually receives little attention. Particularly interesting is the theoretical economic argument in favour of the Fisher Ideal index number. It is perhaps a pity that the author does not go on to describe which practical possibilities are realized in this country, and discuss the published index numbers. Since the national income chapters are so closely tied to the Blue Book, there can be no objection in terms of the book becoming out of date.

In no way can this book satisfactorily stand alone. Part I needs to be supplemented by detailed reading on statistical sources; Part II gives an idea of possible statistical techniques but little help in how to use them if required; Part III reflects the author's own views of index number construction and gives few practical examples. The book is therefore more likely to be used as an additional reference on particular topics, and for this purpose the chapters on index numbers of real family income and real national product are likely to be the most useful.

RITA J. MAURICE.

7.—*Linear Programming: Methods and Application.* By Saul I. Gass. New York and London, McGraw-Hill, 1958. xii, 223 pp. 9″. 50*s*. 6*d*.

The cover sheet of this book claims it to be the first formal text in linear programming and this claim is adequately justified by the contents. It is certainly meant as a textbook to be studied and not as an introduction to the subject to be read casually by someone interested in it. The mathematical background to linear programming is discussed in the

first part of the book and all the necessary theorems are proved with full mathematical rigour. In the second part the computational methods are described and again all the necessary theorems are proved. The last section describes many applications of linear programming.

The treatment of the subject is very comprehensive and the book must become an indispensible part of the library of anyone working on the theoretical aspects of linear programming, and certainly of any teacher of it. In his desire to be comprehensive, however, the author seems not to have made up his mind whether he was in fact writing a textbook for the teacher and research worker or for the undergraduate and student, although he tried to clarify his position in the preface. The former will be irritated at the space taken up in defining a matrix and explaining the most elementary methods of solving simultaneous equations. The latter are likely to find the pace of a book which moves in ten pages from the definition of a matrix to the proof of a theorem in convex sets somewhat breath-taking. The student too may find a certain confusion of symbols in the early part of the book a little puzzling and will not always find the language easily understandable. These are minor criticisms, but the writer of a textbook for students should take extra care on these points.

An able mathematician who wishes to familiarise himself with every aspect of "classical" linear programming will find the book very adequate. The simplex and revised simplex methods of computation are fully discussed. Duality problems are very well explained with a good example, due to Harrison, of their economic meaning. Degeneracy procedures are described sufficiently in view of the author's assertion that only three examples of this difficulty have so far been reported and that those were artificial.

Parametric programming, a speciality of the author, in which the coefficients of the problem are allowed to vary is discussed. Available digital computer codes for linear programming computations are listed. The book ends with a comprehensive survey of applications and a good, but somewhat rushed chapter on the relationship between the theory of games and linear programming.

The adjective "classical" was used above because simple departures from linearity, non-linear programming and dynamic programming are specifically eschewed and discrete programming is but briefly mentioned, although a little known theorem about the discrete nature of the solution to the transportation problem is elegantly proved.

There is, of course, no reference to statistics. Linear programming is at present used in completely deterministic situations. The justification for the statistician taking an interest in the technique is firstly because of its inherent mathematical interest and secondly because it helps solve a type of problem which the industrial statistician in particular may be called upon to handle. This book is really too comprehensive for such a person unless he intends to make a study of the subject but he will find the chapters on the simplex method, and the sections in which computations on practical examples are carefully worked through, very valuable.

The proof reading has been very carefully done and such errors as there are can be detected with a little thought. W. E. DUCKWORTH.

8.—*Allocation in Space. Production, Transport and Industrial Location*. By Louis Lefeber. Amsterdam, North-Holland Publishing Co., 1958. xv, 151 pp. 8¾″. 34*s*.

The three parts of this study proceed from the simple to the more complex in easy steps. Part I considers two locations, each producing a different final commodity from the same two types of input. The latter are available in given quantities at each of the locations. The input can also be transported between the locations, and it is required to maximize the total income from the final goods. In Part II, final goods must be transported to locations of consumption. Finally, in Part III, more than one industry can settle at the same location, and identical goods can be produced at several locations. If the capacity of the transportation network as well as the available input is limited, then all these problems lead to formulations in linear or non-linear programming, dependent on whether the production functions and the transport demand functions are linear or not.

The author mentions these techniques with explanatory remarks which might satisfy a non-mathematical reader. He stresses the implicit derivation of shadow prices (Lagrange multipliers), and describes those features of the solutions of interest to an economist, in terms in which the latter has been brought up to think. Moreover, the differences in the solutions, deriving from various sets of parameters, are well explained in economic terms.

The reviewer found it rather curious, though, that this systematic approach, making convenient use of mathematical shorthand notation and of computational routines, appears only after a geometric representation of "basic cases" has been introduced, a "neoclassical approach" based on calculus has been given, and a "common sense derivation of conditions for optimal allocation" has been exhibited. This seems to be the reverse of a natural method of presentation. It is, of course, impossible to achieve anything beyond the obvious or the trivial by intuitive methods, when the number of variables increases, and it would be more satisfactory to introduce an all-embracing technique first, and then to explain the common-sense interpretation of its application to simpler cases.

The author acknowledges the limitations inherent in his model, and an Appendix generalizes the formal analysis to numerous consumption locations, production locations, final goods, transportable factors, and consumers, to obtain final market prices in a general equilibrium. There is an Index, containing a great number of references to footnotes, thereby incorporating a limited bibliography of further reading. S. VAJDA.

9.—*Elements of Vital Statistics*. By B. BENJAMIN. London, Allen and Unwin, 1959. 352 pp. 9″. 56s.

Newsholme's *Elements of Vital Statistics* was first issued in 1889, at a time when public health activity was increasing rapidly in Great Britain and the United States of America, and this work was in constant demand for a long period. The same author was able to embody 34 years' further experience in a re-written version published in 1923. The lapse of another 36 years has indicated that a fresh revision is due, if not overdue, and Dr. Benjamin, taking heed of many recent developments, has created a completely new text, retaining from past editions little more than a title that is, today, perhaps rather misleading. The book now appearing is not "elementary", and its emphasis is laid much more firmly on health and sickness than on life and death.

Dr. Benjamin has proved himself a worthy successor to Newsholme and has written a very comprehensive guide for all those who are concerned with public health and welfare. His knowledge of statistical inquiries into disability is impressively wide and he is able to quote a long list of references at the end of each chapter. Moreover he begins his account of each different aspect with a detailed history of the origin and advancement of the study of the subject and the collection of data bearing upon it. Although these digressions into the past are perhaps not of great concern to social workers in highly-developed countries today, there is no denying their interest, and they may well have significance in relation to current developments in less fortunate areas of the world.

Chapter 8, though short, seems to be the core of the book. Entitled "The measurement of morbidity", it deals with questions of normality and abnormality in health, showing the results of experiments in which measurements have been made, among fit persons, of characteristics that are usually studied only during illness, and in which wide variations are sometimes found even in the normal state. Like so much of the volume, it deals exclusively with post-1923 work and has no counterpart in Newsholme. Chapter 8 also lists the basic requirements for medical records, and so paves the way for the ensuing eleven chapters, which deal one by one with the various methods of approach to, and regions of special attention in, sickness statistics. At the end of the final chapter, on "Field studies", the text ends without any summary, and thus no attempt has been made, either in advance or in arrear, to sum up the area of enquiry, to explore its limitations or succinctly to compare and contrast the diverse possible forms of approach to the measurement of illness. No doubt this omission was deliberate and constitutes a resistance to a temptation to over-simplify, but it may be wondered whether a guide of this kind, however

tentative, would not have been welcomed at least by the less experienced in matters of public health statistics.

Before reaching Chapter 8 and the long sequence of studies of data bearing on disease that it inaugurates, the reader will encounter first a few introductory pages and then six chapters largely devoted to demography. Two of these deal with census and registration statistics, one with fertility and three with mortality. As is entirely appropriate, these chapters are devoted only to those aspects of demography that are of prime concern to public health workers, and the author hardly touches upon such subjects as marriage experience, population projection or the comparative analysis of actual populations—other than those of Great Britain and the United States. On the topics chosen for exposition, the chapters are, however, fully informative, and that on fertility, for instance, gives a good short introduction to the study of births and replacement. The treatment of mortality is tripartite: first, causes of death and the methods of their medical classification are fully described; then the more mathematical aspects of indexes and life tables are explained, and finally attention is paid to environmental influences on death rates. The exposition of mortality indexes is thorough but condensed and may make considerable demands upon readers new to this subject, especially if they are not mathematically inclined. Much the same is true of the discussion of methods of constructing abridged life tables. These matters were treated by Newsholme in a much more leisurely and elementary fashion.

Chapter 2, entitled "Population census", deals both with the machinery of enumeration and with the kinds of result obtained by this means. It goes so far afield as to include migration records and the Royal Commission of 1944–49. Although the many ramifications of population analysis are notoriously difficult to fit into a clear-cut framework perhaps the author has tried to include too much here, with the result that some sections—for instance that on age-distribution—are so condensed as to be potentially difficult for a non-specialist reader. Chapter 3 seems just right, however, in its summarization of the methods of registration of births, deaths and marriages and the manner of publication of information about these events. It seems strange that in the section, starting on page 106, which is given to the subject of illustrations of mortality experience in terms of "years of life lost", no mention is made of Liebmann Hersch, the great champion of this method. Such omissions are rare, and apart from a few misprints, such as may be expected in a new edition, one can hardly fault the author for errors either in detail or in general approach and emphasis.

The subjects of Chapters 9–19 are: first, statistics of infectious diseases, with a separate and masterly chapter on tuberculosis; then maternity and child welfare, statistics of the health of school children, and other public health statistics; industrial and general incapacity, including the records of friendly societies and systems of national health insurance; four principal modern sources of information—hospitals, general practitioners, and the registration systems for cancer and mental illness; and finally field studies—a special although brief reference to sampling and surveys.

In all these later chapters the author shows his mettle and reveals the width of his knowledge and experience in an inimitable manner. This section of the book alone constitutes a valuable source of reference in a field that has not hitherto been fully charted. The whole will be welcomed by those for whom it is designed, and by many others also.

P. R. Cox.

10.—*Morbidity Statistics from General Practice*. By W. P. D. Logan and A. A. Cushion. General Register Office Studies on Medical and Population Subjects No. 14, Vol. 1 (General). London, H.M. Stationery Office, 1958. 174 pp. $9\frac{1}{2}''$. 15*s*. 6*d*.

This enquiry has added considerably to the knowledge of sickness in the general population. Up to now very little was known about the amount or nature of sickness among the general population. The only enquiry which included samples from the whole population was the Survey of Sickness, 1943–52. This enquiry suffered from serious defects, the chief being self diagnosis and the ability to remember the date and duration of the

sickness and in consequence the enquiry gave only a very broad picture of morbidity. The present investigation, which gives for the first time a representative sample of the sickness in the general population, was made by the College of General Practitioners and the General Register Office and covers the twelve months May 1955 to April 1956. It included the National Health Service lists of 171 doctors (principals) in 106 general practices over the whole country, with 382,829 persons or 0·86 per cent. of the population of England and Wales in 1955.

The number of consultations made by the patients during the year was 1,436,155, a rate of 3·8 consultations per patient. Two-thirds of the patients on the lists consulted their doctor at least once during the twelve months. A larger proportion of females than males, 70·2 and 63·5 per cent. respectively, consulted their doctor, and the females saw their doctor more often, having an average of 4·08 consultations compared with 3·39 for males. The proportion of patients consulting their doctor was largest at ages under 15 years, being slightly greater than the rate for the elderly, ages 65+, but the elderly had on the average twice as many consultations as the children and young adults.

There was a considerable variation between the regions in the amount of sickness. The percentage of female patients seen varied from 61 in the Eastern region to 74 in the South Western region, and for males from 58 in the Eastern to 65 in the Midland and 65 in the London and South Eastern region. The average number of consultations varied, for females, from 3·10 in the Eastern to 4·69 in the South Western region and for males from 2·64 in the Eastern region to 4·00 in Wales.

The diseases and conditions for which patients consulted their doctors have been tabulated in great detail under 261 headings. The most frequent complaints were the common cold, arthritis and rheumatism, bronchitis, psychoneurotic disorders, influenza, and the percentage of patients seen for these sicknesses were 8·1, 6·5, 6·2, 4·6 and 3·8, respectively. The conditions which gave rise to the largest number of consultations were bronchitis, arthritis and rheumatism, psychoneurotic disorders, common cold, influenza with 26·1, 20·4, 16·6, 16·3 and 11·3 per 100 patients, respectively.

Urbanization appears to be a factor of some importance, and for the common cold the number of consultations per 100 patients was 17·6 in the urban, 15·1 in the semi-urban and 13·9 in the rural areas. For bronchitis the percentages were 30·3, 21·2 and 18·9, respectively, but accidents were more important in the rural districts than in the other two areas. Considerable regional variation was recorded. The percentage of patients with a common cold varied from 5·9 in the Southern region to 11·6 in the Midland region, and the percentages for bronchitis varied from 3·5 in the Eastern to 7·1 in the East and West Ridings.

The number of patients who visited their doctor for non-sickness reasons formed 5·3 per cent. of the population. The amount of this service was correlated with urbanization and the percentages varied from 4·1 in the large towns with a population of 100,000 and over to 6·7 in the rural areas.

Admissions to hospital during the year numbered 16,310 persons, or about 11 per 1,000 consultations. The causes with the largest number of admissions were 1,353 for neoplasms (malignant and benign), 1,026 for hypertrophy of tonsils and adenoids, 955 for appendicitis, 682 for hernia of abdominal cavity, 580 for fractures, 532 for psychoses and psychoneurotic disorders.

The second and third volume of this enquiry will deal with the distribution of sickness by occupations and with the more clinical implications of general practitioners' morbidity statistics.

W. J. MARTIN.

11.—*Population Problem in India.* By P. K. Wattal. New Delhi and Simla, Minerva Book Shop, 1958. viii, 228 pp. 8½″. *Rs.* 10

This volume is described as "a census study" but it is a great deal more than that; it is a comprehensive review of the stress of population growth upon the Indian economy as part of a world problem of balance between resources and rapidly growing numbers. The author does not write with political impartiality; his real fear is that the effects of over-

population, unemployment and starvation, will lead to Communist domination in India. However, the facts set out in this review are well documented and are from unimpeachable sources. The implications of population pressure are inescapable. "If present-day trends continue, India will have 410 million people in 1961, 460 million in 1971 and 520 million in 1981. This growth will neutralise all attempts to raise the standard of living by planned economic development."

At its simplest, the situation in India is one of rapid population growth resulting from a steady decline in mortality with only a slight fall in the birth rate. Migration is not a complication of importance. Indian diets are already deficient in proteins, protective foods and fats. Positive measures to remedy this—more efficient exploitation of agricultural manpower and industrialization—will not succeed unless fertility is also reduced. "The family planning movement should set before itself a target of a reduction in the birth rate from 40 to 20 per thousand, within a period of 20 years."

The author admits that "the Government of India today is, with the exception of Japan, the only other government which has given official approval to a programme of family planning", but he is perhaps less than fair in failing to provide any account, in an otherwise detailed book, of the steps being taken to implement this programme. He does, however, touch on a fundamental truth, for in a comment on the financial provision for family planning he says, "mere increase of expenditure is not an indication of successful working. Complaints are made that family planning centres do not attract many people". If the birth rate falls in India, it will not be a result of official policy pronouncements, or of the mere provision of contraceptives, or of books about the dangers of starvation; it will be because the Indian peasant is actually motivated to restrict the size of his family. The problem of the Indian Government will be to achieve this motivation—at village level.

B. BENJAMIN.

12.—*Statistics Essential for Police Efficiency*. By John I. Griffin. Springfield (Ill.), C. C. Thomas; Oxford, Blackwell; 1958. xv, 229 pp. 9″. 52*s*. 6*d*.

This elementary text-book is clearly written, excellently printed and well illustrated. It could very well be studied within all police forces and other organisations that have to do without the help of a professional statistician.

Statistics about crime, whether in the U.S.A. or the United Kingdom, are a fairly top-heavy structure, and their base, that is the methods by which crimes are identified and recorded in the first place, has not been studied as closely as the rest. This book does not repair this omission, and could thus be said to ignore some of the most difficult problems peculiar to police statistics. From the point, however, that basic data are provided, the author describes most clearly and readably all the more usual ways of handling and presenting them; and it should be added that there is also a useful brief discussion of sample survey methods. There is a healthy emphasis on fairness of presentation, particularly with regard to charts and graphs. The number of aspects of statistics, ranging from charts for police reports to the fitting of parabolic trend lines, dealt with in 200 pages is remarkable.

It is inevitable that statistics learnt from a short book should be understood on only a superficial level, and it would be a pity if police or others handling data were to look on this book as more than an introduction to further study or, for senior staff, a survey of what to expect in statistics presented to them or material for a periodical refresher course. For these purposes it is excellent.

There is a bibliography, mainly of American works, some statistical tables, and a good index.

T. S. LODGE.

13.—*An Appraisal of the* 1950 *Census Income Data; by the Conference on Research in Income and Wealth*. A Report of the National Bureau of Economic Research. Princeton, University Press; London, Oxford University Press; 1958. x, 450 pp. 9″. $10.

In the 1950 Population Census of the U.S.A. details about income were collected from one case in every five. This was the source of the "1950 Census income data" which

was studied in great detail at the 1956 Conference on Research in Income and Wealth. The present volume contains many of the papers read at that conference as well as much written comment upon them.

Primarily this is an *aide memoire* for those who attended the conference. Apart from that it is a work for the specialist. The list of the contributors is as long as a biblical genealogy. So many eminent and contemporary economists and statisticians could hardly be expected to agree, and they don't. The volume contains twelve papers each commenting upon some aspect of the income data and upon the other papers. In addition there are seventeen commentaries on those papers.

The volume has three main sections which follow an unusually long introduction. In the first section there are three general papers; one about the general shortcomings of the income data themselves, one about other sources of income data and one giving a history of the census questions. The second section gives the results of the Census income data. The third section gives some detailed comparisons with other income data.

At first sight the use of such a widespread source of information about incomes would seem to be invaluable. For a variety of reasons the harvest of information is not as good as might be expected. For one thing the usefulness of the Census income data is limited by the definitions used which are not the same as those used either for tax purposes or for the national income accounts. After making adjustments for tax differences, adjustments that are necessarily inexact, the data can be compared with those available from other sources. This has been done with the results of field surveys, with the tax returns, with information about the social security payments and with the national income estimates. These comparisons reveal that there was considerable understatement of incomes in the census. The amount of understatement has been estimated as nearly one-tenth of total wages, one-ninth of "non-farm entrepreneurial income", two-ninths of farm income, about a third of rents, social security payments and "military" payments and over three-quarters of interest and dividends. These deficiencies are significantly greater amongst those types of income belonging more commonly to the wealthy than amongst those belonging more commonly to the less wealthy, and so they have the effect of reducing the inequality in the resultant distribution of total incomes. Indeed discrepancies as large as these must raise doubts whether most people know what their incomes really are. This conclusion squares with the common experience of the individual who finds much less left in his pocket than he expects to find. The explanation of a part of the deficiencies may be that income is reported net after the deduction of tax instead of gross before the deduction of tax. The book also shows that if there are many people in the United States who do not know the size of their own incomes even more do not seem to know the size of the family income.

G. PAINE.

14.—*Social Economics*. By Walter Hagenbuch. Welwyn Garden City (Herts.), Nisbet; Cambridge, University Press; 1958. xvi, 320 pp. $7\frac{1}{4}''$. 12*s*. 6*d*.

Social economics, or the study of the problems of mankind in their everyday environment is, in the words of the author, a frontier activity covering as wide a territory as the social economist himself may define. In the pursuit of his study he is constantly crossing and re-crossing the boundaries of neighbouring sciences. The analysis of population changes and their effect on the national economy, for example, rightly fall within his province but he soon finds himself asking such questions as, why do people have smaller or larger families, and why do they marry early in life or live longer? So straight away he moves into fields which are equally the concern of the sociologist, the demographer, the biologist, and many others.

The book, which has ten chapters and an epilogue, approaches the subject matter of social economics against a background of economic theory. The first chapter (which according to the author is "more sober than the rest") is an economic exposition of the scope and meaning of social economics, which can be considered from four points of view —as a branch of applied economics, as a branch of applied statistics, and as studies of the

social causes of economic behaviour, and of the social consequences of economic behaviour. Then follow self-contained chapters on population, housing, working conditions, unemployment, poverty, and the voluntary social services. These in turn are followed by broader-based chapters on the public social services, on national insurance and assistance, and finally on other public services. The theme of the epilogue is the changing aims of the social services.

The construction of each chapter follows much the same lines. Thus the chapter on the public social services opens with an economic analysis of their structure and their objectives. It is remarked that the most valuable social services are not commonly classed as social services at all; these are those aspects of economic policy which promote a high level of employment and a high rate of economic progress. This is followed by a section on the financing of the public social services, in which a table shows that the expenditure on the public social services in 1955–56 was £2,501 m., of which £1,002 m. was spent on transfer payments, e.g. retirement pensions and family allowances, the main purpose of which is to raise personal incomes and prevent poverty; £1,144 m. on health and education and other services which are concerned with the re-distribution of opportunity; and £355 m. on subsidies, the purpose of which is to reduce the cost of living (but, the author asks—whose?). Then follows a section, covering 14 pages, on the development and break-up of the Poor Law from the middle ages to the report of the Royal Commission of 1909. This like every other chapter presents a most coherent and satisfying picture.

In the next chapter which traces the development of national insurance and assistance from the National Insurance Act of 1911 to the present, the author pays tribute to the work of the National Assistance Board, whose Annual Report, he says, "breathes an air of human understanding and personal interest . . . ". It is interesting to be reminded in this chapter of the importance attached in the discussions leading to the existing framework of national insurances to the principle of fixed contributions and fixed benefits. Ideas (and arguments) seem to have changed a good deal now—witness the national pensions scheme proposals by the different political parties, with variable contributions and variable benefits.

In different parts of the book, two interesting points are made as to the effects on the national economy of the ageing population, which at times we are led to believe will be almost catastrophic. In the chapter on "Population" it is calculated that in the thirty years between 1947 and 1977 the number of consumers per producer will rise by $4\frac{3}{4}$ per cent., while on the basis of past experience productivity will in the same period rise by some 50 per cent. In the section on the National Health Service the estimates of the Guillebaud Committee are quoted to the effect that over a period of twenty years population changes will increase the cost of the Service by as little as 8 per cent., of which only $3\frac{1}{2}$ per cent. will be attributable to the increase in the proportion of old people. Such estimates put the matter into better perspective.

The book already includes a wide range of topics and it may seem churlish to ask for more especially as the author himself remarks on the exclusion, through lack of space, of education and the distribution of incomes. But at the risk of making the book over-full the inclusion of a chapter on education, which is now making such a surge forward, would have been valuable. Also an analysis of private pension schemes and their relationship to national insurance and national pension schemes would at this time have been opportune and instructive.

A few points of detail on which comment might be made were noticed in reading. In discussing the National Insurance Scheme the author says: "The main departures from Beveridge were four . . . " among which were, that "Family allowances were not provided for the first child of school age . . . ", a statement which seems to be wrong in two respects. In the section on family allowances it might have been said that Beveridge recommended an allowance additional to the existing allowances in kind, of eight shillings a week for all children other than the first whereas the additional allowance actually paid was five shillings. Although the Preface is dated 1958, no mention is made that in 1956 family-allowances were raised to ten shillings a week for the third and subsequent children,

the age limit being extended from 16 to 18 years—two interesting developments. Speaking of the effect of the National Health Service on general practice, the author says " . . . the higher demand for service per head worked itself out in crowded surgeries and less thorough medical attention . . . ". There is certainly no evidence for the latter statement, while Titmuss in his "Essays on The Welfare State" concludes that demand has not in fact increased and may indeed have fallen. Of the seven Tables, three have misprints or arithmetic errors.

This book is excellent in every way and is most warmly commended. It is a worthy addition to a notable series and meets a very real need. The style is clear and lively and most engaging. Throughout the book there is a strong breeze of logic and reason and a broad picture emerges unencumbered by detail. The few tables are most apt. The author says the book is intended for students of economics; it should equally be welcomed by students of sociology, politics, and the like. Those connected in any way with the administration or development of social policy would also find it most rewarding.

E. R. Bransby.

15.—*The Labor Force under Changing Income and Employment.* By Clarence D. Long. (National Bureau of Economic Rescarch, Gen-Series 65.). Princeton, University Press; London, Oxford University Press; 1958. xxiv, 440 pp. 9″. $10.

The author of this study describes it as an attempt to solve the problem as to why people work. It carries forward and elaborates two studies formerly made by Professor Paul Douglas in addition to discussing new conceptions of various aspects of the subject. Professor Long begins by enumerating many factors, including the availability of work, personal assets, and extent of social security benefits, which may influence a person in his decision to work, some of which are not economic and which suggest a challenge to the simple postulate of the classical economists that people work less as wages increase because higher wages enable them to satisfy their needs with less effort. After examining this theory he proceeds to an investigation as to the extent to which the population participates in the labour force according to age, sex, colour, nativity, marital status, child-care responsibility, rural and urban residence, density of population, size of cities, income, school attendance, educational attainment, employment status of wife, hours of work and social security systems. The central questions which he poses are (1) has labour force participation been influenced by changes in income and in employment and if so in what circumstances, (2) are these two influences powerful enough to stand out over other possible influences and do they reveal themselves only when the effects of others are removed, and (3) does any one among the other possible influences furnish an explanation of labour force behaviour or must an explanation be sought in some combination of social, demographic and economic forces? In his examination the author draws on the statistics of the U.S.A., Great Britain, Canada, New Zealand and Germany for periods up to a century or longer.

On the main question as to whether the labour force is influenced by changes in income and employment the author concludes that the answer is incomplete in spite of a detailed search into an unusually rich store of statistical material. This is due to the fact that many influences had to be neglected. These included the effect of differences in family characteristics, changes in the attitude of communities on whether wives and children should work, changing conditions of work, altering intensity of effort required by employers and changes in the strength of trade unions. The absence of information on these and other economic and social forces leads to the conclusion that the anwer to the main question must be hesitant, but the author concludes that changes in income do affect the extent of participation in the labour force provided the other possible influences do not change much.

The book adopts the somewhat unusual plan of stating the summary and conclusions at the outset. This has the advantage of placing the reader in possession of the general arrangement of the treatise before facing a vast amount of detail on a wide variety of subjects.

The chapters that follow the summary deal in detail with the relationship between the

labour force and earnings in different cities, states, nations and income groups at a given time, the labour force of females in relation to the earnings of males in different income groups, the participation of females, and of males, in the labour force over time. There are also chapters on older workers in the labour force, the labour force in severe depressions, short run labour force behaviour, the stable labour force under rising income and high employment and variations within the stable labour force. All these subjects are examined in considerable detail, the available figures for the five countries being presented in tables and charts. There are also elaborate appendices which, in addition to presenting detailed statistics with ample notes as to their origins, include notes on such subjects as the conception of unemployment and labour force and on the content and comparability of the statistical measures of the labour force in each of the five countries.

R. B. AINSWORTH.

16.—*Economic Characteristics of International Migrants: Statistics for Selected Countries* 1918–1954. New York, United Nations, 1958. xv, 314 pp. 11″. 25*s*.

This is primarily a source book of statistics on international migrants, classified by occupations and other economic characteristics and will be found as invaluable as the U.N. Demographic Yearbook is to users of international vital and census statistics. (Incidentally the Demographic Yearbook itself includes some statistics on migrants, but not by economic characteristics.) The classifications shown are mainly by occupation or industry but others are occasionally shown, such as capital owned or whether the passage was subsidized.

Users of migration statistics will be well aware how inferior such statistics normally are compared with other demographic statistics and, expecting very little, they will not be disappointed. In the circumstances, the U.N. staff have made a praiseworthy attempt in the introductory commentary (comprising the first quarter of the volume, the remainder being devoted to national tables) to examine the problems involved in the collection of sound migration statistics and to clarify questions of comparability. Faults in migration statistics are also discussed frankly.

For users of international migration statistics this volume fills a vital need, but anyone connected with the collection of such statistics might well benefit from a careful study of the first quarter of the book, for the authors have written with an appreciation of the practical difficulties deriving from the circumstances holding at frontier posts, from the prospective migrant's ignorance of his future and from incentives to supply incorrect information to obtain entry when migration is restricted. N. H. CARRIER.

17.—*Corporate Bond Quality and Investor Experience*. By W. Braddock Hickman. Princeton, University Press, 1958. London, Oxford University Press. xxix, 536 pp. 9″. $10.

This volume is the second of a series of three, reporting on the findings of the bond research programme of the National Bureau of Economic Research, New York. The data analysed throughout the programme were compiled from a complete census of all domestic (U.S.) issues in the financial manuals from 1900 to 1943, and it is the purpose of the analysis to derive from that vast body of heterogeneous data useful deductions as to investors' overall experience and the validity of the various methods of classifying bonds which are used by, or legally imposed upon, American investors.

Two techniques are used to examine the data. The default rate is used to test the proportion of new bonds issued in a given period which subsequently go into default and to test the proportion of all bonds current at the beginning of a stated period which defaulted during the period. Redemption yields are used to estimate the promised yield, assuming the terms of the offer are honoured, and to estimate the realized yield taking account of call premiums paid and defaults. The difference between these two yields is the loss rate.

It will be seen that the design of the investigation suffers from two disadvantages:

the attempt to cover too long a period, giving too great a degree of heterogeneity which is not adequately met by subsequent divisions of data; and the almost complete failure to use statistical tests of significance. With the results set out in some 116 charts and tables it is disconcerting to find that yields are often given to two significant figures only and the resulting loss rate, being a difference between two yields, is often given to only one significant figure with a possible substantial proportionate error due to rounding alone. As one is told that the third volume in the series will present the data in more detail than the present volume, giving notes on methods of derivation, this criticism could be unfair, but certainly, taken in isolation, the present report is far from satisfactory.

The American bond market is subject to indirect statutory restraint to a degree unknown in the U.K. Legislation varies between states but, in many cases, mutual savings banks are restricted in their purchases to bonds falling within narrowly defined categories and meeting rigid statutory standards; similarly, commercial banks and life companies are under some pressure to confine purchases to bonds rated highly by the various rating agencies. Sales of bonds which cease to comply with these requirements are sometimes, but not always, compulsory. The tests used establish that each of these restrictions tend to reduce both the proportion of defaults suffered and the yield obtained on a portfolio of bonds. It is also shown that sales on default tend to reduce yields to a lower level than those earned when defaulted bonds are retained, but this result is of limited value in other countries since it is, in part, due to the depressed prices of defaulted bonds resulting from enforced sales under the legislation the effect of which is being considered.

One might conclude that this book records an immense amount of research which has merely established what was otherwise obvious, but that would be too superficial a view. The field of investment abounds in axioms upon which thought and policy are based but which have never been rigorously tested, and this work is a valuable reconnaisance into this uncharted territory. After such a far-ranging general survey, the ground is now ready for detailed studies of small sections of the field using smaller, but more homogeneous, groups of data and more sophisticated tools of analysis. If, later, the realm of inquiry spread into the field of ordinary shares, there would be found an immense field for statistical analysis.

H. C. COTTRELL.

STATISTICAL NOTES

(1) British Official Statistics

The index of retail prices compiled by the Ministry of Labour and National Service which was 110 in December, 1958 (prices at January 17th, 1956 = 100) remained at the same level in January, February and March. Calculated to one place of decimals the figures were 110·2, 110·4, 110·3 and 110·3. The detailed figures of the weights used in calculating the index and the indices for different commodity groups were as follows:

	Food	*Alcoholic Drinks*	*Tobacco*	*Housing*	*Fuel and Light*	*Durable Household Goods*	*Clothing and Footwear*	*Transport and Vehicles*	*Miscellaneous*	*Services*	*All Items*
Weights	350	71	80	87	55	60	106	68	59	58	1,000
Dec. 16th, 1958	109·2	105·8	107·8	125·1	116·6	99·9	102·7	113·1	113·6	115·4	110·2
Jan. 23th, 1959	109·8	105·9	107·8	125·8	116·6	100·1	102·2	113·4	113·6	114·8	110·4
Feb. 17th, 1959	109·1	105·9	107·8	126·2	116·9	100·1	102·2	113·8	113·6	114·9	110·3
Mar. 17th, 1959	108·9	105·9	107·8	126·2	117·0	110·1	102·3	113·9	113·7	115·6	110·3

The Ministry of Labour index of weekly wage rates, calculated on the basis of January 31st, 1956 = 100, showed that in December the level was 116·2. It rose to 116·3 in January, 116·6 in February and 116·7 in March. In manufacturing industries alone the figures were 116·0, 115·9, 116·0 and 116·1. The principal classes of workers whose rates of wages were increased in the period included building and civil engineering employees, Post Office employees and iron and steel workers.

The six-monthly enquiry into average earnings showed that in October, 1958, the average earnings per week in the industries covered by the enquiries, i.e. the manufacturing and some of the principal non-manufacturing industries and services, were 256*s.* 8*d.* for men, 133*s.* 11*d.* for women, 112*s.* 0*d.* for youths and boys and 86*s.* 9*d.* for girls. These figures represented increases since April, 1947 of 108, 99, 137 and 116 per cent. For all workers combined the average was 217*s.* 4*d.*, an increase of 110 per cent. This compares with an estimated rise of about 81 per cent. in weekly full time *rates* of wages in the same industries and services. In manufacturing industries alone the average weekly earnings in October, 1958, were 265*s.* 5*d.* for men and 134*s.* 5*d.* for women. The corresponding hourly earnings were 67·3*d.* and 39·2*d.* The average hours worked per week in all the industries and services were 46·0, the lowest for any date since April, 1952. For the first time an analysis is published in the *Ministry of Labour Gazette* for April showing the variations in average earnings according to size of establishment in each of the main industries. The figures show that for both average weekly earnings and average hourly earnings there is a tendency to rise according to the size of the establishment.

The total working population and the numbers in civil employment in the four months ended February, 1959, were as follows:

(Thousands)

Date	*Total Working Population*			*Numbers in Civil Employment*		
	Males	*Females*	*Total*	*Males*	*Females*	*Total*
Nov., 1958 . .	16,161	7,965	24,126	15,231	7,812	23,043
Dec., 1958 . .	16,124	7,863	23,987	15,164	7,707	22,871
Jan., 1959 . .	16,124	7,841	23,965	15,139	7,681	22,820
Feb., 1959 . .	16,087	7,816	23,903	15,135	7,662	22,797

The number of persons on the unemployment registers of the Employment Exchanges rose by 89,000 in December and fell by 12,000 in January and 58,000 in February. The total for February represented 2·5 per cent. of the number of employees in Great Britain. The percentages in the different Regions ranged from 1·5 in London and the South-East to 4·6 in Wales and 4·8 in Scotland.

The following is the sex analysis of the figures:

Number of Unemployed Persons on the Registers of Employment Exchanges

Date	*Men and Boys*	*Women and Girls*	*Total*
Dec. 8th, 1958 . . .	377,075	154,652	531,727
Jan. 12th, 1959 . . .	453,699	167,087	620,786
Feb. 9th, 1959 . . .	443,343	165,320	608,663
Mar. 9th, 1959 . . .	395,566	154,979	550,545

Of the total of 550,545 in March, 89,893 had been unemployed for less than 2 weeks, 124,600 for 2 to 8 weeks and 286,382 for over 8 weeks, while 49,670 were temporarily stopped. In the four weeks ended March 4th, 129,788 vacancies were filled by the Employment Exchanges and the number unfilled at that date was 180,549.

In the week ended February 21st it is estimated that in manufacturing industries 132,300 manual workers were on short-time. At the same date 1,304,800 workers were reported to be working overtime.

The number of insured workers absent from work owing to illness, including self-employed as well as employed, was 915,200 in December, 1,074,100 in January, 1,342,800 in February and 1,171,900 in March. The numbers of employed persons absent owing to industrial injuries were 57,800, 64,100, 64,000 and 61,900.

The *Ministry of Labour Gazette* for February, 1959, contains estimates of the total number of employees (employed and unemployed) at May, 1958, based on the insurance cards exchanged in the subsequent three months supplemented by returns from employers. The totals show, for the United Kingdom 22,290,000 (14,518,000 males and 7,772,000 females) and for Great Britain 21,820,000 (14,220,000 males and 7,600,000 females). A full analysis of the figures by industries is given.

(2) Other Statistics

The Economic Commission for Europe's annual *Economic Survey of Europe* for 1958 (Geneva, 18*s*.), while following the general pattern of previous surveys in giving full and detailed accounts of the western European economy in 1958 and of recent economic development in eastern Europe (including U.S.S.R.)—accounts which it is difficult to summarize in a short note but which are unusually comprehensive—focuses attention this year on two longer-term trends which are considered to be "immediately relevant to the current formulation of economic policies in European Countries". These are an analysis of the relation between economic expansion and changes in balances of payments in western European countries, and a review of the structure and trends of private consumption in the eastern Europe group of countries on the one hand and of consumption trends in western Europe on the other. Owing to the system of centrally planned supplies and the smaller part played by consumer choice in the former group, it is necessary that any consideration of consumption trends must start from a standpoint rather different from the one for the latter group. The survey for the eastern European group is particularly valuable in that it

analyses data not readily available or available in languages not often familiar to statisticians and economists. Some interesting figures of increase in personal consumption have been computed. They show that, taking 1953 as 100, the increases to 1957 were 41 per cent. for Bulgaria, 38 per cent. for Czechoslovakia, 46 per cent. for Hungary and 47 per cent. for Poland: in west European countries the increases in this period were (with the exception of western Germany 59 per cent.) somewhat smaller, ranging from 3 per cent. in Ireland to 11 per cent. in the United Kindgom and 40 per cent. in Austria. Interesting analyses are also given of housing conditions, expenditure on and stock of consumer's durable goods, health and education, where the position of western European countries is much more favourable.

The wealth of data in over 80 tables, the complete list of sources and notes on methods make this volume a model of presentation and arrangement.

OBITUARY

F. A. Burchardt 1902–1958

Dr. F. A. Burchardt, director of the Oxford University Institute of Statistics, died in December, 1958. He was elected a Fellow of the Society in 1942. His academic career at Kiel and Frankfurt had been interrupted by the advent to power of the Nazis: he worked for a time as financial editor of the *Frankfurter Zeitung* and then came to Oxford in 1935.

During the war he was one of a brilliant team working under Dr. A. L. Bowley at the Institute of Statistics: this team included a number of economists from Europe, such as M. Kalecki, E. F. Schumacher, J. Steindl and T. Balogh as well as English economists such as David Worswick. Although Burchardt had not the outstanding originality of some others in the team, his strength lay in his balanced judgment, his wide experience and his all round ability as an economist. His editorial experience was particularly valuable in building up the Institute's bulletin, and in producing as a cooperative work, *The Economics of Full Employment*: Burchardt's own chapters in this book stand up well to the test of re-reading after nearly fifteen years.

By the end of the war Burchardt was in all but name deputy-director under Bowley, who resided in Oxford for only three or four days in each week. Not long after Bowley's retirement in 1945, Burchardt was made deputy director, and finally became director in 1949.

As director he obtained funds for and sponsored a number of research projects in economic statistics and applied economics: Of these the best known is perhaps the enquiry into savings. In planning many of these projects he drew on experience gained in the United States which he had recently visited.

Burchardt, despite his different background, fitted excellently into Oxford University: as well as directing the Institute and playing a full part in University administration he was a Fellow of Magdalen College, where he had great success as an excellent and devoted teacher. But his statistical activities were by no means confined to Oxford during the post-war years. In addition to visiting the U.S.A. he worked for one year with Professor Myrdal at E.C.E. and he did valuable work on various committees of the National Institute of Social and Economic Research. Fellows will remember that in February, 1958 he seconded the vote of thanks for Professor M. G. Kendall's paper on "Research in Statistics" and showed characteristic wisdom and balance of judgment in his comments on the respective needs for lone wolves and for teams in statistical research, and again in stating the arguments for and against separating a research institute from allegiance to any particular faculty.

Dr. Burchardt was one of the very few economists who combined sound theoretical ability with excellence in descriptive economics and wide experience and keen interest in the application of statistical methods.

D. G. Champernowne.

Arthur Cecil Pigou 1877–1959

Professor Pigou, who died on March 7th, 1959, was elected a Fellow of the Society in 1900.

The stature of Pigou as an economist can be measured only against that of Alfred Marshall and Maynard Keynes, with both of whom he was closely associated; with Marshall as pupil and with Keynes as colleague and as protagonist in a one-sided controversy. Compared with these two, Pigou as an economist suffered from certain obvious

handicaps: his training in mathematics was inferior to that of Keynes or Marshall; he lacked Marshall's passionate concern with practical and human problems; nor did he have Keynes's brilliance and intuitive sense for picking out the key relations in an economic situation. Pigou's strength lay in his sure grasp of logical relations and in his fanatical intellectual honesty. Had Pigou written *The Principles of Economics* or *The General Theory of Employment*, they might have been less attractive works, but there would have been far less ambiguity left for lesser economists to resolve.

Pigou's self-appointed task was to clarify and extend Marshall's theoretical apparatus and already before 1914 his *Wealth and Welfare* set out a theoretical justification for many of those aspects of the welfare state which are today accepted by all parties alike. Statisticians owe to him a lucid exposition of the concept of the national dividend which has served as a cornerstone for most of the work on national income which has been done since. Amongst his minor contributions to the statistical method is an ingenious technique which he suggested for estimating the elasticity of demand for a commodity from time series of its price and quantity.

Great as an economist, Pigou was memorable as a man. He had an astonishing capacity for simplifying life and all its important issues. His outstanding characteristics were imperturbable integrity and an utter disregard for all kinds of frills. There can be few men of equal intelligence who have known so clearly what was the right life for them to lead and who have lived it so successfully. His ability to carry this off was partly due to his great achievements as a young man: at Harrow School he was as a god among mortals: at Cambridge, he won all round success, becoming President of the Union and attaining academic distinction in the moral sciences: at the early age of 31 he succeeded Alfred Marshall as Professor of Political Economy. For the rest of his life, he was happily engaged, now in Cambridge, now at his cottage in Buttermere, in writing a large number of books and in performing his duties as Professor. His refusal to be diverted by any other ambitions enabled him to live a completely uncomplicated life and to dispense altogether with pretence as a weapon. With his background of successes, he had complete confidence in his judgment and acquired great influence in such matters as fellowship elections, where impartial judgment was essential. As a chairman of committees, his main concern was to get through the business quickly so as to release the members for more worthy occupations.

So gentle and consistent a man must have seemed pompous and priggish had it not been for the very human absurdities in which he gaily indulged: no picture of him would be fair without an account of his idiosyncrasies. There were, for example, three classes of person whom Pigou would seldom take seriously or discuss except in terms of light banter. These were politicians, females and foreigners. The failing which in his eyes was common to all three was an incapacity for intellectual integrity and a ridiculous concern with finesse and artful persuasion.

In his lectures to undergraduates he taught that the main purpose of learning economics was to be able to see through the bogus economic arguments of the politicians. He would speak gallantly of the lovely Mrs. Smith, the gorgeous Mrs. Brown and the beautiful Mrs. Jones: but the photographs around his room proclaimed that his eye for beauty was rather concerned with mountains and men: his more candid remarks about the ladies likened them to that variety of spider which acquires a mate and in due course devours him. His equal abhorrence of Americans and other foreigners was attributable to their eagerness to seek him out and talk shop for supposedly inadequate reasons.

In all these three cases Pigou allowed exceptions. Honeymoon couples were welcome to his beautifully situated cottage in Buttermere: during 1940 he would listen on the radio with unfeigned admiration to the speeches of Winston Churchill: on one occasion he gave tea to two eminent economists of foreign origin who had walked from Langdale to Buttermere and taken the precaution of climbing both the Gables en route.

Conventional honours were regarded by Pigou with humorous contempt. During 1914/18, through his activities driving ambulances in France, he had acquired various medals and ribbons. These he used to confer on visitors to his cottage to reward achieve-

ments in hill-walking and rock-climbing: particularly remembered is a large rectangular metal plate covered with inscriptions, which he occasionally awarded for "distinguished incompetence".

His contempt for Adolf Hitler was extreme. On the night after war was declared in 1939 the Fellows of King's College Cambridge were shepherded into the air-raid shelter in the small hours of the morning: in Cambridge was concentrated a high proportion of the best scientific talent in the country, so that it was an obvious target for an intelligent enemy. But hours passed and the all-clear sounded with the University still intact: blinking, the Fellows emerged into the daylight and their eyes were rewarded by the sight of Pigou, sitting in a deck chair on the grass of the Front Court, reading a newspaper, his head protected by a large flannel hat.

D. G. CHAMPERNOWNE

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

British Journal of Preventive and Social Medicine—

Vol. 12 (1958), No. 4—Married women who work: their own and their children's health; *A. Cartwright* and *M. Jefferys*. Sickness, change of residence and death. I. General results of follow-up in two population groups; *D. J. Thompson* and *A. Ciocco*. Seasonal swing in mortality in England and Wales; *A. P. Kanellakis*.

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Vol. 21 (1959), No. 1—Econometric forecasts for 1959; *R. J. Ball*, *A. Hazlewood* and *L. R. Klein*. The reasons given for striking: an analysis of official statistics, 1945–1957; *W. McCarthy*. Some international comparisons of consumers' durable goods; *F. Knox*. Capital formation in Nigeria and Ghana, 1946–1955; *E. K. Hawkins*. Measuring productivity in the absence of records; *L. R. Amey*. A "human needs" diet, autumn 1958; *T. Schulz*.

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Vol. 1 (1959), No. 4—Ten years of computer development; *The Rt. Hon. The Earl of Halsbury*. A class of non-analytical iterative processes; *J. H. Wensley*. Computers and commerce: 4. Management and control; *A. S. Douglas*.

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Vol. 26 (1959), No. 101—British monetary statistics; *H. G. Johnson*. Builders' wage-rates, prices and population: some further evidence; *E. H. Phelps Brown* and *S. V. Hopkins*.

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Vol. 9 (1959), No. 2—Recent developments in American economic statistics; *J. W. Knowles*. A note on a method for determining confidence intervals for the median; *D. J. G. Farlie*. The statistician's future; *R. Brech*.

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AUSTRALIA—

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Vol. 34 (1958), No. 69—New aspects of Australia's industrial structure; *B. Cameron.*

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No. 22 (1958)—Unidimensionality in psychological tests; *W. M. O'Neil.* Some hows and whys of distribution-free statistical methods; *H. S. Konijn.*

INDIA—

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Vol. 8 (1958), Nos. 30 and 31—Study of some aspects of economic growth of India; *P. K. Pani* and *N. S. Sivasubramanian.* A note on a method of estimating birth and death rates from registration data; *S. Biswas.* A note on orthogonality in experimental designs; *B. V. Shah.* A note on stratified random sampling with multiple characters; *S. P. Ghosh.* Gain due to sequential sampling from gamma population; *S. R. Adke* and *S. W. Dharmadhikari.* A note on the order of urgency of the different items of the family budgets in Calcutta; *D. Ghosh.* Seasonal distribution in the yield of tea crop in Assam and West Bengal; *P. Ranjan.* Probability selection of the constituent items of a composite item in the construction of cost of living index numbers; *K. S. Banerjee.*

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Vol. 20 (1958), Parts 1 and 2—On moments and cumulants of systems of statistics; *G. S. James.* A comparison of serological and somatometrical methods used in differentiating between certain East African racial groups, with special reference to D^2 analysis; *D. A. East* and *L. Oschinsky.* Science and national planning; *P. C. Mahalanobis.*

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Vol. 26 (1958), No. 4—The new consumer price index; *H. M. Stoker.*

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Mathematical Tables and other Aids to Computation—

Vol. 12 (1958), No. 61—Approximate relations between series and integrals; *P. A. P. Moran*.

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Vol. 73 (1959), No. 1—A modern treatment of the Ricardian economy. I. The pricing of goods and of labor and land services; *P. A. Samuelson*. Free trade in "corn": a statistical study of the prices and production of wheat in Great Britain from 1873 to 1914; *M. Olson, Jr.* and *C. C. Harris, Jr.*

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Vol. 41 (1959), No. 1—On the predictive value of consumer intentions and attitudes; *J. Tobin*. Factors associated with stock ownership; *M. E. Kreinin*. Education and income; *H. S. Houthakker*. A note on Professor Mahalanobis' model of Indian economic planning; *R. Komiya*. Recurrent objections to the minimax strategy; *A. Y. C. Koo.*

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Vol. 24 (1958), No. 8—L'isolement relatif dans une région rurale; *M.-T. Watelet.*
Vol. 25 (1959), No. 1—La conjoncture économique de la Belgique; *M. Woitrin.*

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Acta mathematica sinica—

Vol. 8 (1958), No. 1—The values at the fixed moment of generalized stochastic processes; *K. Urbanik* (Chinese text).

Vol. 8 (1958), No. 2—On the estimation of regression coefficients of a random field with homogeneous residual; *Wang Shou-jen* (Chinese text, English summary). Harmonizable stochastic process and linear translatable stochastic functional equations; *Cheng Shaw-lian* (Chinese text, English summary).

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Vol. 8 (1958), No. 4—Fonctions aléatoires d'intervalle; *F. Zitek*. On a property of normal distributions of any stochastic process; *J. Hájek* (Russian text, English summary).

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Vol. 20 (1957–58), Part 1, Session 111—Natality in Dublin in the year 1955; *J. F. Knaggs*. Marriage and fertility in nineteenth century Londonderry; *J. H. Johnson*. An analysis of the Irish census of distribution, 1951; *B. F. Shields*. The volume of press display advertising in Ireland, 1957; *W. V. Kingston* and *B. J. Prendiville*.

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Vol. 42 (1958), Heft 4—Zur Methode des internationalen Preis- und Kaufkraftvergleichs; *A. Jacobs*. Das Unternehmen als Objekt der Statistik; *W. Herrmann*. Betriebliche Marktforschung und amtliche Statistik; *G. Luda*. Das Problem der Berücksichtigung qualitativer Veränderungen im Rahmen der Preisstatistik; *P. Deneffe*. Zeitreihenzerlegung vom praktischen Standpunkt der Konjunkturforschung; *O. Anderson, Jr*.

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Vol. 5 (1958), No. 2—On the new random dice and their applications; *M. Kogure*. How to rationalize the physical materials sampling in plants; *K. Ishikawa*. Band-score control charts. I; *H. Toda*. Estimation of variance and standard deviation based on range; *S. Tsukibayashi*.

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Vol. 13 (1959), No. 1—Een nomogram voor betrouwbaarheidsgrenzen van quantielen en overschrijdingskansen bij normaal verdeelde grootheden; *C. Levert*. Het examen Statistisch Analyst, 1958 (Algemeen gedeelte). Mean and variance of a function of two random variables; *J. F. E. Wallin*. Attributenkeuring in theorie en praktijk; *H. C. Hamaker*. Internationale statistieken; *G. Goudswaard*. The statistics of systems of simultaneous eocnomic relationships; *H. Theil* and *T. Kloek*. Enkele opmerkingen over de intensiteit van lokaal berichtenverkeer; *A. van der Putten* and *H. C. Swaagman*.

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Teoriya Veroyatnosteĭ i ee Primeneniya—

Vol. 4 (1959), No. 1—One-dimensional continuous strong Markov processes; *E. B. Dynkin* (Russian text, English summary). General theorems of the factorization of infinitely divisible laws. II; *Yu. V. Linnik* (Russian text, English summary). A comparison of the optimal properties of the Neyman-Pearson and the Wald sequential probability ratio tests; *S. A. Aivazyan* (Russian text, English summary). A note on Stepanow's tests for Markov chains; *L. A. Goodman*. A commutativity problem in the theory of Markov chains; *D. Vere-Jones* and *D. G. Kendall*. On one class of optimal resolving functions for a binomial family of distributions; *I. N. Kovalenko* (Russian text, English summary). Information theory with an abstract alphabet; *A. Pérez* (Russian text, English summary). Generalization of a theorem for branching processes; *V. P. Chistyakov* (Russian text, English summary).

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Vol. 13 (1958), No. 6—Zamechanie o propusknoĭ sposobnosti statsionarnogo kanala s konechnoĭ pamyat'yu; *I. P. Tsaregradskiĭ.*

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LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part I, 1959, the Society has received the publications enumerated below.

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The Comparison of Survival Curves

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[Read before the Royal Statistical Society, February 18th, 1959, the President, Sir Harry Campion, C.B., C.B.E., in the Chair.]

SUMMARY

The asymptotic relative efficiencies (A.R.E's) are examined of four methods for comparing survival-time distributions, when these are exponential and individuals enter the study at a uniform rate during the interval $(0, T)$, the analysis taking place at time T. The methods are (a) maximum likelihood; (b) the sign method, in which individuals entering together are paired and their survival times compared; (c) a comparison of the proportions of survivors at "age" τ ("age" being measured from entry), using only those individuals entering in $(0, T-\tau)$; and (d) an actuarial method. The A.R.E's are expressed as functions of λT, where λ is the death rate per unit time. As $\lambda T \to 0$, the A.R.E. of the sign method, as compared with maximum likelihood, approaches unity, and is higher than that of either of the other two methods. The sign method is particularly suitable for sequential analysis.

1. Introduction

The investigation reported in this paper arose out of a consideration of the possibility of using sequential methods in the design and analysis of clinical trials for treatments of chronic diseases in which the main criterion of success is length of survival after treatment. I have discussed the sequential aspects of this problem elsewhere (Armitage, 1958), and although I shall return to this topic briefly in §7, the mathematical results presented here do not concern sequential analysis. The main problem considered is that of the relative efficiencies of different methods of comparing the survival-time distributions in two treatment groups, with the hope of finding a method which is reasonably efficient and which can be adapted easily to sequential analysis.

We consider, then, clinical trials in which patients enter serially in time, and in which the criterion by which treatments are to be compared is the time elapsing between treatment and some manifestation of the disease. This may be death, in which case we are concerned with survival times after treatment; or it may be recurrence or appearance of some par-

ticular symptom. In both these cases, of course, the aim is to lengthen the time, but there may be other situations in which we measure the duration of symptoms after treatment, in which case the aim is to shorten this time. There is a close analogy with problems in the life-testing of manufactured items (Bartholomew, 1957), although it would perhaps be unusual in an industrial experiment to arrange that items are put on test serially over a long period of time, as happens in the analogous situation in clinical trials. I shall use the terms "patient", "death" and "survival" without implication that the results are restricted to this situation.

If a certain number of patients have been randomly allotted to two treatment groups and have been followed until death, a natural statistical procedure is to compare the two means, or some other location parameters, of the distributions of survival time in the two groups. To avoid measuring exceptionally long survival times the distributions may be truncated at the upper end.

It may be desirable to make a comparison of the two groups at a time when a substantial proportion of the individuals are still alive. This may be during the period of intake of patients into the study, or at a relatively early stage after the end of the intake period. At any instant of time some patients will already have died, their survival times being known, and some will still be alive after varying observation periods. If one knew, or were willing to postulate, that the distribution of survival times had some particular functional form, it might be feasible to use an efficient method of analysis, such as maximum likelihood, taking into account the differing truncation points for different patients. Such a method I shall call "parametric maximum likelihood" to distinguish it from non-parametric methods. Alternatively, some sort of distribution-free method could be used; actuarial methods are traditionally applied to data of this type (Merrell and Shulman, 1955).

Suppose, however, that we wish to apply some sort of sequential analysis to the results. On the face of it, this type of data is not particularly amenable to sequential analysis, for if a decision is made to stop the trial at any instant, a certain amount of information must become available subsequently, as more and more patients die. On the other hand, ethical considerations may make it desirable to examine the results as they become available and to stop the intake of patients if the results of the two treatments differ sufficiently.

However, neither parametric maximum likelihood nor the actuarial method appears to be readily adaptable to any of the standard sequential techniques. In this paper I describe a simple device (the sign method) which under certain circumstances can be applied sequentially, and consider the asymptotic relative efficiencies of this and other methods when the survival curve is exponential. The possible use of the sign method in sequential analysis is discussed briefly in §7, and at greater length in a separate paper (Armitage, 1958) where an artificial realization of the sequential procedure is described. In the present paper we are concerned mainly with the relative efficiencies of various methods in samples of fixed size.

The situation considered here is essentially that postulated by Littell (1952), who compared the relative efficiencies of parametric maximum likelihood and of various actuarial procedures. To a large extent I retain Littell's notation.

To compare the relative efficiencies of different methods, assumptions must be made about (a) the rate of entry of patients, as a function of time, and (b) the true form of the survival time distributions. We shall assume:

(a) That during the interval $(0, T)$ patients enter the trial at a constant rate $2n/T$ per

unit time. An analysis of some sort is to be performed at time T. As we shall consider only asymptotic results as $n \to \infty$, there is no need to distinguish between "random entry", in which the $2n$ entry times are rectangularly distributed in $(0, T)$, and "systematic entry", in which new patients enter regularly at intervals $T/2n$ or in groups of (finite) size k at intervals $kT/2n$.

(b) That the survival time distributions are exponential, the probabilities of death before time t, after treatments A and B, being respectively

$$\left.\begin{aligned} F_A(t) &= 1 - e^{-\lambda t} \\ \text{and} \qquad F_B(t) &= 1 - e^{-\lambda' t} \end{aligned}\right\}, \tag{1}$$

where $\lambda' - \lambda = \delta\lambda$.

We shall consider the asymptotic relative efficiency (A.R.E.) of various methods for testing the difference between λ and λ', as $\delta\lambda \to 0$ and $n \to \infty$. Each test yields, asymptotically, an expected normal deviate, χ, which can be expressed in the form

$$\chi = (\delta\lambda/\lambda)\, n^{\frac{1}{2}}\{\psi(\lambda T)\}^{\frac{1}{2}}.$$

It is useful to exhibit the efficiency of each method as a function of λT by tabulating the *efficiency index*

$$\psi(\lambda T) = \chi^2(\lambda/\delta\lambda)^2\, n^{-1}.$$

The A.R.E. of two tests is the ratio of the two values of $\psi(\lambda T)$, and is itself a function only of λT.

2. Parametric Maximum Likelihood Solution

This has been given by Littell (1952) and (without the assumption of a uniform entry-rate) by Bartholomew (1957). Consider the n patients treated with A. Suppose that d deaths have occurred, at periods t_j after treatment $(j = 1, 2, \ldots, d)$, the $n - d$ survivors at time T having been observed for periods T_i since treatment $(i = 1, 2, \ldots, n - d)$. Then the log likelihood is

$$L = -\lambda \sum_i T_i + d \log \lambda - \lambda \sum_j t_j,$$

$$\frac{\partial L}{\partial \lambda} = -\Sigma T_i + \frac{d}{\lambda} - \Sigma t_j,$$

and the maximum likelihood (m.l.) estimate of λ is given by

$$\frac{1}{\hat{\lambda}} = \frac{\Sigma T_i + \Sigma t_j}{d}. \tag{2}$$

We find

$$\partial^2 L/\partial\lambda^2 = -d/\lambda^2,$$

and

$$E\left(\frac{\partial^2 L}{\partial \lambda^2}\right) = -\frac{n}{\lambda^2 T}\int_0^T (1 - e^{-\lambda(T-t)})\, dt$$

$$= -\frac{n}{\lambda^2} + \frac{n}{\lambda^3 T}(1 - e^{-\lambda T}).$$

Hence, asymptotically for large n,

$$\text{var}\,(\hat{\lambda}) \sim \frac{\lambda^2}{n[1-(1-e^{-\lambda T})/\lambda T]}.$$

Suppose that $\lambda' = \lambda + \delta\lambda$, and let $\hat{\lambda}$ and $\hat{\lambda}'$ be m.l. estimates of λ and λ'. Then a large-sample test for the difference between λ and λ' would yield, asymptotically, an expected normal deviate

$$\chi = \frac{\delta\lambda}{\lambda}\left\{\frac{n}{2}\left[1-\frac{1-e^{-\lambda T}}{\lambda T}\right]\right\}^{\frac{1}{2}}.$$

Hence, the way in which the asymptotic power of a test based on m.l. estimates depends on λT is indicated by the efficiency index

$$\psi(\lambda T) = \frac{\chi^2}{n}\left(\frac{\lambda}{\delta\lambda}\right)^2 = \frac{1}{2}\left\{1-\frac{1-e^{-\lambda T}}{\lambda T}\right\}.$$

As $\lambda T \to \infty$, $\psi \to \frac{1}{2}$; and as $\lambda T \to 0$, $\psi \sim \frac{1}{4}\lambda T$. Values of ψ for various values of λT are shown in Table 1 and Fig. 1.

3. The Sign Method

This is an application of the non-parametric "sign test" (Dixon and Mood, 1946). Suppose that patients enter the trial in pairs at a rate of n/T pairs per unit time, and that the members of each pair are allocated at random, one to each treatment group. We assume that pairing is random and unrestricted, but some increase in efficiency might be achieved by a randomized block design, the blocks being pairs of patients of similar prognosis.

For each pair, we observe which of two survival times is greater. Let t_A and t_B be the survival times for patients on treatments A and B, respectively. A pair for which $t_A > t_B$ will be said to give an "A preference", one for which $t_A < t_B$ a "B preference", and one for which $t_A = t_B$ (if this is possible) or for which t_A and t_B are both greater than the observation period (so that neither patient has died) will be said to give "no preference".

Consider first the use of the sign method without invoking conditions (a) and (b) of §1. Let t_A and t_B have differentiable distribution functions, $F_A(t) \equiv P(t_A \leqslant t)$ and $F_B(t) \equiv P(t_B \leqslant t)$, with $f_A(t) = F_A'(t)$ and $f_B(t) = F_B'(t)$. This restriction implies $P(t_A = t_B) = 0$.

The probability that a preference occurring at time t after treatment is an A preference is

$$\theta = \frac{f_B(t)\{1-F_A(t)\}}{f_A(t)\{1-F_B(t)\}+f_B(t)\{1-F_A(t)\}}.$$

A necessary and sufficient condition that θ should be independent of t, so that the preferences form a random binomial sequence with constant probability parameter, is that $f_A(t)\{1-F_B(t)\}/[f_B(t)\{1-F_A(t)\}]$ is a constant, k, where $\theta = 1/(1+k)$. Since

$$F_A(0) = F_B(0) = 0,$$

this implies

$$1-F_A(t) = \{1-F_B(t)\}^k. \tag{3}$$

That is, one survival curve must be a constant power of the other. This condition is satisfied in particular if the survival curves are exponential.

Assuming, as in condition (a) of §1, a uniform rate of entry, but still assuming no particular functional form for $F_A(t)$ and $F_B(t)$, the probability density for a preference at time t after the start of the study is

$$dG(t)/dt = (n/T)\int_0^t [\{1 - F_A(t')\} f_B(t') + \{1 - F_B(t')\} f_A(t')]\, dt'$$
$$= (n/T)[1 - \{1 - F_A(t)\}\{1 - F_B(t)\}]$$

and

$$G(t) = n[(t/T) - (1/T)\int_0^t \{1 - F_A(t')\}\{1 - F_B(t')\}\, dt'],$$

since

$$G(0) = 0.$$

Note that $G(t)$, the expected number of preferences obtained in $(0, t)$ is a function of the geometric means of the two survival rates at times t', where $0 < t' < t$. If, in addition, $F_A(t)$ and $F_B(t)$ satisfy (3), we have

$$G(t) = n[(t/T) - (1/T)\int_0^t \{1 - F_A(t')\}^{1+1/k}\, dt']. \tag{4}$$

A number of different sequential procedures are available for binomial sequences (Wald, 1947; Bross, 1952; Armitage, 1957), and the sign method could be used in conjunction with any of these.

We now consider the efficiency of the sign method in samples of fixed size, under conditions (a) and (b) of §1. From (1) and (4), with $\delta\lambda \to 0$ (whence $k \to 1$), we have

$$G(t) = n[(t/T) - (1 - e^{-2\lambda t})/2\lambda T]. \tag{5}$$

The hypothesis that $\lambda = \lambda'$ may be tested at time T by testing the difference between the observed proportion, $\hat{\theta}$, of A preferences and its null value $\frac{1}{2}$, using the binomial distribution. Now, the expected proportion of A preferences is

$$\theta = \lambda'/(\lambda + \lambda') = \tfrac{1}{2} + \delta\lambda/4\lambda + o(\delta\lambda),$$

and on the null hypothesis

$$\operatorname{var}(\hat{\theta}) = 1/4G(T).$$

Hence,

$$\chi = (\theta - \tfrac{1}{2})/\{\operatorname{var}(\hat{\theta})\}^{\frac{1}{2}}$$
$$= (\delta\lambda/2\lambda)\, G^{\frac{1}{2}}(T),$$

and

$$\psi \equiv (\chi^2/n)(\lambda/\delta\lambda)^2$$
$$= G(T)/4n = \frac{1}{4}\left\{1 - \frac{1 - e^{-2\lambda T}}{2\lambda T}\right\}.$$

As $\lambda T \to \infty$, $\psi \to \frac{1}{4}$; and as $\lambda T \to 0$, $\psi \sim \frac{1}{4}\lambda T$. Comparison of these results with

those for m.l. shows that the asymptotic efficiency of the sign method, relative to m.l., tends to 50 per cent. as $\lambda T \to \infty$ and 100 per cent. as $\lambda T \to 0$. Values of ψ for various values of λT are given in Table 1 (together with the A.R.E.), and in Fig. 1.

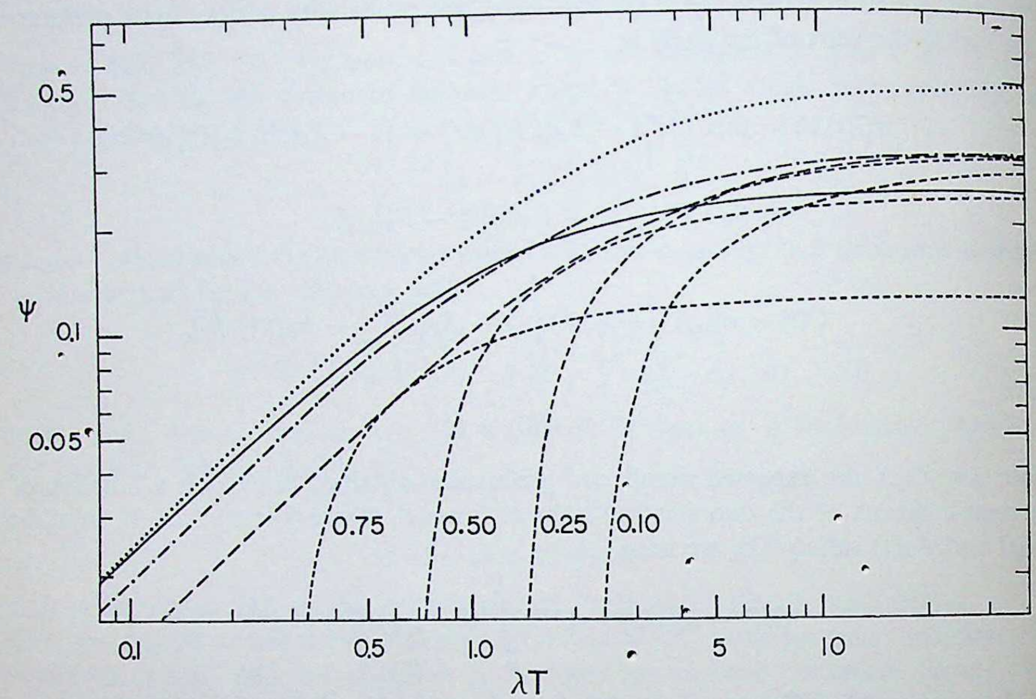

FIG. 1.—Efficiency index, ψ, as a function of λT (the product of the death rate, λ, and the total observation period T), for various methods of analysis. ········ parametric maximum likelihood; ——— sign method; — — — direct comparison of proportions of survivors after optimal period τ; - - - - - direct comparison of proportions of survivors at values of exp $(-\lambda\tau)$ shown; -·-·-·- actuarial (product-limit) method, for optimal τ.

The asymptotic efficiency of the sign method, when λT is small, is at first sight surprising. A heuristic reason is as follows. As $\lambda T \to 0$, the m.l. estimator of λ will (from (2)) converge in probability to $d/(\Sigma T_i + \Sigma T_j)$, where T_j is the full observation period (from entry to time T) for the patient dying at time t_j. But the denominator $\Sigma T_i + \Sigma T_j$ is asymptotically equal to $\frac{1}{2}nT$, a non-random variable, and consequently a comparison of $\hat{\lambda}$ and $\hat{\lambda}'$ is asymptotically equivalent to a comparison of their numerators, d and d'. But for small λT the probability of death of both members of a pair can be ignored, and consequently a test based on $d - d'$ is equivalent to one based on the difference between the numbers of A and B preferences.

4. Direct Comparison of Proportions of Survivors

Another inefficient but simple method is to compare the proportions of survivors, at time τ after treatment, amongst those patients who have been observed for at least this period. This method is also suitable for sequential analysis, since sequential methods for comparing proportions are available (Wald, 1947; Armitage, 1957). Only those patients can be used in the comparison who entered during the interval $(0, T - \tau)$, and these will number $m = n(1 - \tau/T)$ on each treatment (any random fluctuations in m being asymptotically negligible for large n). Then, the expected normal deviate, χ, and the efficiency

index, ψ, are given by

$$\chi^2 = \frac{(\delta\lambda)^2\, m\left(\dfrac{\partial}{\partial\lambda}(e^{-\lambda\tau})\right)^2}{2e^{-\lambda\tau}(1-e^{-\lambda\tau})} = \frac{m(\delta\lambda)^2\,\tau^2\,e^{-\lambda\tau}}{2(1-e^{-\lambda\tau})},$$

and

$$\psi = \left(1-\frac{\tau}{T}\right)\frac{(\lambda\tau)^2\,e^{-\lambda\tau}}{2(1-e^{-\lambda\tau})}. \tag{6}$$

Values of ψ are tabulated in Table 1, and shown in Fig. 1, for various values of λT and for $e^{-\lambda\tau} = 0\cdot10$, $0\cdot25$, $0\cdot50$ and $0\cdot75$. Clearly $\psi = 0$ for $T = \tau$ and is undefined for $T < \tau$ (since the method is not then applicable).

For fixed T, let the maximum value of ψ be $\tilde{\psi}$, occurring when $\tau = \tilde{\tau}$. Putting

$$\partial\psi/\partial\tau = 0$$

we have

$$T\{(2-\lambda\tilde{\tau}) - 2e^{-\lambda\tilde{\tau}}\} = \tilde{\tau}\{(3-\lambda\tilde{\tau}) - 3e^{-\lambda\tilde{\tau}}\}. \tag{7}$$

As $T \to \infty$, (7) has the solution

$$2 - \lambda\tilde{\tau} - 2e^{-\lambda\tilde{\tau}} = 0$$

or

$$e^{-\lambda\tilde{\tau}} = 1 - \tfrac{1}{2}\lambda\tilde{\tau},$$

whence

$$\lambda\tilde{\tau} = 1\cdot5936 \text{ and } e^{-\lambda\tilde{\tau}} = 0\cdot20319.$$

This is the well-known result that for quantal-response estimation of an exponential curve the optimal proportion of survivors is $20\cdot3$ per cent. (cf. Fisher, 1949). For this value of $\tilde{\tau}$ we find $\tilde{\psi} = 0\cdot324$, and a comparison with the value of ψ for the m.l. solution (Table 1) shows that the survival-proportion method has an efficiency of 65 per cent.

It is clear from (6) that for finite T the optimal value, $\lambda\tilde{\tau}$, is less than $1\cdot5936$, since $1-\tau/T$ is a decreasing function of τ, and consequently a decrease in $\lambda\tau$ below the critical value of $1\cdot5936$ will initially cause an increase in ψ. To obtain the optimal value of $\lambda\tau$ for various values of λT, the simplest plan is to solve (7) for λT. Thus,

$$\lambda T = \frac{\lambda\tilde{\tau}\{3 - \lambda\tilde{\tau} - 3e^{-\lambda\tilde{\tau}}\}}{2 - \lambda\tilde{\tau} - 2e^{-\lambda\tilde{\tau}}}.$$

Substitution of various values of $\lambda\tilde{\tau}$, and inverse interpolation, gives the solutions shown in Table 2. The values of $\tilde{\psi}$ are shown also in Table 1 (together with the A.R.E.) and in Fig. 1.

As $\lambda\tilde{\tau} \to 0$, $\lambda T \to 2\lambda\tilde{\tau}$ and $\tilde{\psi} \sim \frac{1}{8}\lambda T$. Comparison with the results for m.l. and for the sign method (Table 1) show that the direct comparison of proportions of survivors has, asymptotically as $\lambda T \to 0$, at best a relative efficiency of 50 per cent. The sign method is asymptotically more efficient than the optimal direct comparison of proportions of survivors for all $\lambda T < 4\cdot3$ (or $e^{-\lambda T} > 0\cdot014$).

TABLE 1

Values of the Efficiency Index, ψ, for Various Values of λT and for Different Methods of Analysis. The Asymptotic Efficiency (A.R.E.) Relative to Parametric Maximum Likelihood is Given in Parentheses, Expressed as a Percentage

			Direct Comparison of Survival Proportions at Time τ After Treatment					*Actuarial (P.L.) Comparison of Survival Proportions*
			$\exp(-\lambda\tau)$					
λT	*Parametric Maximum Likelihood*	*Sign Method*	0·10	0·25	0·50	0·75	*Optimal* $\tau = \tilde{\tau}$	*(Optimal τ)*
0 .	$\sim\frac{1}{4}\lambda T$ (100·0)	$\sim\frac{1}{4}\lambda T$ (100·0)	—	—	—	—	$\sim\frac{1}{8}\lambda T$ (50·0)	$\sim 0{\cdot}2036\lambda T$ (81·5)
0·1 .	0·0242 (100·0)	0·0234 (96·8)	—	—	—	—	0·0122 (50·4)	0·0195 (80·7)
0·2 .	0·0468 (100·0)	0·0440 (93·9)	—	—	—	—	0·0238 (50·8)	0·0374 (79·9)
0·5 .	0·107 (100·0)	0·0920 (86·3)	—	—	—	0·0527 (49·5)	0·0552 (51·9)	0·0828 (77·7)
0·8 .	0·156 (100·0)	0·125 (80·4)	—	—	0·0321 (20·6)	0·0795 (51·0)	0·0822 (52·8)	0·118 (75·8)
1·0 .	0·184 (100·0)	0·142 (77·2)	—	—	0·0737 (40·1)	0·0884 (48·1)	0·0981 (53·3)	0·137 (74·6)
2·0 .	0·284 (100·0)	0·189 (66·5)	—	0·0983 (34·6)	0·157 (55·3)	0·106 (37·4)	0·157 (55·5)	0·200 (70·4)
5·0 .	0·401 (100·0)	0·225 (56·2)	0·159 (39·7)	0·231 (57·8)	0·207 (51·6)	0·117 (29·2)	0·236 (58·9)	0·265 (66·1)
10·0 .	0·450 (100·0)	0·237 (52·8)	0·227 (50·4)	0·276 (61·3)	0·224 (49·7)	0·121 (26·8)	0·276 (61·3)	0·293 (65·1)
20·0 .	0·475 (100·0)	0·244 (51·3)	0·261 (54·9)	0·298 (62·8)	0·232 (48·8)	0·122 (25·8)	0·299 (62·9)	0·308 (64·8)
∞ .	0·500 (100·0)	0·250 (50·0)	0·295 (58·9)	0·320 (64·1)	0·240 (48·0)	0·124 (24·8)	0·324 (64·8)	0·324 (64·8)

TABLE 2

Optimal Values of τ, and the Corresponding Efficiencies, for the Direct Comparison of Survival Proportions and for the Actuarial (P.L.) Comparison of Survival Proportions, at Various Values of λT

	Direct Comparison of Survival Proportions					*Actuarial (P.L.) Comparison of Survival Proportions*				
λT	$\lambda\tilde{\tau}$	$e^{-\lambda\tilde{\tau}}$	$\tilde{\tau}/T$	ψ	%*A.R.E.*	$\lambda\tilde{\tau}(=u\tilde{\gamma})$	$e^{-\lambda\tilde{\tau}}$	$\tilde{\tau}/T$ $(=\tilde{\gamma})$	ψ	%*A.R.E.*
0 .	$\sim\frac{1}{2}\lambda T$	1·000	0·500	$\sim\frac{1}{8}\lambda T$	(50·0)	$\sim 0{\cdot}715\lambda T$	1·000	0·715	$\sim 0{\cdot}2036\lambda T$	(81·5)
0·1 .	0·0494	0·952	0·494	0·0122	(50·4)	0·0702	0·932	0·702	0·0195	(80·7)
0·2 .	0·0975	0·907	0·487	0·0238	(50·8)	0·138	0·871	0·688	0·0374	(79·9)
0·5 .	0·234	0·792	0·468	0·0552	(51·9)	0·323	0·724	0·646	0·0828	(77·7)
0·8 .	0·358	0·699	0·448	0·0822	(52·8)	0·483	0·617	0·603	0·118	(75·8)
1·0 .	0·434	0·648	0·434	0·0981	(53·3)	0·576	0·562	0·576	0·137	(74·6)
2·0 .	0·739	0·478	0·369	0·157	(55·5)	0·908	0·403	0·454	0·200	(70·4)
5·0 .	1·168	0·311	0·234	0·236	(58·9)	1·279	0·278	0·256	0·265	(66·1)
10·0 .	1·375	0·253	0·138	0·276	(61·3)	1·434	0·238	0·143	0·293	(65·1)
20·0 .	1·485	0·227	0·074	0·299	(62·9)	1·514	0·220	0·076	0·308	(64·8)
∞ .	1·594	0·203	0	0·324	(64·8)	1·594	0·203	0	0·324	(64·8)

5. THE ACTUARIAL METHOD

Actuarial methods, of which there are many varieties, would probably be widely regarded as the most appropriate for data of the type which we are considering. The essential feature is the formation of a life table for each treatment. The estimate of the proportion of survivors at time τ after treatment differs from that obtained in §4, since patients entering during the interval $(T - \tau, T)$ contribute information on the proportions of survivors at times $\tau' < \tau$ after treatment, and hence also on the total probability of survival to time τ after treatment.

In most applications of actuarial techniques the interval $(0, \tau)$ is subdivided into (usually equal) sub-intervals, and the probabilities of survival through each sub-interval are estimated directly from the data. Kaplan and Meier (1958) have defined the *product-limit* (P.L.) estimate of the proportion of survivors at time τ as follows: arrange the n observations for any one treatment in order of increasing "lifetime" (the interval between treatment and either death or end of observation, whichever is the shorter), and denote these by $0 \leqslant t_1' \leqslant t_2' \leqslant \ \ldots \ \leqslant t_n'$. Then $\hat{P}(\tau) = \prod_r (n-r)/(n-r+1)$, where r assumes those values for which $t_r' \leqslant \tau$ and for which t_r' measures the time to death. This is shown by Kaplan and Meier to be the unique non-parametric maximum likelihood estimator of $1 - F(\tau)$, and to coincide with the usual life-table estimate if the grouping intervals for the latter are chosen so that no sub-interval contains both a death and a withdrawal (i.e. the end of an observation period for a surviving patient). The product-limit may, therefore, be regarded as the limiting form of the familiar life-table estimator, as the lengths of the grouping intervals are reduced. According to Kaplan and Meier the product-limit was used as early as 1912 by P. E. Böhmer.

The asymptotic variance of $\hat{P}(\tau)$, for the n observations on treatment A, may be obtained (following Kaplan and Meier) as a limiting form of the formula due to Greenwood

(1926). The expected number of survivors at a period t after treatment is

$$n_t \sim n(1 - t/T)\, e^{-\lambda t}.$$

The probability that a survivor at time t after treatment will die during a further interval $(t,\ t + dt)$ is

$$q_t = 1 - e^{\lambda dt} \sim \lambda dt$$

and

$$p_t \equiv 1 - q_t \sim 1 - \lambda dt.$$

From Greenwood's formula,

$$\operatorname{var}\{\hat{P}(\tau)\} \sim \{1 - F(\tau)\}^2 \int_0^{\tau} \frac{q_t}{n_t p_t}$$

$$\sim e^{-2\lambda\tau} \int_0^{\tau} \frac{\lambda T\, e^{\lambda t}\, dt}{n(T - t)}.$$

Let $y = \lambda(T - t)$. Then

$$\operatorname{var}\{\hat{P}(\tau)\} \sim (\lambda T/n)\, e^{\lambda(T-2\tau)} \int_{\lambda(T-\tau)}^{\lambda T} y^{-1}\, e^{-y}\, dy,$$

$$\chi^2 = (\delta\lambda)^2 \left\{\frac{\partial}{\partial\lambda}(e^{-\lambda\tau})\right\}^2 \Big/ 2 \operatorname{var}\{\hat{P}(\tau)\}$$

$$\sim \frac{(\delta\lambda)^2\, n}{2\lambda^2} \left[\frac{(\lambda\tau)^2\, e^{-\lambda T}}{\lambda T \displaystyle\int_{\lambda(T-\tau)}^{\lambda T} y^{-1}\, e^{-y}\, dy}\right],$$

and

$$\psi = \frac{\lambda T\, e^{-\lambda T}(\tau/T)^2}{2 \displaystyle\int_{\lambda(T-\tau)}^{\lambda T} y^{-1}\, e^{-y}\, dy}. \tag{8}$$

An upper bound for the integral in (8) is

$$\lambda^{-1}(T - \tau)^{-1} \int_{\lambda(T-\tau)}^{\lambda T} e^{-y}\, dy = \lambda^{-1}(T - \tau)^{-1}\, e^{-\lambda(T-\tau)}\{1 - e^{-\lambda\tau}\},$$

and it follows immediately that the right-hand side (R.H.S.) of (8) exceeds the R.H.S. of (6). Hence the P.L. method is more efficient than the direct comparison of proportions of survivors, as would be expected.

Rather than examine the behaviour of (8) as a function of λT, for various values of $\lambda\tau$, we proceed immediately to maximize (8) with respect to τ for a given value of T. To simplify the notation, let $\lambda T = u$, $\tau/T = \gamma$. Then

$$\partial\psi/\partial\gamma = 0$$

when

$$\{\gamma/(1-\gamma)\}\, e^{-u(1-\gamma)} = 2\int_{u(1-\gamma)}^{u} y^{-1}\, e^{-y}\, dy \tag{9}$$

$$= 2\{-\,\mathrm{Ei}\,(-\,u(1-\gamma)) + \mathrm{Ei}\,(-\,u)\}, \tag{10}$$

where $-\,\mathrm{Ei}\,(-\,x)$ is the exponential integral tabulated, for example, by Works Project Administration (1940*a*, *b*).

For various values of u, the value of γ satisfying (10), $\tilde{\gamma}$, has been found numerically, and the corresponding values of ψ obtained from (8). These are given in Tables 1 and 2 and shown in Fig. 1. The A.R.E. of the P.L. method (for optimal τ) varies between 65 per cent. and 82 per cent.

As $u \equiv \lambda T \to 0$, the integrand in (9) is asymptotically equivalent to y^{-1}, and in the limit (9) may be written

$$\tilde{\gamma}/(1-\tilde{\gamma}) = -\,2\log(1-\tilde{\gamma}),$$

whence $\tilde{\gamma} = 0{\cdot}7153$ and $\psi \sim (\lambda T)\,\tilde{\gamma}(1-\tilde{\gamma}) = 0{\cdot}2036\,\lambda T$. The asymptotic efficiency relative to m.l. is $0{\cdot}2036/0{\cdot}2500 = 0{\cdot}815$.

As $u \equiv \lambda T \to \infty$, use of an asymptotic series for $-\,\mathrm{Ei}\,(-\,x)$ yields the asymptotic solution

$$e^{-u\tilde{\gamma}} = 1 - \tfrac{1}{2}u\tilde{\gamma},$$

whence $u\tilde{\gamma}(\equiv \lambda\tilde{\tau}) = 1{\cdot}5936$ and $e^{-u\tilde{\gamma}} = 0{\cdot}20319$. This is exactly the same limiting solution as was obtained for the direct comparison of proportions of survivors in §4, and the limiting values of ψ also are the same. This result would be expected, for as $\lambda T \to \infty$ the proportion of patients surviving at time T becomes negligible, and the P.L. estimate coincides with the direct estimate.

It is of some interest that the optimal value of γ lies between zero and $0{\cdot}715$ (Table 2). This result strictly applies only when the survival curve is exponential and when entry is uniformly distributed; but for moderate departures from the exponential form it may be a good working rule that estimated survival curves should be compared at a period after treatment not greater than about $\frac{3}{4}$ of the total observation period, and that as the latter is increased the comparison should be made relatively earlier, so that the mean proportion of survivors exceeds, but approaches, 20 per cent.

In both (6) and (8), $\psi \to 0$ as $\tau \to T$. This is because both the direct and the P.L. estimates of the proportions of survivors at time τ after treatment have variances which are $O(1/n)$ for $\tau < T$, but not for $\tau = T$. This singularity at $\tau = T$ would not apply to the estimator obtained by the usual actuarial method, with the time period divided into sub-intervals of finite length, and it is interesting in this connection to note that Littell (1952) found the usual estimator to have a variance which increased with the number of sub-intervals. (The P.L. method, as already observed, corresponds to a high degree of sub-division of the interval $(0, \tau)$.) The usual estimator, of course, does not have the property which is possessed by the P.L. estimator, of maximizing the likelihood over the class of all survival time distributions.

Dr. Paul Meier has pointed out to me that an alternative actuarial method is to compare the values obtained, from the two series, for the "mean life limited to a time T" (cf. Irwin, 1949). From results given by Kaplan and Meier (1958) it may be seen that, for this com-

parison, $\psi \to \frac{1}{4}\lambda T$ as $\lambda T \to 0$; the method is, therefore, fully efficient in this limiting situation.

6. AN ILLUSTRATIVE EXAMPLE

In another paper (Armitage, 1958) I have described an imaginary clinical trial, carried out by random numbers, giving an artificial realization of a sequential procedure based on the sign method.

TABLE 3

Assumed Proportions of Survivors at Selected Times after Treatment

Years after Treatment	Proportion of Survivors, $1 - F(t)$: A	B	Geometric Mean
0·5	0·71	0·51	0·60
1·0	0·59	0·35	0·45
2·0	0·45	0·20	0·30

Empirical values were given to $F_A(t)$ and $F_B(t)$ bearing some resemblance to the sort of survival curves observed for certain forms of malignant disease. The proportions of survivors for selected values of t are shown in Table 3. These are not exponential curves: the proportions of survivors at high values of t are higher than would be expected from exponential curves, a discrepancy often observed in this type of data. The curves were so chosen, however, that

$$1 - F_B(t) = \{1 - F_A(t)\}^2,$$

and it follows from §3 that, in the sign method, $k = \frac{1}{2}$ and the A and B preferences form a binomial sequence with $\theta = 2/3$.

It was supposed that patients entered the trial at a rate of 300 a year. A sequential procedure based on the sign method caused the intake to be stopped after 1·15 years, when the total number of patients in each treatment group was 174. The number of preferences obtained was 100.

We now give the results of applying some of the tests considered in this paper, to the two groups of 174 patients. We have $T = 1{\cdot}15$ and $n = 174$. There is some difficulty in ascribing a value to λ, of course, since the survival curves are not exponential. If exponential curves are fitted to the assumed proportions of survivors between 0 and 1 year, the approximate values of λ are 0·60 and 1·20, for A and B, respectively. In applying the theoretical results we shall therefore take $\delta\lambda = 1{\cdot}20 - 0{\cdot}60 = 0{\cdot}60$, and $\lambda = \frac{1}{2}(0{\cdot}60 + 1{\cdot}20) = 0{\cdot}90$. Thus, $\lambda T = 0{\cdot}90 \times 1{\cdot}15 = 1{\cdot}035$.

TABLE 4

Observed and Expected Values of χ, the Normal Deviate Obtained by Applying Various Tests at $T = 1{\cdot}15$ Years, in the Artificial Trial

	Value of χ: Observed	Expected
Max. likelihood	3·96	3·82
Sign method	2·80	3·34
Direct comparison of survival proportions at $\tau = 0{\cdot}5$	2·33	2·79
Actuarial comparison of survival proportions at $\tau = 0{\cdot}5$	3·19	3·21

The observed normal deviates are compared in Table 4 with those given by the theoretical formulae of §§2 — 5. Several points should be emphasized here:

(a) The theoretical formulae are based on the assumption of exponential survival curves, and the divergence from this form will presumably affect the situation.

(b) The theoretical normal deviates are *expected* values, and are subject to sampling variation. The sampling fluctuations in the values of χ for different tests are probably rather highly correlated. In general, the two series of results were rather more similar than would be expected on the average, and we should expect to find values of χ rather smaller than the theoretical values.

(c) The theoretical values are asymptotic, on the assumption that $\delta\lambda/\lambda$ is infinitesimal. The value of $\delta\lambda/\lambda$ here (0·67) may be sufficiently big to cause an appreciable disturbance.

(d) The theoretical results for the life-table analysis (§5) refer strictly to the P.L. method, whereas the analysis actually done was of the more conventional form, with grouping intervals of 0·1 year.

The parametric maximum likelihood solution (§2) is not really appropriate in the non-exponential situation, but it might have been applied in practice if the departures from exponentiality were not too obvious. The values of λ are estimated from (2), and their standard errors estimated as $\lambda/\sqrt{d}$, where d is the number of deaths in each group. The estimates are $\lambda = 0{\cdot}635 \pm 0{\cdot}092$ for A, and $1{\cdot}304 \pm 0{\cdot}142$ for B, giving $\chi = 0{\cdot}669/0{\cdot}169 = 3{\cdot}96$. The expected value, 3·82, is obtained from the appropriate formula in §2.

For the sign method (§3) there were 64 preferences for A, and 36 for B (as against the expected numbers 66·7 and 33·3). The observed χ is therefore $(0{\cdot}64 - 0{\cdot}50)/\{(0{\cdot}5)^2/100\}^{\frac{1}{2}} = 2{\cdot}80$. The expected χ is given by the formula in §3.

In the direct comparison of proportions of survivors (§4), and in the actuarial method (§5), estimates of the proportion of survivors 6 months after treatment were compared. Thus $\tau = 0{\cdot}5$ and $\lambda\tau = 0{\cdot}45$. This value of $\lambda\tau$ is not too far from the optimum for each method, when $\lambda T = 1{\cdot}035$ (cf. Table 2). For the method of §4, we have 99 patients on each treatment, observed for at least 0·5 year. The numbers of deaths on each are 31 and 47. The observed χ is obtained from the customary test for the comparison of two proportions, and the formula for the expected χ is given in §4.

In the actuarial method the estimates of the 6-month survival proportions are $0{\cdot}682 \pm 0{\cdot}038$ and $0{\cdot}506 \pm 0{\cdot}040$, giving an observed $\chi = 3{\cdot}19$. The formula for the expected χ is given in §5.

7. Discussion

The results in §§2–5 suggest that the sign method is reasonably efficient, particularly during the early stages of a trial when relatively few deaths have taken place. The effect of departures from exponentiality is unknown. The artificial example of §6 gave results in good accordance with the theory, even though the survival curves were not exponential and the difference between the curves was large, but of course a single example of this sort provides no grounds for generalization.

The sign method provides a distribution-free test of the null hypothesis, and is particularly suitable for sequential analysis. A sequential analysis based on this method could provide a stopping-rule, but it must be remembered that a final analysis of the experiment must await a fairly complete follow-up of all the subjects and will not be available until some time after the intake has stopped. This final analysis would probably

be done by life-table methods, unless there were strong reasons for assuming a particular functional form for the survival curves.

It might perhaps be argued that, since the sequential scheme will not provide the final analysis, there is little point in insisting on a sequential design which controls the probabilities of hitting certain boundaries. Other types of stopping-rule could perhaps be based on more efficient criteria. One possibility, for instance, would be to use the exponential model of §2, and stop when the likelihood of the most likely separate values of λ and λ' reached a certain multiple of the likelihood of the most likely common value. There are perhaps two arguments against this type of approach:

(a) It is not distribution-free; it is not clear how to apply such a method to the life-table analysis.

(b) It would be useful to control directly the probability of stopping prematurely when there is really no difference between treatments.

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Discussion on Dr. Armitage's Paper

Dr. D. R. Cox: Dr. Armitage has mentioned that the comparison of survival curves arises in fields other than the medical one, with which the paper is primarily concerned. I should like to begin by elaborating this point with a few examples.

First, comparisons of survival curves arise in actuarial work and in bioassays in which response times are measured. As just one biological example there is Dr. Comfort's work comparing the survival curves of different species, the aim being to obtain information about the ageing processes involved.

Those tensile strength tests in which two or more types of failure can occur provide another example. As the load increases the specimen survives until a load corresponding to the smallest of the separate breaking loads is reached. The situation is closely parallel to that of the paper, when pairs of individuals are considered and the one with the smaller

lifetime noted. In this particular application to strength testing each test specimen corresponds to a pair of individuals and load corresponds to time.

In cloud-chamber experiments there are similar problems of estimating particle mean life-times from incomplete data; see, for example, Bartlett, *Phil. Mag.*, **44** (1953), 249. There may be several types of particle under comparison. Incomplete data are obtained when a particle passes through the chamber without "dying" or "dies" between the plates of a multiple-plate chamber.

In interpreting some simple learning experiments Dr. Audley (*Q. J. Exptl. Psychol.*, **9** (1957), 12) has considered a probability model very similar to that used by Dr. Armitage in his analysis of the sign test. Audley imagines two independent Poisson sequences, one of correct responses and one of incorrect responses. The process to produce an event first determines both the nature of the response and the time at which it occurs (latency). In this way Audley is able to combine the interpretation of the changes during learning in the latencies and in the proportions of correct responses.

Another field is industrial life-testing. I hope we shall hear more about this in the discussion.

Having noted the resemblances between these applications, it is of course equally important to note the differences. I do not suggest that Dr. Armitage's results are immediately applicable in more than a few of these problems. However, I do think that his general approach, the methods he has used, and the general qualitative conclusions he has reached will be most stimulating and valuable to anyone who has to work with this sort of data. His paper is very much to be welcomed on this account, as well as for the specific conclusions he has reached for his particular problem.

Next I wish to comment on a more technical matter, namely the construction of sequential tests based on maximum likelihood estimates, using a parametric formulation (section 1 of the paper). Suppose that at the n^{th} step t_n is any estimate of the parameter θ of interest. Then a standard likelihood ratio test of $\theta = \theta_1$ against $\theta = \theta_0$ is obtained by considering the inequality

$$A < \frac{p(t_1, \quad . \quad . \quad . \quad , t_n; \ \theta_1)}{p(t_1, \quad . \quad . \quad . \quad , t_n; \ \theta_0)} < B, \tag{1}$$

sampling being continued only so long as (1) is satisfied. Here $p(\mathbf{t}; \ \theta)$ is the density function of $\mathbf{t}$ given θ. Provided that (1) is independent of nuisance parameters and that the scheme ends with probability one, the usual formulae for determining A and B can be used.

As a general procedure this is quite impracticable, because of the difficulty of finding the joint density of many random variables. However if t_n is sufficient, the inequality (1) reduces to

$$A < \frac{p(t_n; \ \theta_1)}{p(t_n; \ \theta_0)} < B.$$

Now the maximum likelihood estimate $\hat{\theta}_n$, while not in general sufficient, is asymptotically sufficient (Wald, *Trans. Amer. Math. Soc.*, **54** (1943), 426; LeCam, *Calif. Publ. Statist.*, **1** (1953), 277). Further the distribution of $\hat{\theta}_n$ is asymptotically normal with mean θ and variance a known function of θ and of nuisance parameters; let nuisance parameters be replaced by consistent estimates.

We thus have a simple general procedure. It would be interesting to examine in detail the following approximations involved in the argument:

(*a*) asymptotic sufficiency is used instead of exact sufficiency;
(*b*) the asymptotic density is used;
(*c*) nuisance parameters are replaced by estimates;
(*d*) Wald's approximate formulae for the limits are used.

Now Dr. Armitage has suggested a different type of sequential procedure, which he calls a restricted procedure, as more appropriate for his applications than those based on

(1). Tests of this type also can be based on maximum likelihood estimates. For a maximum likelihood estimate is well known to be asymptotically an average of independent identically distributed random variables; hence the stochastic process $n(\hat{\theta}_n - \theta)$, considered as a function of n, is asymptotically of the simple diffusion type, and the arguments that Dr. Armitage used in his paper (*Biometrika*, **44** (1957), 9) can be used. In fact a whole family of such restricted schemes can be found. For the same argument can be applied to $n[g(\hat{\theta}_n) - g(\theta)]$, where $g(x)$ is a monotone function of x. A similar argument can be used to examine properties of rather complicated sequential procedures such as the sequential t and F tests.

Dr. Armitage's paper is of theoretical interest and of value in several substantive fields. It is a great pleasure to propose a cordial vote of thanks.

Dr. J. W. Boag: When I was invited to second this vote of thanks I was a little hesitant about accepting. Remembering the traditions of the Society, I doubted whether I should find sufficient grounds for criticism of the paper to warrant my seconding the vote of thanks.

I shall have to go a little outside the defined limits of the paper to find a point of attack. My interest in the subject lies in the field which Dr. Armitage has used as an example, namely the statistics of cancer therapy. I am doubtful about the form of distribution which he has postulated to test the efficiency of the different methods of analysis. In my experience the exponential curve will often fit quite adequately the distribution of survival times of unsuccessfully treated cases, that is, those who subsequently die with the disease present. About ten years ago I examined a number of survival time distributions of this kind taken from several different cancer centres and I found that they were all well fitted by the lognormal curve, which is a close companion to the exponential, and, as Dr. Irwin (*J. Hyg., Camb.*, **42**, (1942), 328) has shown, extensive data are required to distinguish between these two distributions. I have been collecting more recent data on survival times and again I find that in all cases the lognormal, and in some the exponential, provide a very good fit for the data. The point I wish to make, however, is that these distributions all apply only to the unsuccessfully treated group. There may be some sites, such as bronchus, where the proportion of successes is negligibly small. In all the data I have seen for cancer of the breast, cervix, mouth and throat, there is a very significant flattening off in the distribution of survival times if all the treated patients are included. This arises from the long term survivors, who may justifiably be reckoned as cures. If their survival times are included in the distribution there is a significant departure not merely from the exponential but even from the lognormal curve. One might say that there is a lump on the tail of the distribution which refuses to be smoothed out. There is no single obvious criterion of the success of treatment in this disease, but the fact that some patients do appear to be permanently cured is perhaps the most hopeful feature of present treatment methods and this should, I think, be given great weight in assessing the relative value of different treatments. If we set out to determine the k of the best fitting exponentials we may miss this feature, and in the "sign" method there would seem to be some danger that the decision between two treatment methods would depend more on the length of palliation achieved in the unsuccessful cases than on the proportion of long-term successes. One way of dealing with this might be to use Dr. Armitage's "sign" method but to base the comparison within each pair on symptom-free time instead of on survival time. I should be glad to have Dr. Armitage's opinion on this alternative criterion. It is true that we have much fewer data on the distribution of symptom-free time than on the distribution of survival time. Nevertheless, its departure from an exponential would probably not be so great as to invalidate the general conclusions about the efficiency of the method which Dr. Armitage reaches in his paper.

I should like to congratulate Dr. Armitage on his use of Monte Carlo methods in this field of statistics. The collection of actual clinical data takes a great deal of time, and if one is merely wishing to test different statistical methods on typical data I think much time can be saved by constructing artificial random samples from populations having known

parameters. It may also be extremely educative to our medical colleagues if we demonstrate that statistical random selection can produce distributions which are indistinguishable from those obtained from clinical records. Sceptical clinicians can be invited to give their opinion on which are the genuine and which the artificial distributions of survival time.

Dr. Armitage has dealt exhaustively and very elegantly with the particular problem he set himself, and I am sure that this paper will, because of its elegance, attract the interest of other statisticians to this rather neglected corner of the statistical vineyard. I congratulate Dr. Armitage on his paper and have great pleasure in seconding the vote of thanks.

The vote of thanks was put to the meeting and carried unanimously.

Professor A. BRADFORD HILL: I am glad that Dr. Armitage has turned his attention so helpfully to a problem that can be very troublesome in clinical medicine. As an example I may take a problem with which I have recently had to struggle. It concerns a new treatment for a disease of the central nervous system, a disease for which there is no known effective treatment, which is usually prolonged and always distressing, and of which phases of quiescence and relapses are characteristic. The new treatment is very unpleasant and not wholly devoid of risk to the patient. Not unnaturally then, it has been developed somewhat haphazardly, without a controlled trial, by trying it out on a number of patients and judging the results in relation to past clinical experience with the disease. Weighed in that balance it appears to be of value in postponing relapses and in preventing deterioration, though for how long is quite unknown. Its promise has, however, become known to patients and, at least in some centres, they are asking for it. At this late date and in this situation can one ethically or practically promote a controlled trial of its efficacy? It is most important to do so. But can the physician now randomize the patients to new treatment and not new treatment and follow them up to record their progress?

It is obviously extremely difficult. The solution (or, more accurately, partial solution) that I have proposed is that pairs of patients, as far as possible similar in their clinical features and prognosis and in *both* of whom the disease is in its quiescent phase, should be accepted into the trial. One, chosen by lot, should be treated immediately; the other would be held off temporarily, (1) on the grounds that his disease is quiescent, and (2) on the promise that he will be given this new treatment immediately should he relapse. With each such pair one will be able to observe which of the two, the treated or the untreated, first suffers a relapse. Is the "survival" time more often longer on one side than the other? In this setting the untreated patient can be given the new treatment immediately if he himself relapses *or* if his treated pair relapses. In either case the longer survival has been determined.

Such a method certainly cannot show how long an advantage is reaped by the treated patients. But it should show whether *any* advantage is being reaped.

A minor difficulty will certainly arise in practice. Pairs of similar patients will not often be available at the same time. What we shall have to do is to accept a patient, of a previously defined type, and allocate him at random to treated or not treated. We shall then have to await a pair to him, who will automatically be allocated to the reverse group, whichever it may be. We may have to await some days, or even weeks, but beyond exercising patience I imagine that does not upset Dr. Armitage's scheme.

One weakness is that the doctor will sometimes, perhaps frequently, know before admitting a patient to the trial, whether that patient will be treated or not. That, we know from experience, may bias his judgment or lead him to over-compensate for bias. As an alternative scheme, supposing that one could foresee that in, say, 3 months one would admit 30 patients; could one then randomize as they came in to treatment or not treatment (with perhaps the proviso that in every 10 there should be 5 treated and 5 not treated) and then *after* the patients have thus entered the trial make the most efficient pairing that is possible for the subsequent follow-up? One could keep a running census of the

entries so that the pairing could be done without too much loss of time and it could (and should), of course, be done with no knowledge whatever of any events subsequent to the date of entry to the trial.

To the solution of this type of problem Dr. Armitage's paper makes a most valuable contribution and I am glad to add my acknowledgments to those of the proposer and seconder.

Dr. C. C. SPICER: I think that for a general purpose comparison, especially in dealing with a situation in which one has no *a priori* knowledge of the distribution of survival times, the actuarial method is the most generally useful. It cannot be applied in a sequential way but it does provide a fairly simple method of dealing with the familiar situation where patients are lost sight of or die from other causes. It is to all intents and purposes distribution-free, because it is applied to short intervals where the mortality curve can be regarded as linear or exponential, and one is not necessarily confined to making one's comparison solely on the basis of a proportion surviving at some given time. In most cases where the patients are followed for a fair length of time the expectation of life can be calculated for an artificial life table constructed for a group who are free of any interfering factors such as being lost sight of or dying from other causes. This expectation of life can be provided with a standard error. I have not had time to compare the efficiency with that of the methods of Dr. Armitage's paper but I gather it might be of the same order as that of Kaplan and Meier's. One case where λT would be small, but which does not yet occur in cancer research, is in diseases where there is a complete cure. Here Dr. Armitage's test should be efficient in detecting the superiority of the treatment at an early stage.

There is one other point which arises from this paper. A number of stochastic models have been devised for survival after diagnosis of cancer and tuberculosis and it seems to me that these are rather unsatisfactory if one wants to get results because they lead to exceedingly complicated estimating equations and one does not get any extra information out of them when one has finished. I should like to see some kind of *a priori* explanation of why there is such an extremely good fit of cancer survival curves to lognormal distributions.

Mr. J. HAJNAL: I should like to make two points, one a small technical one, the other more general. My first point concerns the condition for the sign test embodied in formula (3) of the paper, i.e., that one of the two survival curves is a fixed power of the other. This is, I believe, precisely equivalent to the statement that the death-rate at each duration after entry is, in one group, a fixed multiple of what it is in the other group. This may be seen as follows:

Let the risk of death between times x and $x + dx$, if the patient is alive at x, be $\lambda(x)\,dx$. In actuarial terminology $\lambda(x)$ is the force of mortality at time x. $\lambda(x)$ may vary with x, i.e. it is not assumed that the survival curve is exponential. The proportion surviving at time t may then be written

$$\exp\{-\int_0^t \lambda(x)\,dx\}.$$

Now let us write $\lambda_A(x)$ and $\lambda_B(x)$ for the force of mortality under treatments A and B respectively. Equation (3) then amounts to the following equation

$$\exp\{-\int_0^t \lambda_A(x)\,dx\} = \exp\{-k\int_0^t \lambda_B(x)\,dx\}.$$

This can hold for all t if, and only if, $\lambda_A(x) = k\lambda_B(x)$ for all x.

The condition for the sign test is thus fulfilled exactly if the new cure reduces the risk of death at each duration after treatment by some fixed proportion.

My second point is purely a commonsense one. It relates to circumstances where λT is large, so that almost all patients die before the end of the observation period. It struck me that a commonsense statistician with only elementary knowledge of statistical techniques would in these circumstances calculate the mean survival time for those patients in each treatment group who have died, and test the difference by a t-test. The calculation of the mean must give almost the same results as the maximum likelihood method, except that the information on the length of survival for those who have not died at the end of the observation period is lost. The use of the mean is also closely akin to the actuarial method in the modification which has been suggested, namely measuring the average life time or expectation of life at entry; a suitable actuarial method would not lose the information provided by those patients still living at the end of the observation period.

This consideration gives the clue to a possible sequential procedure for the case where λT is large. Is there any objection to a sequential t-test? One might pair the patients, as for the sign method; but wait for the second member of the pair to die and then take the difference between the survival times as a single observation for a sequential t-test. This procedure would have the advantage over the sign method of making use of the information provided by the length of time lived by the surviving member of a pair after the death of the first. It would presumably be more efficient than the sign method when λT is large.

Mr. J. C. Tanner: I should like to refer to Dr. Armitage's statement that it would perhaps be unusual in an industrial experiment to arrange for items to be put on test serially over a long period of time. A situation of this sort arises in connection with studies of the lives of road surfaces, which have been made at the Road Research Laboratory. It we wish to study types of surface that have been laid during the last few years we can use the actuarial method to construct the life-distribution of each type, but difficulties arise in assessing the significance of the differences between the average lives or the probability of survival for a certain length of time. The basic difficulty is that we do not have samples of independent units: the data consist of mileages of road which "die" after different periods of time. We therefore cannot use any of the methods which Dr. Armitage has described for testing of significance and we have to use a crude alternative. If, for example, we have two types of surface to compare, we divide up the data for each type into, say, four or five independent parts (which may come from different parts of the country) and for each part of the data construct a survival curve by the actuarial method; all we do then is use a t-test to compare the percentages surviving to a certain age with the two types of surface. If the survival curves were nearly complete then we should complete them (using methods which may not be orthodox) and then we could compare the mean lives by means of a t-test. I was glad to see from Dr. Armitage's paper that significance tests based on the actuarial method, using the percentage surviving to a certain age, such as we have used, are likely to be fairly efficient.

Mr. C. B. Winsten: The paper we have just heard from Dr. Armitage, and Dr. Cox's comments on it, prompt me to raise the questions: what is the essential difference between medical and industrial trials from the statistician's point of view—what particular aspect most contrasts the two? I know very little about medical trials, but a previous paper by Dr. Armitage stimulated me to the following thoughts about costs, and I wondered what he might think about them.

In industrial trials costs are involved in simply carrying out the experiments. But the costs in medical trials occur in a different way. Treatment has to be given anyway, so the cost is incurred in giving the "wrong" or less effective treatment. It does not matter if the trial goes on rather longer, provided that one is giving the "right" or more appropriate treatment. Thus in this sense the costs involved are associated with giving one of the treatments rather than the other.

This reasoning suggests that it may be possible to vary the proportion of patients given a particular treatment over the course of a trial. I show in a note to be published else-

where that it is possible to construct a sequential technique by which this can be done. Triples and quintuples of one treatment can be matched against a single one of the other. By this device it may be possible to lessen the numbers given the "wrong" treatment whilst obtaining a given amount of information about the relative effectiveness of the two treatments under trial.

Professor G. A. BARNARD: I am prompted to enquire whether Dr. Armitage has considered a slight extension of the method of §4 which would be attained by using the test which Kolmogorov introduced, to see whether two samples can be said to be drawn from the same population. According to this test the empirical cumulative frequency functions (F_1 and F_2) are plotted for each of the two samples, and the point at which these two functions differ most is found. The critical value of $| F_1 - F_2 |$ is a simple function of the two sample numbers. I think we owe it to Dr. Kiefer for drawing the attention of statisticians in the NATO countries to the fact that this test is much more powerful than one would expect a non-parametric test to be. In effect we are testing the significance of the *maximum* difference between the survivor rates and one would expect therefore that the test would at least be better than that of §4, which tests the difference between the survivor rates half way through the period.

The following contribution was received in writing after the meeting:

Dr. D. J. BARTHOLOMEW: Some interesting problems arise when the actual survival times of the patients who have died are not known. This situation is hardly likely to arise in a carefully supervised clinical trial but it is quite common in other applications of the theory. Sometimes the data have to be obtained from past records which were originally collected for some other purpose. For example, if one wishes to estimate the length of service (survival) distribution for men employed in a factory it may only be possible to discover when men were recruited and whether or not they are still employed. Any man may be regarded as having been "exposed to leaving" for a period T_i and having responded by leaving or remaining. The problem is, therefore, of the quantal response type. If the survival curve is known to be normal or lognormal, the parametric maximum likelihood solution is the familiar probit analysis. For the exponential case, considered in the paper, λ is obtained from the equation

$$\sum_{i=1}^{n} \frac{a_i T_i}{e^{\lambda T_i} - 1} = \sum_{i=1}^{n} (1 - a_i) T_i$$

where the time since the i^{th} individual joined is T_i;

$$\begin{aligned} a_i &= 1 \text{ if he has left} \\ &= 0 \text{ if he has not left.} \end{aligned}$$

The sign method and the direct comparison of proportions of survivors is clearly inapplicable in this case.

I have recently been investigating the possibility of obtaining a non-parametric estimate analogous to Kaplan and Meier's product limit. This is obtained by maximizing

$$l = \prod_{i=1}^{n} [F(T_i)]^{a_i} [1 - F(T_i)]^{1-a_i}$$

subject to the restraint $0 \leqslant F(T_1) \leqslant F(T_2) \leqslant \ldots \leqslant F(T_n) \leqslant 1$ where

$$T_1 \leqslant T_2 \leqslant \ldots \leqslant T_n.$$

The solution is obtained by finding the unrestrained maximum, when $F(T_i) = a_i$, and then "smoothing" these values by an averaging process. I hope to publish details of this method later. It would be interesting to compare the relative efficiencies in this case with those given by the author when the survival times are known.

Dr. ARMITAGE: I should like to avail myself of the opportunity to reply later to some of the points which have been raised. I am very grateful to Dr. Cox for his examples of analogous problems in other fields, and also for his interesting remarks about the use of maximum likelihood estimators in sequential analysis. I hope he will publish details showing how his general approach would be applied to the present problem.

Dr. Boag has pointed out that frequently lognormal distributions give at least as good a fit as exponential curves to cancer survival data, and I must admit to having used the exponential curve in this paper largely for mathematical convenience. He has himself put forward a model (*J. R. Statist. Soc.*, B, **11** (1949), 15) according to which a certain proportion of patients are effectively cured. It might be of interest to mention one approach which has been proposed fairly recently to the problem of estimating or defining the proportion of patients cured. S. J. Cutler and his colleagues in the United States have suggested that one might follow a group of patients treated in some way and look at the survival proportions from one year to the next. Initially, these are much lower than for the general population, but as time goes on the chances of survival, say from the ninth year to the tenth year, gradually increase, and in fact approach that of the general population. When these survival proportions are effectively the same as for the general population one can then say that the remainder of the patients are cured in the sense that their survival follows the same pattern as that of the general population.

Dr. Boag pointed out that the sign method might be influenced particularly by the early part of the survival curves, and if these crossed over the method might be misleading. It is true that a really fair comparison can be made only when patients have been followed for a long time. The possible value of the sign method lies, I think, in its adaptability to sequential analysis. If there were a very appreciable difference between the two treatments in their effect on the early part of the survival curve, then even though that experience might be reversed at longer periods after treatment, one would not feel justified in continuing to randomize.

Professor Bradford Hill described an interesting example of a trial in which the sign method would be appropriate. He discussed the desirability of pairing patients with a similar severity of disease and prognosis. One must beware of attempting too much in the way of pairing, because the more effort one makes to match the patients in a pair, the longer it will be before a suitable pair comes along, and what is gained in reduction of random error is lost by waste of time. I would not try to match pairs unless the matching was based on a criterion closely associated with survival.

I share Dr. Spicer's preference for the comparison of the expectations of life, rather than the survival proportions. At the end of §5 I mention Dr. Meier's observation that the expectation of life is fully efficient when $\lambda T \to 0$. It is also fully efficient when $\lambda T \to \infty$, for it is then equivalent to the maximum likelihood estimator—both are equivalent to the mean survival time. It looks a tricky problem to work out the efficiency in general, but I would conjecture that a comparison of the expectations of life is at least as efficient as a comparison of survival proportions. Dr. Spicer suggested that in typical cancer data λT would not usually be small because unfortunately survival proportions are not very high. This may be true of a long trial, with data collected over several years, but λT will be low during the early stages of such a trial and will increase as the trial proceeds. If there were a big difference between treatments one would like to know as soon as possible, and it is in this sort of situation that λT may be fairly small.

Mr. Hajnal has pointed out that equation (3) is equivalent to saying that the force of mortality in one group is a constant multiple of that in the other group. A similar result, in a slightly different problem, is given by C. F. Stevens (*I.R.E. Transaction on Reliability and Quality Control*, PGRQC—12, 1957).

I am grateful to Mr. Tanner for his description of the use of the actuarial method in another field, and to Professor Barnard for suggesting the possible use of the Kolmogorov-Smirnov two-sample test. Mr. Winsten has grappled with the thorny problem of assessing costs in clinical experiments, which I have always found rather intractable. He suggests that as the trial proceeds the proportions of subjects allotted to different treatment groups

should be adjusted, so that the treatments which seem to be coming out best should receive the most patients. This is clearly attractive, but one might have difficulty in preserving the advantages of a strictly randomized experiment. If the type of patient entering the trial, or some external environmental factor affecting the prognosis, changed with time, the comparison of the treatment groups might be biased.

Dr. ARMITAGE subsequently replied in writing as follows:

I agree with Dr. Hajnal that if one would be unlikely to contemplate stopping until λT were large, then a sequential t-test on the survival times would be a very reasonable approach. I do not know how sensitive to non-normality sequential t-tests are, but I should conjecture that the effect would not be serious.

It would be quite possible to use the two-sample Kolmogorov-Smirnov test, as suggested by Professor Barnard. I have not investigated its efficiency, but it would be interesting to know whether it is appreciably better than a comparison of survival rates. The test, like that of §4, could be performed on the entrants in $(0, T - \tau)$, for various τ. For any choice of τ, the distribution of survival times would be censored at the upper end, at a value τ. One could enquire what is the optimal value of τ, for any value of λT.

The situation discussed by Dr. Bartholomew may, in spite of his disclaimer, have an analogue in medical trials. If the response is death it will usually be possible to record the exact day on which this occurs. If the response is the recurrence of a symptom the doctor may see his patients at set intervals, and may have to rely on the patients' reports which may not fix the time at all precisely. In some instances the response may be detected only by a periodic examination (say by X-rays), in which case one would merely know whether or not the response had taken place between successive examinations.

In my earlier comments I was less than fair to Mr. Winsten's proposal to change the proportions allocated to different treatments. It would be quite possible to divide the period of the trial into sub-intervals, and to arrange that subjects are allocated randomly to treatments, the allocation probabilities remaining constant within a sub-interval but varying between sub-intervals. The results for any two treatments could be compared by forming a weighted mean of the contrasts within sub-intervals. I hope he will be able to consider the problem of discovering the optimal method of allocation.

As a result of the ballot taken during the meeting the candidates named below were elected Fellows of the Society:

Domingo Cruz Alonzo
Lionel Wolfe Arbis
John Vernon Chelsom
Ian Grant Cumming
Laurence Stanley Goddard
Sidney Gould
John Selwyn James
Louis B. Kahn
John Herbert Kilton
Norman Down Stewart Smith
Ernest Tampin
George Thomas Watson

Corporate representative

Richard Stanley Smith *representing* The Library, University of Nottingham.

ERNIE—A Mathematical and Statistical Analysis

By W. E. Thomson

Post Office Research Station

[Read before the Royal Statistical Society, 18th March, 1959, the President, Sir Harry Campion, C.B., C.B.E., in the Chair]

Contents

SUMMARY

In conducting the monthly prize draws for Premium Savings Bonds, use is made of a machine usually known as ERNIE (Electronic Random Number Generator Indicator Equipment), which generates and prints automatically a random sample of bond numbers. The paper discusses this machine, and some related topics, from the point of view of the mathematician and statistician.

The machine contains a number of random digit generators, which rely for their randomness on electrical noise in neon tubes; deterministic operations convert the continuous noise waveform into a sequence of random digits. An analysis of these generators, partly theoretical, partly experimental, backed up by conventional random digit tests, shows that these generators should, for practical purposes, be indistinguishable from perfect generators. It is also necessary, for the machine as a whole to be satisfactory, that these generators be independent; this also has been investigated. Nine random digits are required to form a complete bond number, but the machine uses ten random noise sources, combined in such a way that the output is not prejudiced by the malfunctioning of any one source.

Statistical analyses of certain operations in the draw are made each month, with the object of having some assurance that the results are con-

sistent with random behaviour. In planning a draw, account must be taken of the fact that the number of bond numbers to be printed by ERNIE, and the time taken to print them, are both random variables. Routine tests of ERNIE include special tests of the random features of the digit generators.

1. Introduction

The Premium Savings Bond scheme (General Post Office, 1956), which was introduced in 1956, involves a monthly prize draw, the first draw occurring in June, 1957. The prize draw is equivalent to the random drawing of counterfoils from a hat, but the scale of the operation (10^4 prizes from 10^8 counterfoils are the orders of numbers currently involved), among other things, requires a more refined practical method of determining the prize list. The operation involves an electronic machine widely known as ERNIE (acronym for "Electronic Random Number Indicator Equipment"). This paper discusses those features of ERNIE which are of interest to mathematicians and statisticians.

Press articles and B.B.C. programmes have often included ERNIE among the electronic digital computers which have made such an impact on scientific and commercial calculations during the last decade. ERNIE certainly uses hardware and carries out processing of digital data in a way similar to many computers. The main difference lies in the input. In ordinary computers, data are fed in from outside, processed, and passed to the output. With ERNIE, the noise waveforms of the random generators constitute the input data; they are generated within ERNIE, but they are not susceptible to control, and act independently.

Technical descriptions of ERNIE are given elsewhere (Hayward and Bubb, 1957; Hayward *et al.*, 1957), but it has been necessary, at various points in the paper, to give some account of the physical processes and components involved. It must be emphasized that these accounts are intended as background for the mathematical analysis and hence ignore various features which, though technically important, are logically irrelevant.

2. General

2.1. *Requirements for a Prize Draw*

Each month, a certain number of Premium Savings Bonds are eligible to take part in a draw and a certain number of prizes are to be given (General Post Office, 1956). Basically then, the requirement is that a sample, equal in number to the number of prizes, and ideally perfectly random, has to be drawn from the population of eligible bond numbers. The term "bond numbers" is used rather than "bonds", because of the existence of multiple bonds; a £100 bond, say, has 100 bond numbers associated with it, and each of these numbers must have its individual chance in the draw. In any one draw, a bond number can win only one prize, i.e. the sampling must be "without replacement". Further, the prizes are not equal in value. The sample is therefore to be ordered, the bond numbers at the head of the list getting the biggest prizes.

It is not proposed to discuss the various ways—ranging from purely manual to completely automatic—in which this desired sample might be drawn. The solution adopted involves a two-stage process. In the first stage, ERNIE generates and prints automatically, in order, a sample with replacement from a population which includes the eligible bond

numbers. This sample, which is ideally perfectly random and whose size is not fixed beforehand, forms a preliminary list which then goes through the second stage.

Reference is made manually to the records, and bond numbers which are ineligible, or which have already appeared higher up in the list, are deleted. The remaining numbers form the prize list and the process is stopped when this list reaches the required size.

A particular bond number is specified by nine characters, e.g. 2KF 999 999. The first character indicates a tens-of-millions digit; the second (with 23 possibilities) a denomination, and the third (with ten possibilities) a unit-millions digit; the remaining six indicate hundreds-of-thousands down to units in the usual way. Thus the number quoted corresponds to serial number 22 999 999 of denomination K. The special form of the first three characters has been chosen to make reading and copying easier. When the tens-of-millions digit is zero the first character becomes a space on the bonds themselves and in published lists, but is printed as zero by ERNIE.

2.2. *Operation of ERNIE*

The nine characters of a bond number are generated separately. Random digit generators, of the type discussed in Section 3, are used to generate what are effectively ten random digits and these are then compounded (Section 4) to give the nine random digits specifying the nine characters; the object of having the extra digit is to cater for the possible malfunctioning of one of the ten sources.

Of the nine digits, the last seven are in the scale of ten. The first digit is, in principle, in the scale of ten, but time-wasting generation of ineligible numbers can be reduced by taking advantage of the fact that, since bonds are issued to selling points in numerical order, not all ten digits occur among eligible bonds. In the first draw, in June, 1957, only the digits 0, 1, 2 were needed and so a scale-of-three source was used. Since July, 1958 a scale-of-four source, to give digits 0, 1, 2, 3 has been used. The second character has 23 possibilities but for technical reasons a scale-of-24 source is actually used. When the 24th character appears, it is rejected and a new number generated. This is done by the same mechanism by which other ineligible numbers are rejected, as will now be explained.

The number-generating equipment is capable of generating $24n \times 10^7$ different numbers, n being the range of the tens-of-millions digit, currently four. The number of eligible bond numbers varies from draw to draw, but is currently of the order of 10^8. As each number is generated, the first four characters are examined, thus determining a block of 100,000 numbers within which the generated number lies. If the block is known to contain no eligible numbers, the number is rejected and a new number generated; if the block contains at least one eligible number, than any number from such a block is printed, the ineligible numbers being rejected subsequently. In the June, 1957 draw, 72×10^7 different numbers could be generated. Of the 7,200 blocks of 100,000, 1,181 contained eligible numbers. Of the 118,100,000 printable numbers, 48,487,854 were eligible. Thus roughly 5/6 of the generated numbers were rejected within ERNIE and roughly 3/5 of the printed numbers rejected by reference to the records. In November, 1958 these proportions were 4/5 and 1/4 respectively.

ERNIE can be thought of as being divided into two parts, the Number Generator and the Printer, which, apart from the interchange of control signals, act autonomously. While the generating and testing process that has just been described is going on within the Number Generator, the Printer is printing the previous number, along with a serial number.

Usually, the Number Generator finds an acceptable number before printing is finished. This number is then stored until printing is finished. A control signal then causes the stored number to be passed to the Printer, and the Number Generator then starts testing numbers to find the next one.

Since it is a matter of chance how many numbers may have to be rejected before an acceptable number is found, it sometimes happens that the Number Generator is still searching when the Printer is ready; the Printer then remains idle till an acceptable number has been found. The consequence is that the rate of printing varies randomly; this is further discussed in Section 5.1.

3. Random Digit Generation

3.1. *General*

Random digit generation in ERNIE uses essentially the same method as in the Post Office Artificial Traffic Machine (Broadhurst and Harmston, 1950; 1953), which handles certain stochastic problems by what would nowadays be called a Monte Carlo method.

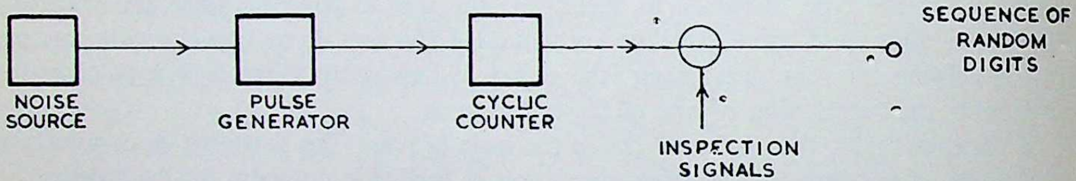

FIG. 1.—Components of random digit generator.

The randomness arises from a suitable natural phenomenon which, while capable of being harnessed to produce digits, remains variable, uncontrollable, unpredictable. One could conceivably use such things as the wind or the waves, but the most convenient phenomenon is that of electrical "noise". Because of the ultimate atomic nature of electricity, a current which is nominally constant is in fact varying slightly all the time. These variations constitute the noise (so called because, if passed to a transducer such as a loudspeaker, the sound produced is a kind of noise) and they are commonly regarded as statistically random. Some electrical components are more suitable than others for use as noise sources; ERNIE uses neon discharge tubes, ten of them. These tubes, physically part of ERNIE and similar in appearance to other components, are of prime importance and play a logically different role. The bond numbers produced in any run by ERNIE depend on the particular noise waveforms which these ten neon tubes happen to produce during the course of the run. They provide an independent input to the rest of the equipment, which is designed to act in a purely deterministic way to convert the noise waveforms into printed bond numbers.

Fig. 1 shows the parts of a digit generator. Each time the noise waveform passes upwardly through a certain fixed level, the pulse generator emits a short pulse of about two microseconds duration, except that after the production of each pulse there is a dead time of about 30 microseconds before the noise can instigate another pulse. A train of pulses, randomly spaced in time, is thus passed to the n-position cyclic counter, each pulse stepping the counter one position onwards; from the nth position the counter steps to the first. From time to time, in practice at regular intervals of about 1/6 second, the counter

is examined and the digit indicated passed to the output as one of a sequence of random digits.

This method is very similar to that used by Kendall and Babington Smith (1938), who used a disk, divided into ten equal sectors, spinning uniformly and examined at random intervals, the sector opposite a fixed pointer indicating the output digit.

The essential difference is that the ERNIE method corresponds to examination at regular intervals of a disk spinning at a randomly varying rate. This difference is not trivial: for the same ratio of average spinning rate to examination rate the latter method is vastly superior, at any rate for exponentially distributed intervals (between examinations in the one case, and between advances from one digit to the next in the other). This point is further discussed in Appendix 1.

3.2. *Theoretical Analysis*

A mathematical model might begin with the random noise waveform but this soon leads to insoluble problems. The next stage is the random pulse train. Fig. 2 shows a typical experimental histogram for the distribution of intervals between successive pulses; it is roughly an exponential distribution truncated below at about 30 microseconds (the

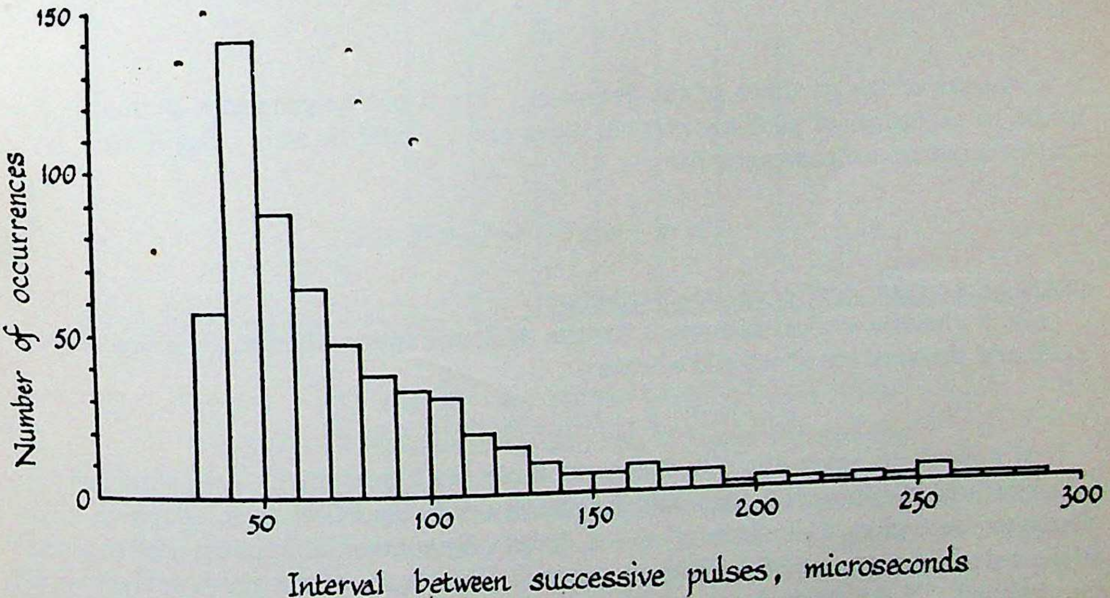

FIG. 2.—Typical histogram for pulse intervals in random digit generator.

average is about 76 microseconds and the standard deviation about 44 microseconds). The 30 is fixed by technical requirements, but the average can be varied. It can be shown (see Appendix 2) that, if the distribution is exponential, and if the time-intervals are independent, a 3 : 1 ratio, of average to truncation point, gives the nearest approach to perfect randomness. Certain indirect evidence, however, indicates that the intervals are not independent; one would not, in fact expect them to be, because of the autocorrelation in the noise waveform. The 3 : 1 ratio will probably be fairly near the optimum and generators are consequently set up to have a nominal average of 90 microseconds. The actual average is not at all critical.

The operation of the counter is now considered. Let u_r represent the sequence of output digits. Then

$$u_r \equiv u_{r-1} + m_r \qquad (\text{mod } n) \tag{1}$$

where m_r (with $0 \leqslant m_r \leqslant n-1$) is the effective number added to the counter during the r^{th} interval. We have

$$m_r \equiv M_r \qquad (\text{mod } n) \tag{2}$$

where M_r is the total number of pulses passed to the counter, and is a random variable whose frequency function is, in principle, derivable from the properties of the sequence of time intervals. The frequency function for the random variable m (the distributions are assumed to be independent of r) is

$$P_m = \sum_{j=0}^{\infty} \Pr\{M = m + jn\} \qquad 0 \leqslant m \leqslant n-1. \tag{3}$$

Let

$$P_m = \frac{1}{n} + \epsilon_m. \tag{4}$$

Then the quantity

$$v = \sum_{m=0}^{n-1} \epsilon_m^2 \tag{5}$$

is a measure of the goodness of the generator. For a perfect generator, both u_r and m_r would be sequences of perfectly random digits and v would be zero. Appendix 3 shows that an approximate expression for v is

$$v \approx \frac{2}{n} \exp(-4\pi^2 \mu_2/n^2) \tag{6}$$

where μ_2 is the variance of the M-distribution.

Fig. 3 shows a typical histogram for the M-distribution. The mean count is about 1,850 and the variance about 400 whence

$$v \approx 10^{-70},\ n = 10; \quad v \approx 10^{-13},\ n = 24. \tag{7}$$

It is difficult to assess the significance of these small quantities. One approach is to consider how the imperfections could be shown up by statistical tests. Suppose an investigator were given a sequence of $N+1$ digits. By subtraction modulo n of successive digits a derived sequence of N digits corresponding to the m_r of the above analysis would be obtained. If the original sequence were perfectly random so too would be the derived sequence. The chi-squared test might be used to test the null hypothesis that the digits of the derived sequence were equiprobable, using a significance level of, say, 1 per cent. With digits produced from the type of generator discussed, in which the digits of the derived sequence are not equiprobable, the probability of making the correct decision, namely to reject the null hypothesis, will depend on N and v. For constant v, this probability will rise from 1 per cent. when N is zero towards unity as N increases indefinitely.

Actual figures can be obtained from Patnaik's (1949) results on non-central chi-squared. Using his approximation, that $\{2\chi^2(\nu+\lambda)/(\nu+2\lambda)\}^{\frac{1}{2}}$ is normally distributed with mean $\{2(\nu+\lambda)^2/(\nu+2\lambda)-1\}^{\frac{1}{2}}$ and unit variance, with, in this case, $\lambda = Nnv$ and $\nu = n-1$, we find that for $n = 24$ and $v = 10^{-13}$, the probability of rejecting the null hypothesis is

about 50 per cent. for $N = 10^{13}$ and over 90 per cent. for $N = 3 \times 10^{13}$. Since the current rate of use of digits is of the order of 10^6 per year, it seems that the values of v obtained are satisfactorily small.

It has now been shown that the digits m_r, from which the output sequence u_r is derived by the relation $u_r \equiv u_{r-1} + m_r \pmod{n}$, are, for practical purposes, equiprobable. We must also consider their independence. This will be related to independence in the sequence

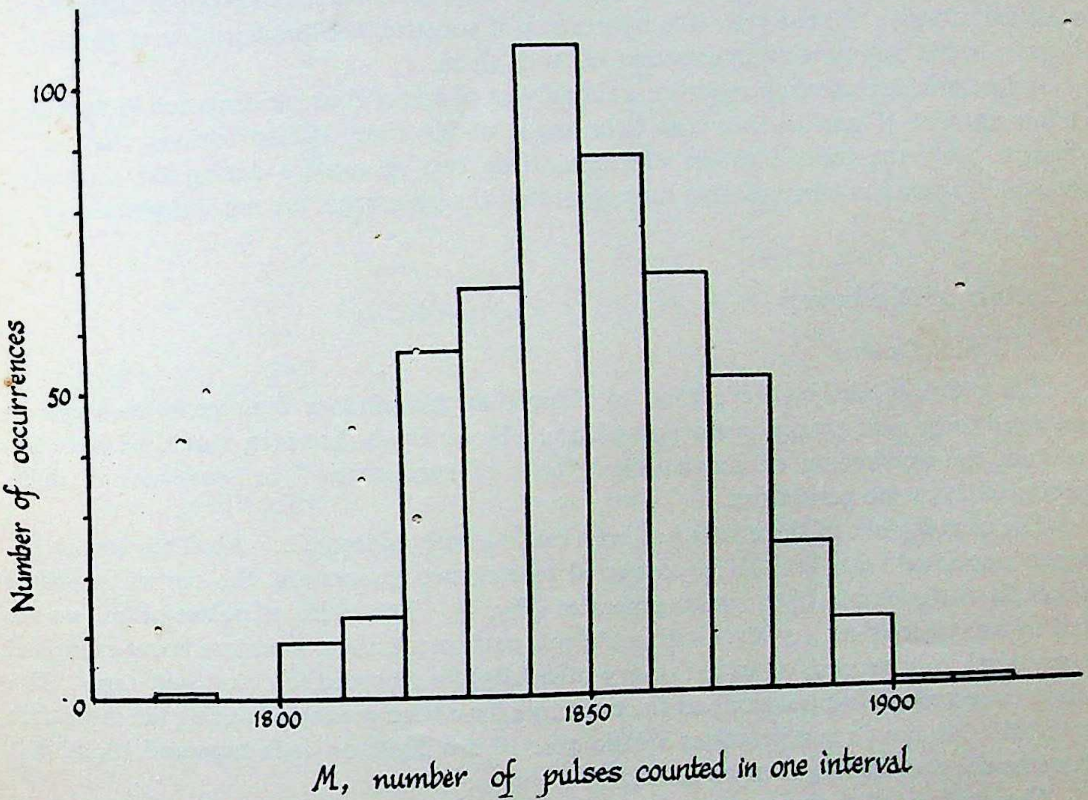

FIG. 3.—Typical histogram for pulse counts in random digit generator.

M_r. Thus the joint probability distribution for any two digits in the m sequence is derived from the corresponding M-distribution by

$$\Pr\{m_r = h,\ m_s = k\} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \Pr\{M_r = h + in,\ M_s = k + jn\} \qquad 0 \leqslant h,\ k \leqslant n - 1. \quad (8)$$

Although no formal result is available, it is obvious that, in the same way that a considerable M-variance is reduced to a negligible m-variance, departures from equiprobability in the joint distribution for M_r, M_s will be evened out and the m_r, m_s distribution will become practically uniform, i.e. the m's will be practically independent.

Although imperfections in the sequence m_r obviously imply imperfections in the sequence u_r, the relations are not simple. In discussion of the random cyclic walk (Feller, 1950), it has been shown that, if the m_r are independent, the u_r are equiprobable; lack of equiprobability among the m_r leads to dependence among the u_r. In the present case, where

the m_r are very nearly independent, one would expect a tendency towards the same effect, i.e. the u_r would be more nearly equiprobable than the m_r. For practical purposes the u_r should be equivalent to a perfectly random sequence, the departures being so small as to require millions of digits to show them up.

In order that sequences of bond numbers should be perfectly random, it is also necessary that the separate digit generators which are used be independent. This independence is not amenable to theoretical analysis. The neon tubes are assumed to produce independent noise waveforms; for the rest, it is a question of constructing the assembly of generators so as to avoid any possible interaction between them.

Independence between generators is the subject of one of the tests discussed in Section 4. A fair amount of analysis has also been made of bivariate M-distributions, the pair of variates being the total numbers of pulses from two generators during the same time interval. There has been nothing to suggest that the generators are not independent.

3.3. *Tests of Randomness*

3.3.1. *General*

The previous sections have tried to show that the random digit generators can be expected to be near enough perfectly random. No analysis, however, would be complete without the application of conventional "tests of randomness" to sequences of digits produced from the generators.

The carrying out of these tests has been made largely automatic. Ancillary test equipment associated with ERNIE is connected to any two generators, the connexion being taken from the output of the pulse generator (Fig. 1). The trains of pulses picked up are fed to two scale-of-ten counters within the test equipment, these counters being examined every sixth of a second, as usual; every other digit is recorded on punched tape. The digits in corresponding positions on the two tapes have been generated during the same time interval. An hour's run produces a little over 10,000 digits on each tape and 10,000 has become the normal length to be tested.

The analysis of these two tapes is then carried out automatically on the Pegasus electronic computer, for which a programme to carry out several well-known tests has been prepared. Two sequences of 10,000 digits take about 15 minutes to run through. Table 1 shows the analysis for two such sequences. The computer programme includes arrangements to print suitable headings and so on, as well as the numerical results; Table 1 reproduces the material and layout as actually printed by the computer. Some of the headings of the table, and the probabilities associated with the chi-squared values, are not included in the computer results sheet; these additions have been set in italic characters. Under several of the sub-headings will be seen a string of characters such as "ERNIE/10000/165MS/11·5KC/G3/29" for sequence A. This is a reference which identifies the particular sequence; it is punched on the data tape, preceding the digits, and is reproduced on the results sheet.

The tests used on the individual sequences are the four introduced by Kendall and Babington Smith (1938), namely the Frequency, Serial, Gap, and Poker Tests; and also the Coupon Collector Test. Some notes will now be given. The reader is assumed to be acquainted with the four classical tests.

TABLE 1

Sequence A

RANDOM DIGIT TESTS

COUPON COLLECTOR

ERNIE/10,000/165 MS/11·5 KC/G 3/29

Check 9,980 + 20 = 10,000.

RANGE	OBS.	EXP.
10–19	75	60·5
20–22	43	45·3
23–25	51	46·6
26–28	27	42·5
29–31	36	35·9
32–34	29	28·9
35–37	26	22·6
⩾38	62	66·8
Total	349	349·0

Chi-squared = 10·5. 7 DF. $P = 0{\cdot}16$.

SERIAL/FREQUENCY/GAP

ERNIE/10,000/165 MS/11·5 KC/G 3/29.

GAP

RANGE	OBS.	EXP.
0–1	1,943	1,898·1
2–4	2,150	2,192·9
5–7	1,606	1,598·6
8–10	1,189	1,165·4
11–13	819	849·6
14–16	600	619·3
17–19	442	451·5
⩾20	1,241	1,214·5
Total	9,990	9,990·0

Chi-squared = 4·9. 7 DF. $P = 0{\cdot}67$.

SERIAL + FREQUENCY

	9	8	7	6	5	4	3	2	1	0	*T*
9	97	94	93	107	90	95	100	102	92	100	970
8	87	100	100	105	100	89	98	101	112	104	996
7	90	81	99	107	94	113	93	95	100	100	972
6	99	96	102	105	111	121	95	103	107	97	1,036
5	98	103	95	97	91	96	101	97	97	96	971
4	95	97	110	82	98	110	106	112	100	116	1,026
3	103	97	100	104	79	89	100	106	100	106	984
2	94	115	91	109	101	107	87	107	105	104	1,020
1	110	96	83	120	111	90	111	98	96	97	1,012
0	97	117	99	100	96	116	93	99	103	93	1,013
										Grand Total	10,000

Serial

Chi-squared * = 61·9. 90 DF. $P = 0{\cdot}98$.

Frequency

Chi-squared = 5·5. 9 DF. $P = 0{\cdot}80$.

POKER

ERNIE/10,000/165 MS/11·5 KC/G 3/29.

	OBS.	EXP.
ABCD . . .	1,271	1,260·0
AABC . . .	1,066	1,080·0
AABB . . .	63	67·5
AAAB . . .	95	90·0
AAAA . . .	5	2·5
Total . . .	2,500	2,500·0

Chi-squared = 0·6. 3 DF. *P = 0·90.*

Sequence B

RANDOM DIGIT TESTS

COUPON COLLECTOR

ERNIE/10,000/165 MS/11·5 KC/G 7/28
Check 9,954 + 46 = 10,000

RANGE	OBS.	EXP.
10–19 . . .	58	58·0
20–22 . . .	48	43·5
23–25 . . .	45	44·7
26–28 . . .	42	40·8
29–31 . . .	30	34·5
32–34 . . .	29	27·8
35–37 . . .	17	21·7
$\geqslant$38 . . .	66	64·1
Total . . .	335	335·0

Chi-squared = 2·2. 7 DF. *P = 0·95.*

SERIAL/FREQUENCY/GAP

ERNIE/10,000/165 MS/11·5 KC/G 7/28

GAP

RANGE	OBS.	EXP.
0–1 . . .	1,913	1,898·1
2–4 . . .	2,194	2,192·9
5–7 . . .	1,617	1,598·6
8–10 . . .	1,121	1,165·4
11–13 . . .	831	849·6
14–16 . . .	624	619·3
17–19 . . .	484	451·5
$\geqslant$20 . . .	1,206	1,214·5
Total . . .	9,990	9,990·0

Chi-squared = 4·9. 7 DF. *P = 0·67.*

SERIAL + FREQUENCY

	9	8	7	6	5	4	3	2	1	0	*T*
9	95	92	97	106	105	101	119	120	115	117	1,067
8	87	96	79	80	85	100	94	109	118	100	948
7	98	109	102	79	97	108	103	78	95	100	969
6	116	93	92	101	118	91	97	100	90	83	981
5	113	85	113	99	112	103	99	103	94	90	1,011
4	118	98	92	88	103	82	101	98	107	98	985
3	97	88	89	104	96	106	98	117	96	104	995
2	108	90	95	113	93	87	107	100	116	105	1,014
1	131	102	103	110	105	113	84	90	112	99	1,049
0	104	95	107	101	97	94	93	99	106	85	981
									Grand Total		10,000

Serial
Chi-squared * = 98·2. 90 DF. $P = 0{\cdot}27$.

Frequency
Chi-squared = 11·8. 9 DF. $P = 0{\cdot}22$.

POKER

ERNIE/10,000/165 MS/11·5 KC/G 7/28

	OBS.	EXP.
ABCD . . .	1,255	1,260·0
AABC . . .	1,102	1,080·0
AABB . . .	55	67·5
AAAB . . .	86	90·0
AAAA . . .	2	2·5
Total . . .	2,500	2,500·0

Chi-squared = 3·1. 3 DF. $P = 0{\cdot}37$.

A and B

INDEPENDENCE TEST

ERNIE/10,000/165 MS/11·5 KC/G 3/29. (A)
ERNIE/10,000/165 MS/11·5 KC/G 7/28. (B)

A \ B	9	8	7	6	5	4	3	2	1	0
9	101	109	84	104	99	105	91	81	85	111
8	99	90	74	93	118	92	91	109	130	100
7	105	93	89	103	102	93	115	95	91	86
6	109	100	102	86	104	119	98	107	108	103
5	110	93	110	96	99	92	99	99	88	85
4	102	80	98	106	104	101	111	121	112	91
3	104	99	96	89	89	100	97	100	109	101
2	109	97	96	123	87	104	95	103	123	83
1	101	97	117	92	109	95	91	105	103	102
0	127	90	103	89	100	84	107	94	100	119

$N = 10{,}000$

Chi-squared = 112·5. 99 DF. $P = 0{\cdot}17$.

Coupon collector (Greenwood, 1955).—The sequence to be analysed is split into "collections", which are sub-sequences in which each of the digits 0–9 appears at least once. The last digit of the sub-sequence, which completes the collection, appears for the first time. The next digit of the main sequence begins the next collection.

The first line of the results, beginning "CHECK", indicates, taking sequence A for example, that the first 9,980 digits formed a number of complete collections, the last 20 forming an incomplete collection. The analysis that follows shows that 349 complete collections were obtained, the lengths of these collections, grouped into eight ranges, having a histogram corresponding to the OBS(erved) column. The number of ranges is, of course, arbitrary; eight is a very convenient number for Pegasus. The EXP(ected) column is obtained by multiplying the observed total by the range probabilities, obtained from the frequency function for the length of a collection.

Additional information is obtained from the fact that the number of collections from a sequence of N perfectly random digits is asymptotically normally distributed with mean N/μ and variance $N\sigma^2/\mu^3$ where μ and σ^2 are the mean and variance of the length of a collection equal, for denary digits, to $29\cdot 29$ and $125\cdot 69$ respectively. For $N = 10{,}000$ this gives $341\cdot 4$ and $7\cdot 07$ for the mean and standard deviation of the number of collections.

Gap.—The analysis lumps together the gaps for each individual digit 0–9, grouped into eight ranges. Provided each digit appears once, the total number of gaps from N digits in the scale of n is $N-n$; this number is multiplied by $p_r = (n-1)^r/n^{r+1}$, the usual geometric probability for a gap of length r. There must be some dependence between the gaps for the different digits, and hence the weighted sum of squares for the grouped distribution cannot be distributed exactly as chi-squared with 7 degrees of freedom; the assumption that it is so distributed is probably near enough right for N large, although some doubts are cast on this by the results given in Table 2, which is discussed in Section 3.3.2.

It can in fact be shown that the expected number of gaps of length r from N digits is

$$(N-r-1)(n-1)^r/n^{r+1} \tag{9}$$

and the expected total is the sum from $r = 0$ to $r = n-2$ which reduces to

$$N - n + n(1-1/n)^N. \tag{10}$$

The differences are trivial when N is large.

Serial and frequency.—The printed two-way table is primarily a serial analysis, in which the N^{th} digit is associated with the first to form the N^{th} pair; each entry shows the number of cases in which the digit at the left-hand end of the row is followed by the digit at the head of the column. Row or column totals of such a table give the frequency analysis; only the row totals are printed. The frequency chi-squared is as usual; the asterisk after "Chi-squared" for the Serial figure indicates that the quantity recorded is

$$\frac{100}{N}\sum_{rs}\left(O_{rs}-\frac{N}{100}\right)^2 - \text{(frequency chi-squared)} \tag{11}$$

which Good (1953) has shown to be distributed as chi-squared.

Poker.—The N digits are divided into $N/4$ groups of 4 (the programme also caters for N not divisible by 4) and classified as "ABCD", i.e. 4 different digits, and so on. In calculating chi-squared the class AAAA is grouped with AABB; hence 3 degrees of freedom.

Independence test.—In addition to these tests on the individual sequences, an "Independence Test" is carried out on the pair of sequences; a frequency analysis is made of the

digit-pairs formed by the r^{th} digit of one sequence and the r^{th} digit of the other. These digits, it will be remembered, are generated during the same time interval.

3.3.2. *Interpretation of Tests*

Kendall and Babington Smith (1938) introduced their tests as acceptance tests for sequences of digits which were classified as "locally random" or not. Not all sequences from a perfectly random generator would pass the test; and pseudo-random sequences generated by deterministic processes can pass the tests.

The present application is quite different. The tests are best regarded as means of summarizing the evidence of sequences of digits with the object of answering the question

TABLE 2

Random Digit Tests

(Probabilities of exceeding observed value of chi-squared)

Sequence No.	*Coupon Collector*	*Gap*	*Serial*	*Frequency*	*Poker*	*Independence*
0	0·64	0·57	0·40	0·69	0·78	0·85
1	0·73	0·20	0·62	0·20	0·51	
2	0·79	0·29	0·39	0·32	0·83	0·17
3	0·84	0·63	0·17	0·08	0·78	
4	0·90	0·32	0·64	0·05	0·64	0·05
5	0·83	0·99	0·55	0·05	0·43	
6	0·62	0·97	0·77	0·25	0·99	0·32
7	0·35	0·90	0·41	0·76	0·04	
8	0·30	0·35	0·55	0·33	0·18	0·45
9	0·18	0·89	0·43	0·25	0·27	
10	0·82	0·95	0·52	0·47	0·13	0·95
11	0·79	0·88	0·38	0·70	0·73	
12	0·72	0·12	0·60	0·69	0·96	0·94
13	0·05	0·55	0·64	0·86	0·80	
14	0·45	0·29	0·14	0·23	0·51	0·40
15	0·06	0·25	0·44	0·27	0·76	
16	0·67	0·88	0·04	0·17	0·43	0·53
17	0·14	0·83	0·44	0·07	0·25	
18	1·00	0·65	0·59	0·52	0·57	0·52
19	0·83	0·89	0·60	0·55	0·24	
20	0·75	0·37	0·26	0·02	0·19	0·08
21	0·33	0·55	0·89	0·88	0·36	
22	0·60	0·50	0·24	0·84	0·78	0·63
23	0·51	0·98	0·08	0·81	0·20	
24	0·47	0·87	0·26	0·74	0·59	0·74
25	0·37	0·49	0·66	0·33	0·40	
26	0·63	0·80	0·97	0·21	0·78	0·41
27	0·45	0·93	0·87	0·50	0·19	
28	0·95	0·67	0·27	0·22	0·37	0·17
29	0·16	0·67	0·98	0·80	0·90	
Grand Test	0·77	1·00	0·13	0·08	0·76	0·41

(Distribution of the above figures (excluding the Grand Test))

Range	*Number of occurrences*	*Range*	*Number of occurrences*
0·0–0·1	12	0·5–0·6	18
0·1–0·2	13	0·6–0·7	19
0·2–0·3	19	0·7–0·8	16
0·3–0·4	15	0·8–0·9	20
0·4–0·5	16	0·9–1·0	17

Average number of occurrences: 16·5

"does this sequence suggest that the random generator is not behaving as it was designed to behave?" Abnormal chi-squared values would suggest investigation to look for faults in theory or construction. If no such faults could be found, one would be extremely reluctant to condemn a generator on the evidence of the tests alone.

Table 2 shows the probabilities associated with chi-squared for 15 pairs of 10,000-digit sequences, i.e. 300,000 digits altogether. The distribution of these figures in 10 per cent. ranges is also shown. The deviations from the expected uniform distribution do not seem excessive.

The table also shows the results of a series of grand chi-squared tests; those for the Gap Test warrant some discussion. The actual value of chi-squared is 159·6, with, nominally, 210 d.f., so that chi-squared is exceptionally small, the probability of its being exceeded being 99·5 per cent. It is believed that this result is, partly at any rate, due to the form of Gap Test used (see Section 3.3.1) in which correlation has been ignored in assigning the degrees of freedom. It will be noted that the Coupon Collector Test, which covers much the same features of a sequence of digits as the Gap Test, does not result in an abnormal grand chi-squared.

4. Compounding of Digits

The nine characters of a bond number are generated separately, using digit generators of the type discussed in Section 3. The simplest way would be to have nine generators, one for each character, and this would be perfectly satisfactory in principle. A more refined scheme is actually used, the object being to cater for the possible failure of generators.

Each character depends on two generators. The two digits are passed to a combiner, which produces one output digit to indicate the character. Various methods of combination are possible; the essential requirement is that, with one input digit fixed, the n different possible digits at the other input lead to n different digits at the output. This leads to the following specification: enter the output digits in a $n \times n$ table in which rows and columns are associated with the different possible digits from the two inputs. Then each row and column must contain all n digits, i.e., the table must form a latin square. The method actually used is equivalent to

$$u \equiv u_1 - u_2 \pmod{n}, \tag{12}$$

u being the output digit, u_1 and u_2 the input digits.

One effect of this operation is a nearer approach to equiprobability. This form of combining is equivalent to a single generator where the digit added is

$$M = M_1 - M_2. \tag{13}$$

The variance of this M-distribution is thus the sum of the variances of the M_1 and M_2 distributions, and the m-distribution is thus more nearly equiprobable.

This improvement, however, is not the prime reason for introducing compounding: the advantage is that if either u_1 or u_2 ceases to be random, because of some malfunctioning of the corresponding generator, u is still random.

Each of the nine characters, as has been said, depends on two generators, compounded as just described. There are, however, not 18 generators altogether but only ten, connected as shown in Fig. 4. The mathematical description of this interconnexion is complicated by the fact that seven of the ten output digits are in the scale of ten, one in the

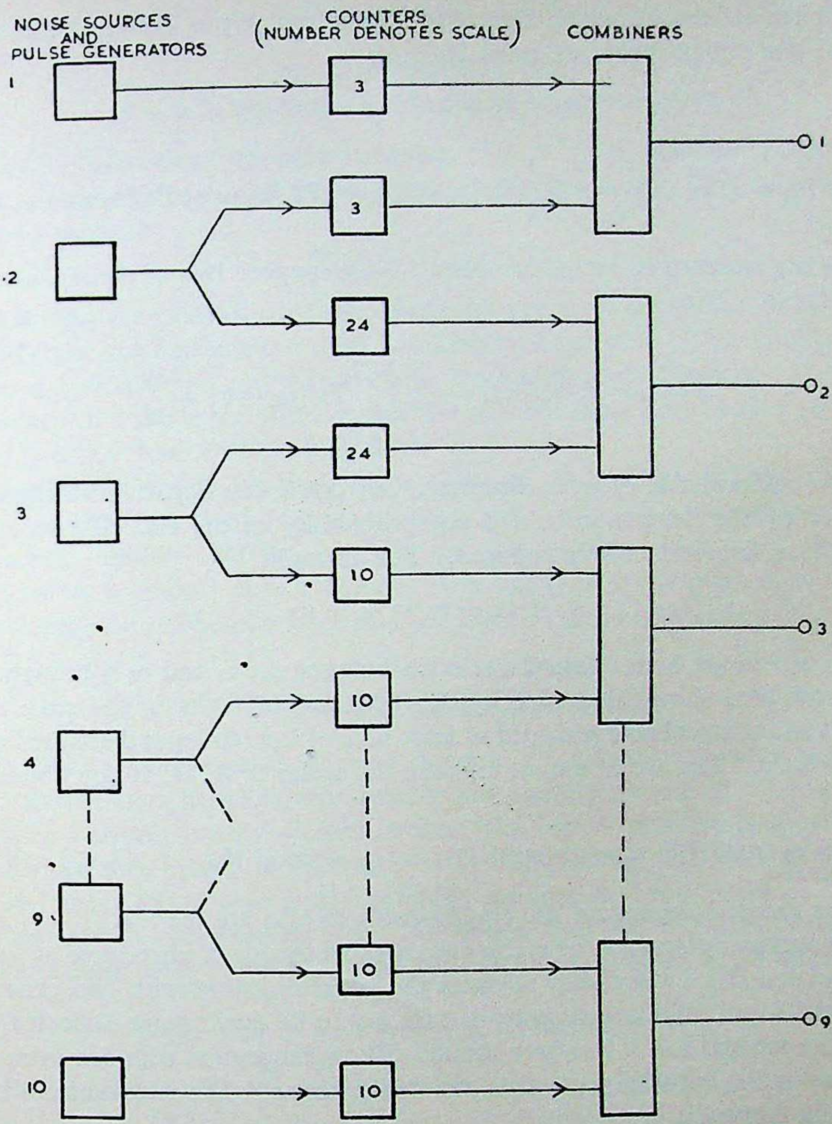

FIG. 4.—Generation of nine random digits from ten random noise sources. (Since July, 1958 the scale-of-three counters have been replaced by scale-of-four counters).

scale of 24, and one in a scale which is currently four, but may change. The simplest way is to imagine that all counters and combiners operate in a scale which is the lowest common multiple, namely 120. This gives nine output digits, v_1 to v_9, related to ten independent input digits, u_1 to u_{10}, by

$$\begin{aligned} v_1 &\equiv u_1 - u_2 \pmod n \\ v_2 &\equiv u_2 - u_3 \pmod n \\ &\vdots \\ v_9 &= u_9 - u_{10} \pmod n \end{aligned} \tag{14}$$

where n is 120.

Consider two of the v's which have a u in common, say v_1 and v_2. For these to be independent and equiprobable we must have:

$$\Pr\{v_1 = i,\ v_2 = j\} = n^{-2} \qquad 0 \leqslant i, j \leqslant n-1. \tag{15}$$

Now

$$\Pr\{v_1 = i,\ v_2 = j\} = \sum_{r=0}^{n-1} \Pr\{u_1 = i + r\} \Pr\{u_2 = r\} \Pr\{u_3 = r - j\} \tag{16}$$

since the u's are assumed to be independent. Now suppose two of the u's, u_1 and u_3 say, are equiprobable. Then

$$\Pr\{v_1 = i,\ v_2 = j\} = \frac{1}{n^2} \sum_{r=0}^{n-1} \Pr\{u_2 = r\} = \frac{1}{n^2}. \tag{17}$$

whether u_2 is equiprobable or not. Similarly, any one u can depart from equiprobability without effecting the independence and equiprobability of the v's. If two or more go wrong, however, dependence is introduced. For example,

$$v_1 + v_2 + v_3 = u_1 - u_4 \tag{18}$$

so that if u_1 and u_4 are both affected a relation between v_1, v_2 and v_3 is brought about.

It has now been shown that nine hypothetical output digits in the scale of 120 are independent and equiprobable provided at least nine of the ten input digits are independent and equiprobable. The actual output digits in the scales of 4, 24, 10 are obtained by the relations

$$v_1' \equiv v_1 \pmod 4; \quad v_2' \equiv v_2 \pmod{24}; \quad v_j' \equiv v_j \pmod{10} \qquad (3 \leqslant j \leqslant 9), \tag{19}$$

so that if the v's are independent and equiprobable so also are the v''s.

The arrangement discussed, whereby one extra generator is introduced in such a way that any one can fail, immediately suggests the possibility that with two extra, any two could fail, and so on. Such arrangements turn out to be much more difficult than would appear and no method has in fact been found. The arrangement with one extra generator works for any radix, but with more than one this is not so. The situation is discussed in more detail in Appendix 4.

5. Miscellanea

5.1. *Estimating the Duration of Printing*

In Section 2 it was explained how numbers are generated within ERNIE, of which some are rejected and some printed. Again, some of those printed are rejected, the remainder forming the prize list. This process was described as going on indefinitely until the prize list reached the required size.

In practice, a draw lasts several days and a good deal of staff work is involved in planning the operation. In particular, an estimate is made of the quantity of bond numbers to be printed to yield the required number of prizes and of the time taken to print them.

The prize list is a sample "without replacement" but the sample is such a small fraction of the population that the binomial, rather than the hypergeometric, distribution can be

taken to apply in estimating the quantity of bond numbers to be printed, the probability

$$p_1 = \frac{\text{number of eligible bond numbers}}{\text{number of printable bond numbers}}$$

being known (but changing from draw to draw). Let E_r be the number of eligible numbers found in r printed numbers, and let the number of prizes be k. Then planning is based on a number r such that

$$\Pr\{E_r < k\} \approx 0 \cdot 01;$$

i.e. extra numbers are printed, over and above the average k/p_1, such that the probability of not getting the required number of prizes is about 1 per cent. The number r is readily found from a normal-error approximation to the binomial distribution. In practice, the figures are such that the extra printing over the average takes about half-an-hour, and the total printing takes about two working days.

An estimate, deliberately on the generous side, of the quantity of bond numbers to be printed having thus been obtained, the printing time, which is also subject to chance, is then estimated. Within ERNIE, numbers are generated at about six a second until a printable number is found; this number is then stored until the printing of the previous number is completed. The operation is a sequence of Bernouilli trials with probability

$$p_2 = \frac{\text{number of printable numbers}}{\text{number of generatable numbers}}$$

of success; like p_1, p_2 changes from draw to draw. If a number is found at the n^{th} trial, the search time is thus basically $n/6$ seconds, but about $\frac{3}{8}$ second must be added for the time taken to transmit control signals between the Printer and the Number Generator. For $n \leqslant 13$, the search time is less than the normal printing time of $2 \cdot 56$ seconds; for $n > 13$, the Printer has to wait until a printable number has been found and the effective printing time is equal to the search time. Allowance must also be made for the time taken for five blank lines after every 100 numbers. Planning is based on an average printing time of

$$2 \cdot 569 + (1 - p_2)^{13} (0 \cdot 165/p_2 - 0 \cdot 0675) \text{ seconds,} \tag{20}$$

the variance associated with this being small enough to be ignored.

5.2. *Statistical Analyses of the Results of a Draw*

Some sort of statistical analysis of the results of each draw suggests itself, the object being to check that the prize list is a random sample from the population of eligible numbers. The usual way of doing this is to compare certain properties of the sample with corresponding properties of the population, but this is not practicable in the present case, since the properties of the population are not readily available.

Since it is impracticable to draw conclusions from analysis of the prize list, other analyses have been considered, and one in particular is regularly carried out. It uses data which are available from the records normally kept by the Draw Controller.

The list of printed numbers is handled in sheets of 50 consecutive numbers and a record is kept of the number of prize-winning numbers found in each sheet. If the whole process operates correctly, this number is a random variable with known distribution, strictly, a

hypergeometric distribution (because the sampling is "without replacement") but in practice near enough binomial with

$$p = \frac{\text{number of eligible bond numbers}}{\text{number of printable bond numbers}}$$

(i.e. the p_1 of Section 5.1). Each sheet represents a sample of 50 with this probability of success and about 300 sheets are usually dealt with. The results of such an analysis are shown in Table 3.

TABLE 3

Premium Savings Bonds—October, 1957 *Draw*

(Analysis of prize winners in Master list sheets)

(1) *Prize Winners per Sheet of* 50	(2) *Observed Number of Sheets*		(3) *Expected Number of Sheets*	(4) *Chi-squared*
⩽10	0	9	10·3	0·2
11	1			
16	2			
17	2			
18	4			
19	7		8·2	0·2
20	13		12·5	0·0
21	12		17·5	1·7
22	24		22·6	0·1
23	39		27·0	5·4
24	28		29·8	0·1
25	41		30·4	3·7
26	32		28·7	0·4
27	14		25·1	4·9
28	16		20·3	0·9
29	14		15·1	0·1
30	12		10·4	0·2
31	4		6·6	1·0
32	2	7	7·7	0·1
33	2			
35	3			
⩾36	0			
Total	272		272·2	18·9 (14 d.f.)

Col. (3) was calculated from binomial distribution with

$$p = \frac{\text{number of eligible bond units}}{\text{number of printable bond units}} = \frac{64{,}966{,}407}{131{,}100{,}000}.$$

The chi-squared test does not lead to any doubts that, (1) ERNIE picked a random sample, and (2) the subsequent classification was correctly done.

There was some confusion and misunderstanding among the public about what was to be expected in the prize lists; there was a tendency to assume that fairness implied equal proportions in every respect. For example, there were complaints that bond numbers of denomination "Z" (which are units in £500 multiple bonds) were having more than their fair share of prizes. Now the proportion of bond numbers in any denomination in the population of eligible bond numbers in any one month depends on the vagaries of buying and encashing by the public. It happened that, in the early draws, about one quarter of the eligible bond numbers were of denomination "Z". It will consequently not be surprising to a statistician that about one quarter of the prizes fell to such bonds.

5.3. *Routine Testing of ERNIE*

As with any other electronic equipment, checks of performance can be, and are, carried out by measurement of voltages, currents, waveforms, and so on, at certain points. There are two special forms of testing in addition to these.

The sets of nine random digits generated by the assembly of generators and combiners are handled by the rest of the equipment by a fixed programme by which certain sets of digits lead to no external action, others to the printing of bond numbers, plus a serial number, in a certain layout. To test the operation of this programme, a run is made in which the first four of the nine digits are derived from a cyclic tester which systematically generates all possible combinations of the four digits. Such a run, while testing the general operation of the equipment, tests in particular the operation of the "redundancy table", the section which, by examination of the first four characters of a generated bond number, accepts or rejects the number for printing. The acceptable numbers change from month to month with consequent changes in the redundancy table.

The operation of the random digit generators is the subject of separate tests. The conventional tests of Section 3 might form the basis of routine tests, but a better method is to obtain histograms like that of Fig. 3. These are not only easier to obtain, but are much more sensitive. Variations in the performance of generators which would have a negligible effect on the random digits can be detected by these histograms, thus allowing the anticipation of more serious trouble. In practice, generators of this type (which have been used for many years in the Artificial Traffic Machine (Broadhurst and Harmston, 1950; 1953)) are very reliable.

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APPENDIX 1

A Comparison of Two Basic Methods of Producing Random Digits

In Method 1, a cyclic counter with n positions is advanced one step by each pulse in a Poisson stream of pulses. The counter is inspected at regular intervals such that the

average number of pulses per interval is A. The distribution of the number of pulses in an interval is thus Poissonian, and v, the quantity defined in Section 3.2 as a measure of goodness (it is zero for a perfect generator), is proportional to e^{-A}, as shown in Appendix 3.

In Method 2, a cyclic counter with n positions is advanced one step by pulses occurring at regular intervals. The counter is inspected at random instants, the intervals between inspections being exponentially distributed with average duration A times the pulse interval, so that A is also the average number of pulses in an inspection interval. Consider an inspection interval starting at a time which is a fraction α (of a pulse interval) later than the previous pulse. Then the probability of obtaining M pulses in that interval can be shown to be

$$P_0 = 1 - e^{-(1-\alpha)/A}$$

$$P_M = e^{-(M-\alpha)/A}(1 - e^{-1/A}) \qquad 1 \leqslant M < \infty.$$

From this we obtain

$$F(x, \alpha) = \sum_M P_M e^{2\pi i M x}.$$

In practice, α is not fixed, but has a rectangular distribution between 0 and 1; so we derive $F(x)$, the average value of $F(x, \alpha)$, as

$$F(x) = 1 - A(1 - e^{-1/A}) + \frac{A(1 - e^{-1/A})^2 e^{2\pi i x}}{1 - e^{2\pi i x - 1/A}}.$$

As shown in Appendix 3,

$$v \approx \frac{2}{n} \mid F(1/n) \mid^2.$$

For large A, we find v is proportional to $1/A^2$.

Hence, for the same value of A, Method 1 is very much superior to Method 2.

Appendix 2

Optimum Average Pulse Rate

If the random intervals between pulses in a pulse train have a distribution with mean μ and variance σ^2, and if intervals are independent, the number of pulses in time t is asymptotically normally distributed (Feller, 1950, p. 249) with variance $\sigma^2 t/\mu^3$. Appendix 3 shows that this variance needs to be as large as possible.

Pulse interval distributions with mean $T + T_0$, T_0 being the dead time, and variance RT^2 are considered. (For the truncated exponential distribution, $R = 1$). For R and T_0 fixed and T variable, it is readily shown that the maximum variance is obtained with $T = 2T_0$, i.e. $\mu = 3T_0$.

Appendix 3

Relations Between the M and m Distributions

Lemma.—Let $F(x)$ be a periodic function, period 1, and let its Fourier coefficients be given by

$$a_r = \int_0^1 F(x)\, e^{-2\pi i r x}\, dx \qquad (1)$$

so that

$$F(x) = \sum_{r=-\infty}^{\infty} a_r e^{2\pi i r x}. \tag{2}$$

Then

$$E_{mn} = \frac{1}{n} \sum_{s=0}^{n-1} F\left(\frac{s}{n}\right) e^{-2\pi i m s/n} = \sum_{k=-\infty}^{\infty} a_{m+kn} \qquad 0 \leqslant m \leqslant n-1. \tag{3}$$

Proof: From (1) and (2)

$$F(x) = \sum_r \int_0^1 F(u)\, e^{2\pi i r(x-u)}\, du.$$

Hence

$$E_{mn} = \frac{1}{n} \sum_{s,r} \int_0^1 F(u)\, e^{2\pi i r(s/n-u)-2\pi i m s/n}\, du$$

$$= \sum_{r=-\infty}^{\infty} \int_0^1 F(u)\, e^{-2\pi i r u}\, du \,.\, \frac{1}{n} \sum_{s=0}^{n-1} e^{2is(\pi r-m)/n}.$$

Now

$$\sum_{s=0}^{n-1} e^{2\pi i s h/n} = \begin{cases} 0 & h \neq kn \\ n & h = kn \end{cases} \qquad h,\, k \text{ integral}. \tag{4}$$

Hence

$$E_{mn} = \sum_{r=m+kn} \int_0^1 F(u)\, e^{-2\pi i r u}\, du$$

$$= \sum_{k=-\infty}^{\infty} a_{m+kn},$$

which is the required result.

Let p_M, $0 \leqslant M < \infty$, and P_m, $0 \leqslant m \leqslant n-1$, be the ordinates of the M and m probability distributions. Then

$$P_m = \sum_0^{\infty} p_{m+kn}. \tag{5}$$

Whence, by the lemma

$$P_m = \frac{1}{n} \sum_{s=0}^{n-1} F\left(\frac{s}{n}\right) e^{-2\pi i m s/n} \tag{6}$$

where

$$F(x) = \sum_{M=0}^{\infty} p_M\, e^{2\pi i M x}. \tag{7}$$

The average value of P_m is $1/n$. Let

$$P_m = \frac{1}{n} + \epsilon_m. \tag{8}$$

Then

$$\epsilon_m = \frac{1}{n} \sum_{s=1}^{n-1} F\left(\frac{s}{n}\right) e^{-2\pi i m s/n}. \tag{9}$$

A measure of the departure of the m's from equiprobability is

$$v = \sum_{m=0}^{n-1} \epsilon_m{}^2 \tag{10}$$

$$= \frac{1}{n^2} \sum_{m=0}^{n-1} \sum_{r=1}^{n-1} \sum_{s=1}^{n-1} F\left(\frac{r}{n}\right) F\left(\frac{s}{n}\right) e^{-2\pi i m(r+s)/n}. \tag{11}$$

Equation (4) of the lemma shows that, in the summation over m of the exponential factor, only values of r and s for which $r + s = n$ contribute; whence

$$v = \frac{1}{n} \sum_{r=1}^{n-1} F\left(\frac{r}{n}\right) F\left(\frac{n-r}{n}\right) \tag{12}$$

$$= \frac{1}{n} \sum_{r-1}^{r=1} \left| F\left(\frac{r}{n}\right) \right|^2 \tag{13}$$

since, by equation (2) of the lemma, $F(x)$ and $F(1-x)$ are conjugate complex. In practice, the terms for $r = 1$ and $r = n - 1$ dominate the others; these two terms are of course equal. Hence

$$v \approx \frac{2}{n} \left| F\left(\frac{1}{n}\right) \right|^2. \tag{14}$$

Examples:

Poisson distribution:

$$F(x) = \exp\{-A(1 - e^{2\pi i x})\} \qquad v \approx \frac{2}{n} \exp\left(-4A \sin^2 \frac{\pi}{n}\right).$$

Pascal distribution:

$$F(x) = \{a/(a + 1 - e^{2\pi i x})\}^b \qquad v \approx \frac{2}{n} \left\{1 + \frac{4(1+a)}{a^2} \sin^2 \frac{\pi}{n}\right\}^{-b}.$$

When the M-distribution can be approximated by a normal distribution:

$$F(x) \approx \exp(-2\pi\mu_2 x^2 + 2\pi i \mu_1 x),$$

$$v \approx \frac{2}{n} \exp(-4\pi^2\mu_2/n^2).$$

APPENDIX 4

Compounding of Digits

Consider r output digits $y_1, y_2 \ldots y_r$ related to s input digits $x_1, x_2 \ldots x_s$ by the relations

$$y_i \equiv \sum_{j=0}^{s} a_{ij} x_j \pmod n \qquad 1 \leqslant i \leqslant r.$$

The a_{ij} are integral and $r \leqslant s$.

The problem is, given that the x_j are independent equiprobable random digits, what are the restrictions on the a_{ij} in order that the y_i are also independent and equiprobable?

It is believed that the following process will allow a given matrix $\| a_{ij} \|$ to be tested, but no proof has been obtained.

From the matrix $\| a_{ij} \|$,

$$\binom{s}{r} = k$$

square matrices can be obtained by deleting $s - r$ columns. Let the determinants of these be $D_1, D_2 \ \ldots \ D_k$. Then the y_i are independent and equiprobable if

$$(D_1, D_2, \ \ldots \ , D_k, n) = 1 \tag{1}$$

where (,) means Greatest Common Divisor. Some, but not all, of the D's may be zero.

If one or more (up to $s - r$) of the x's cease to be random, becoming constant say, they can be taken over to the left-hand side and assimilated in the y's. The process is then applied to the reduced matrix $\| a_{ij} \|$, in which the columns corresponding to the non-random x's have been deleted. Hence, if

$$(D_i, n) = \pm 1 \qquad 1 \leqslant i \leqslant k \tag{2}$$

all such reduced matrices satisfy the criterion (1) and any x's (up to $s - r$ in number) can cease to be random.

In the compounding scheme of Section 4,

$$\| a_{ij} \| = \begin{Vmatrix} 1 & -1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & 0 & \ldots & 0 & 0 \\ \cdot & & & & & & \\ \cdot & & & & & & \\ \cdot & & & & & & \\ 0 & 0 & 0 & 0 & \ldots & -1 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 1 & -1 \end{Vmatrix}$$

with $s = r + 1$. The determinants are equal to ± 1 and hence criterion (2) is satisfied for all n. Thus any one x can cease to be random.

No method of extending this scheme to allow more than one x to become non-random, independent of n, has been found. Consider for example five x's and three y's with

$$\| a_{ij} \| = \begin{Vmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{Vmatrix}.$$

All but two of the ten determinants equal ± 1. The other two are ± 2. Thus when x_2 and x_5 cease to be random, and are assimilated in the y's

$$\begin{aligned} y_1 &\equiv x_1 - x_3 && \pmod{n} \\ y_2 &\equiv x_3 - x_4 && \pmod{n} \\ y_3 &\equiv x_3 + x_4 && \pmod{n} \end{aligned}$$

for which $D = 2$. The relation

$$y_2 + y_3 \equiv 2x_3 \qquad (\text{mod } n)$$

holds, whence, if n is even, so also is $y_2 + y_3$ and thus y_2 and y_3 are not independent. But if n is odd, this relation does not prevent all pairs of values for y_2 and y_3 from being equally likely. Thus the scheme is satisfactory for odd n but not for even n.

Discussion on Mr. Thomson's Paper

Mr. A. Stuart: Mr. Thomson has given us a very clear picture of ERNIE in its statistical aspects. A major disability of electronic randomization devices is the absence of a physical, visible method of selection which the customer can understand. It is not surprising that despite its friendly name (no doubt intended to be disarming) ERNIE arouses suspicion in the uninformed and unfortunate. These suspicions are clearly unfounded, and it is of interest to recall for contrast that in a paper in this *Journal* in 1955 Mr. J. H. West showed that in the Southern Rhodesian government lottery, which employs an elaborate physical and public randomizing mechanism, the results of tests of randomness are unsatisfactory. Seeing should not always be believing. After Mr. Thomson's paper, I for one am certain that ERNIE has nothing personal against my Bond numbers: it just misses them monthly by chance.

I am sworn not to make puns on the word ERNIE, so I turn to more serious matters. A good deal of Mr. Thomson's paper is taken up with problems of testing for randomness. I do not think that the practice of applying a battery of tests, whose *joint* performance characteristics are unknown, is a good one, but there is no reasonable alternative at the moment. There are only two theoretical points I should like to raise.

In his section 3.2 Mr. Thomson defines a parameter, which I shall call V, whose departure from zero is an indication of non-randomness. Mr. Thomson obtains an estimate of it, v, which for $n = 10$ is of order 10^{-70}. Now this is as near zero as most of us are likely to require, but Mr. Thomson then goes on to ask the following question (although he phrases it differently, and he discusses the case $n = 24$, whereas most of his counters have $n = 10$): if a chi-squared goodness-of-fit test is carried out at the 1 per cent. significance level, what would be its power to detect a true value, V, of order 10^{-70}? Since the test statistic has a non-central chi-squared distribution when V is not zero, with non-centrality parameter $\lambda = NnV = N \times 10^{-69}$, he naturally requires a sample size N of order 10^{69} to achieve reasonable power. At the present rate N is of the order 10^6 per year, and from this Mr. Thomson concludes that the value of v is satisfactorily small. But, so far as I can see, it establishes only that Mr. Thomson will have to wait about 10^{63} years at the present rate to get a satisfactory test. For the value of N available the test is quite insensitive, with power very close to its significance level (and possibly even below it, since the test is not unbiased so far as I know). Thus we must rely, after all, on the subjective impressiveness of 10^{-70} as an approximation to zero.

My second point is concerned with the interpretation of Table 2 in section 3.3.2. Mr. Thomson suspects the high P-value of 0·995 obtained for the Grand Gap Test, and suggests a possible explanation. He may be right, but it seems worth while saying that *in itself* this P-value is no reason for suspicion: the critical region of the chi-squared goodness-of-fit test is in the upper tail alone, and I am at a loss to understand why so much is made of the importance of not having "too good" a fit to the theoretical distribution. In Mr. Thomson's case there is no question of errors in computation or of "cheating" to make chi-squared small; why, therefore, is he worried about a high P? Let me ask him what he would have done if in section 3.2 he had found his estimated value to be $v = 0$ exactly. Would he reject that hypothesis on the evidence of excellent agreement? And what is the logical difference between the two cases?

Having said this, I must add that there does seem to be some reason to suspect the

Gap Test results. If the 30 Gap Test values in the main body of Table 1 are arranged in a table of 10 per cent. ranges like that below Table 1, we can calculate chi-squared for that table against the expected uniform distribution. I get the result chi-squared $= 18$ for 9 degrees of freedom, whence $\cdot 02 < P < \cdot 05$. P is not very small, but might repay investigation, especially in the light of Mr. Thomson's own suggestions as to the cause.

Finally, I should like to ask Mr. Thomson about a technical point. When the bond scheme was introduced, I wrote an article in *The Observer* in which I explained an interesting feature of the scheme. On being purchased every bond waits for six months, accumulating interest the while, and then enters its first draw, contributing six months' interest to the prize fund. At that same draw, older bonds contributing only one month's interest (accumulated since the previous draw) have exactly the same chance of winning prizes. On the face of it this is unequitable. (In fact a titled gentleman, who wrote to me after my article appeared, refused at first to believe that a Conservative administration could be responsible for such a flagrant piece of levelling!) My question is this: Would it have been technically feasible to give the new bonds at each draw 6 times the probability of selection that the older bonds have? I can think of no way of doing this. However, it would be simple to give new bond-holders a "fair" expected return at their first draw by multiplying any prizes they won by 6 (and adjusting the total prize-money accordingly).

Mr. Thomson has given us an attractive and businesslike paper, and I have great pleasure in proposing a vote of thanks to him.

Dr. E. S. Page: A description of the tests of randomness applied to the digits generated by ERNIE and the results of these tests occupy a substantial portion of Mr. Thomson's paper. It is difficult for me to criticise the selection of tests; these tests are nearly the same as those I have used in testing sets of sampling digits and, in addition, one set of digits produced by ERNIE has brought me a little personal satisfaction. In fact other sets of sampling numbers which have satisfied these tests have given similar satisfaction in other fields. Instead I should like to consider ERNIE as a special purpose computer of a type which might be attached to a general purpose automatic digital computer in order to provide a source of random digits for problems in artificial sampling, operational research and for Monte Carlo methods. Some computers have been fitted with sources of random digits using a physical process similar to that in ERNIE, but such equipment is now the exception rather than the rule. During the last few years several reasons for and against random number generator equipment have been given and their force has changed as advances in equipment and methods have appeared. The first reason against installing separate Ernie-type equipment was that yet another piece of electronic equipment would need service and testing; in the early days of computers—even now little more than ten years ago—it was difficult enough to keep the basic computer working error-free for a long period without adding to the tasks of the engineers. Today the strength of this argument is much less; now computers are reliable and we expect them to work error-free for days and do not hope in vain for a good half-hour of machine time. But as computers have grown more reliable the amount of ancillary equipment for them has multiplied; magnetic tape units, fast printers, graphical output devices, multiple input and output channels have all been attached and the maintenance task has grown again. It may be that the addition of one piece of special purpose equipment would not strain the maintenance staff too much, but at some point in the stream of acquisitions extra staff will have to be sought.

The speed of generation of random numbers may be too slow for some fast computers; ERNIE generates about six nine-digit numbers per second and although this speed can be improved considerably the rate may still be less than computers with operation times measured in microseconds require. On the other hand, those with slow computers may prefer to spend any available money on general purpose equipment designed to speed the whole operation of the machine instead of buying an Ernie-type attachment.

Apart from the maintenance problem there is one of testing the digits produced by the generator. In computer operation this means either recording the digits or testing them

by counting or by a control calculation as they are produced. Recording internally might tax the storage capacity of the machine and external recording might be slow, while the alternative imposes an additional programming task and a possible storage problem. The force of these objections varies from machine to machine according to the facilities available.

A possibility for computers is to record the digits from an Ernie-type generator directly on to a fast input-cum-auxiliary-storage medium like magnetic tape. If this process is performed once, copies of the tapes can be prepared on the computer for other installations. The digits are then available to the computer at the usual rate of reading tape. Both punched cards and paper tape have been used to store random digits but the reading speeds of these media may be less than the speed of using the digits. However, digits so recorded can be tested once and for all.

An alternative method for computers is to generate sampling numbers deterministically; the problems are to find suitable methods of generation and to treat the philosophical difficulties that emerge. The first of these problems seems solved and although the last are still being discussed more or less amicably satisfactory practical results are being obtained. Such methods seem quite inapplicable to the requirements imposed on Mr. Thomson. He has had to show us how a fair and random procedure has been adopted for a special and exacting application and I am pleased to second a vote of thanks to him for his interesting paper.

The vote of thanks was put to the meeting and carried unanimously.

Mr. Berners-Lee: One thing which is becoming quite clear is that there is a big demand for very large quantities of random numbers for simulation work. We have here a solution to the problem which is to take digits from ERNIE and put them on magnetic tape; it seems to be an acceptable solution. I was interested in Dr. Page's remarks on deterministic methods. People have had trouble with deterministic methods; in one case they ran into very serious trouble. Determinate random number generation is important in some rather odd ways. For example, we have recently come across a sorting process which uses a large store in a very efficient way. It will be very useful for large computers in commercial processes. It has one disadvantage; if the data contain long strings of consecutive keys then this particular process, unlike most commercial processes, takes considerably longer to do the sorting, and to make it efficient it is necessary to randomize the data first. Given a compact pseudo-random number generation process, however, the method becomes a practicable one.

On the subject of built-in random number generation, there was a built-in generator in the Mark I computer at Manchester but this did not generate numbers very well and was abandoned.

The tests which Mr. Thomson has used appear to have been the right sort to test a pseudo-random number generation process for ordinary purposes.

As an interesting sidelight on this G. E. Felton has recently computed π to 10,017 places after the decimal point. This is a world record at the moment and one question is whether these digits are in fact random. It is suggested that they may be, within the meaning of the act.

Added in writing to a question concerning Mr. Felton's calculations for π:

The series of 10,017 digits referred to is the correct series, re-calculated but not yet re-published.

Dr. I. J. Good: Holders of Premium Bonds will be glad to hear that the conjecture in Appendix 4 is correct. A fairly short proof can I think be obtained with the help of the multidimensional discrete Fourier transform, which has already had applications in mathematical statistics. (See S. Vajda, "The algebraic analysis of contingency tables", *J. R. Statist. Soc.*, A, **106** (1943), 333–342; I. J. Good, "Random motion on a finite Abelian

group", *Proc. Camb. Phil. Soc.*, **47** (1951), 756–762, and **48** (1952), 368; "The serial test for sampling numbers and other tests for randomness", *Proc. Camb. Phil. Soc.*, **49** (1953), 276–284; "The interaction algorithm and practical Fourier analysis", *J. R. Statist. Soc.*, B, **20** (1958), 361–372.)

I shall replace the r and s of Appendix 4 by upper case R and S, since I am used to using lower case r and s in a different sense in this work. Consider a distribution $p_{\mathbf{r}}$, where $\mathbf{r}$ is the R-dimensional vector

$$\mathbf{r} = (r_1, r_2, \ \ldots \ , r_R),$$

in which each component runs from 0 to $n - 1$ through the integers. The R-dimensional mod n discrete Fourier transform of this distribution is defined as

$$p_{\mathbf{s}}^* = \sum_{\mathbf{r}} p_{\mathbf{r}} \, \omega^{\mathbf{r}.\mathbf{s}}$$

where $\omega = \exp(2\pi i/n)$, and $\mathbf{r}.\mathbf{s} = r_1s_1 + \ \ldots \ + r_Rs_R$. There are formal analogies of the Fourier inversion formula, Parseval's formula; and also of the rule for adding independent random variables, namely that it is equivalent to the multiplication of the components of the Fourier transform.

If x_1 is equiprobable, and if $\mathbf{b} = (b_1, \ \ldots \ , b_R)$ is a vector whose components are integers, then the distribution of $(b_1x_1, \ \ldots \ , b_Rx_1) = \mathbf{b}x_1$ is given by

$$p_{\mathbf{r}} = \begin{cases} 1/n \text{ if } \mathbf{r} \text{ is an integer times } \mathbf{b}, \\ 0 \text{ otherwise.} \end{cases}$$

It follows at once that $p_{\mathbf{s}}^* = 1$ if $\mathbf{b}.\mathbf{s} \equiv 0 \pmod{n}$, and is 0 otherwise.

Suppose $\mathbf{y} = (y_1, \ \ldots \ , y_R)$, where $y_i = \sum_{j=1}^{S} a_{ij}x_j \pmod{n}$, and $x_1, \ \ldots \ , x_S$ are independent and each is equiprobable, so that $\mathbf{y}$ is the sum of S independent vectors

$$\mathbf{y} = \begin{bmatrix} a_{11}x_1 \\ a_{21}x_1 \\ \cdot \\ \cdot \\ \cdot \\ a_{R1}x_1 \end{bmatrix} + \ \ldots \ + \begin{bmatrix} a_{1S}x_S \\ a_{2S}x_S \\ \cdot \\ \cdot \\ \cdot \\ a_{RS}x_S \end{bmatrix}.$$

It follows that the Fourier transform of the distribution of $\mathbf{y}$, being the product of the transforms of the S separate vectors, is 1 when $A \,.\, \mathbf{s}$ is the zero vector (where $A = \{a_{ij}\}$), and is 0 otherwise. But it is easily seen that the components of an R-dimensional vector are equiprobable and independent if and only if the Fourier transform is so to speak the Dirac function, i.e. vanishes except when $\mathbf{s} = \mathbf{0}$ and is then 1. So a necessary and sufficient condition for $y_1, y_2, \ \ldots \ , y_R$ to be equiprobable and independent is that

$$A \,.\, \mathbf{s} \not\equiv \mathbf{0} \text{ whenever } \mathbf{s} \not\equiv \mathbf{0}.$$

If n were a prime number the author's conjecture would now follow immediately since the usual theorems about determinants and ranks of matrices could be used directly. Since n may be composite there is a little more to be proved.

We are given that $(D_1, D_2, \ \ldots \ , D_k, n) = 1$. Assume $A \,.\, \mathbf{s} \equiv \mathbf{0} \pmod{n}$. We require to prove that $\mathbf{s} = \mathbf{0} \pmod{n}$.

If p is a prime number that divides n, we have $(D_1, \ \ldots \ , D_k, p) = 1$, so that the rank of A mod p is R and therefore the condition $A \,.\, \mathbf{s} \equiv \mathbf{0} \pmod{p}$ implies that $\mathbf{s} \equiv \mathbf{0} \pmod{p}$, i.e. $\mathbf{s}$ is of the form $\mathbf{s} = p\mathbf{s}'$. Therefore $A \,.\, \mathbf{s}' \equiv \mathbf{0} \pmod{n/p}$. In this manner

we may successively divide away the factors of n and we finish up with the result that s must be **0**, as required.

Concerning the serial test, if

$$\chi^2 = \frac{10}{N} \Sigma \left(O_r - \frac{N}{10}\right)^2$$

and

$$\psi^2 = \frac{100}{N} \Sigma \left(O_{rs} - \frac{N}{100}\right)^2,$$

then $\psi^2 - \chi^2$ and $\psi^2 - 2\chi^2$ both have asymptotically chi-squared distributions, with 90 and 81 degrees of freedom respectively. The latter is essentially independent of χ^2 and therefore, if χ^2 is one statistic used, it is better to use $\psi^2 - 2\chi^2$ than $\psi^2 - \chi^2$ for the serial test.

A fairly simple justification can be given for the parameter v, defined in equation (5). Suppose we have a true non-null hypothesis, H, that the digits are produced randomly, but not perfectly randomly, the true probabilities being P_m ($m = 0, 1, \ldots, n-1$). If the observed frequencies are O_m, then the weight of evidence in favour of H against the null hypothesis of perfect randomness, is

$$\sum_{m=0}^{n-1} O_m \log (nP_m),$$

in natural units. (The weight of evidence, or log-factor, is defined as the logarithm of the Bayes factor on the odds of H, which for simple statistical hypotheses is equal to the logarithm of the likelihood ratio.) The expected weight of evidence is

$$N \sum_{m=0}^{n-1} P_m \log (nP_m),$$

so that the expected weight of evidence per digit is simply related to the entropy of the distribution. (Cf. Good, *Probability and the Weighing of Evidence*, 1950, London and New York, p. 75, where however the last sentence contains an error.) When the P_m's are close to $1/n$ we see that the expected weight of evidence is approximately $\frac{1}{2}Nnv$. When this is less than say 1 the total evidence that the sequence is not perfectly random is negligible.

Mr. A. J. Mayne: I should be interested and grateful if Mr. Thomson, when replying to the discussion on his paper, could give a brief specification of the stochastic process producing the electrical noise in devices like ERNIE or, at least, give a short list of relevant references to the literature on stochastic processes and random noise.

It is possible to suggest examples of disturbances which if superposed on the random noise would destroy the randomness of the device. To take an extreme example (which I should imagine is far worse than anything likely to arise in practice), a spiked periodic waveform added to the noise could produce most of the deviations from mean level beyond a certain size and thus lead to a more nearly regular output of pulses supposed to be random. I do not know how plausible it is to suppose that such disturbances might occur in practice but it does at least suggest that more general cases of random noise should be investigated mathematically, in addition to the special case which leads to a "Poisson stream" and which was used by Mr. Thomson.

It seems that where such large disturbances to the noise occur their effects could be greatly reduced by random, instead of regular, inspection of a cyclical counter advanced one step by each pulse. It is necessary to ensure that such a procedure does not reduce the quality of the randomness too much when there is no disturbance of the noise.

If there is a "Poisson stream" of pulses this leads to the following mathematical results. Let the length t of the interval between two successive inspections of the counter have

cumulative distribution function $G(t)$ and, if it exists, probability density function $f(t)$. Then, in Mr. Thomson's notation,

$$p_M = \int_0^\infty (M!)^{-1} (At)^M e^{-At} \, dG(t),$$

which is equal to

$$\int_0^\infty (M!)^{-1} (At)^M e^{-At} f(t) \, dt,$$

if $f(t)$ exists. Then

$$F(x) = \int_0^\infty \exp \{At (e^{2\pi i x} - 1)\} \, dG(t),$$

which is equal to

$$\int_0^\infty \exp \{At(e^{2\pi i x} - 1)\} f(t) \, dt,$$

if $f(t)$ exists. Finally calculate

$$v \approx \frac{2}{n} \left| F\left(\frac{1}{n}\right) \right|^2,$$

which is Mr. Thomson's measure of the goodness of the randomizer.

If $G(t) = 0$ for $t < 1$, 1 for $t \geqslant 1$, which is the case of regular inspection, then $v \approx 2n^{-1}E$, where $E = \exp \{-4 A \sin^2 (\pi/n)\}$, as obtained by Mr. Thomson.

If $f(t) = Be^{-Bt}$ for $t \geqslant 0$, 0 for $t < 0$, then $v \approx 2B^2 n^{-1} \{B^2 + 4A(A + B) \sin^2 (\pi/n)\}^{-1}$, so that the quality of randomness is much worse than for regular inspection.

If $f(t) = 1/(b - a)$ for t in $[a, b]$, 0 for other values of t, then

$$v \approx \frac{E^b - 2E^{\frac{1}{2}(a+b)} \cos [A(b - a) \sin (2\pi/n)] + E^a}{2nA^2(b - a)^2 \sin^2 (\pi/n)},$$

so that the quality of randomness is very roughly the same as for regular inspection; thus this choice of $f(t)$ seems to be satisfactory.

With reference to the combination of several random sources, it is probably well-known that r sources can be combined in such a way that randomness is preserved even if only one of the original sources remains random; if most of the sources are nearly random this process will result in a much closer final approximation to randomness. The simplest process of this type has r stages: stage 1 is the output from source 1; stage i is the result of combining the outputs from stage $(i - 1)$ and source i $(i = 2$ to $r)$; the output of stage r is used as the combined random source. The combination operation can be any "Latin square" operation; for example, sum or difference (mod n) if n different digits are used. The chief disadvantage of a simple combination method like this is that it is more wasteful of digits than a more complicated method would be; its efficiency is only $1/r$.

Dr. K. D. Tocher: In reading Mr. Thomson's paper I had mixed feelings of admiration and envy. Admiration, because Mr. Thomson has been associated with a random number generator which works, whereas I have been associated with one which will not yet work. Envy, because the specification for Mr. Thomson's machine was so much easier than the specification for our machine. ERNIE produces about 50 decimals per second, equivalent to about 200 bits per second, whereas my application needs something like 10,000 bits per second. It is natural to ask what possible application can require random numbers at this rate. The demand arises in connection with experimental work in "black box" theory but this is not the occasion to describe this in any detail.

It is of some interest to see the difficulties caused by requiring a very fast random number generator. In common with ERNIE and all other random number generators, we use the cyclic counting method and since we require binary random numbers we use binary cyclic counters or flip-flops. These, as in ERNIE, are fed from a source of random pulses and sampled at fixed intervals. The rate of sampling is about once every 20 microseconds, the dead time of the pulse generator used in ERNIE. To give comparable performance with ERNIE, we would need 10–20,000 pulses in that 20 microseconds which would imply a very high frequency response of amplifiers. Thus, the machine must be satisfied with a much smaller average number of pulses between sampling periods, and in fact, this is restricted to about 30.

To compensate for this small number of pulses counted, we use a comparative method which I first described in a paper to the Research Section of the Society in 1954 (*J. R. Statist. Soc.*, B, **16** (1954), 39–61). This technique compares the states of two counters which have been set up from statistically identical pulse sources, ignoring those cases in which the counters are in the same state and using the state of one of the counters to supply the digit when they differ. In that same paper I argued the case that there could be no satisfactory conclusion to a process of testing the output of a random number generator to see if the results were random, because at any stage it was always possible to devise yet another test which on any set of data would establish some non-random effect. A better approach, I claimed, was to incorporate sufficient transformation devices which would "guarantee" mathematically that the output was random.

It is an ironical comment that the first time our random number produced any bits, about 10,000 of them, the first thing I did was to count through to see how many ones I had got, i.e. I immediately made a very simple test on their randomness.

It now seems to me that in the process of proving a machine of this kind, tests are valuable and with the enormous output that our present machine will have, this raises its own problems. Our requirement is for 10 digit binary numbers all equally likely and correlation between successive digits would not be particularly harmful. Thus the test is whether the random digits arranged in groups of 10 form numbers which have a uniform distribution over all possible values. This could, of course, be tested by the χ^2 test, but this is a blanket test and odd peculiarities in a distribution would be masked. There is some need for a display of the distribution which enables systematic departures from uniformity to be easily detected. One of my colleagues, J. E. Cannell, has written a computer programme for "Pegasus" which gives such a visual table. The 1,024 possible cells are arranged in a 32×32 square and in each cell the departure of the frequency of occurrences from expected is marked. These are not, however, given as actual numbers, but by symbols for various ranges of departure. The symbols used are arranged so that the larger the departure the more ink is put on the paper and so heavy departures are brought to one's attention as large symbols in a pattern which is often useful in diagnosing the cause of the trouble. However, once the generator is functioning correctly, other methods of testing the machine are required.

I agree with Mr. Thomson that the actual noise generators themselves are not the most likely cause of error; this lies in the control and recording mechanisms. We therefore require an operational test of the machine which proves that *all* its parts (including the generators) are functioning properly. We can achieve this in our generator by altering one or two connections in such a way that the output is a completely deterministic pattern. The possible patterns are all ones, all noughts, or a set of p ones followed by a set of p units ($p = 1(1)10$).

I much admire the procedure used in ERNIE for eliminating the printing of a large number of ineligible bond numbers. I presume the reason why all ineligible bond numbers are not inhibited is due to the prohibitive amount of space required to store the information recording those numbers. However, this technique of rejecting all bond numbers whose leading digits correspond to blocks containing no eligible numbers could be extended. A similar list could be made for trailing numbers listing control numbers which do not occur in any block, and by making a test on the leading digits and then a test on the trailing

digits (which involves a table occupying double the storage used at present). The percentage of ineligible numbers printed would be reduced to 1–2 per cent. This is an application of the method of Zato-coding which is becoming increasingly popular in the field of mechanized literature searching.

Mr. Thomson raises a most interesting mathematical problem concerning the combination of the output of generators to form a robust output. I should like to make a few remarks concerning the extension of this problem when it is required that the output be random with two or more generators being faulty. If we restrict ourselves to linear functions of the basic generators, then there is no loss of generality in assuming that the linear functions are simple sums. For if $a_1x_1 + a_2x_2$ is to be random when x_2 is non-random, we may take x_2 as constant when we require a_1x_1 to be random. This implies that $a_1 = 1 \pmod n$ and similarly $a_2 = 1 \pmod n$. If we can allow p generators to fail, every compound must consist of at least $p + 1$ generators.

Mr. Thomson's proof in his paper that the digits of the outputs will be independent is not complete, since this only shows that the digits are pairwise independent. However, the result stated is, of course, true.

Consider the problem of two generators failing. Each compound must be of at least three basic generators. Consider two compounds $A_1 + A_2 + A_3$ and $B_1 + B_2 + B_3$. If these two sets have two generators in common, say $A_1 = B_1$ and $A_2 = B_2$, then in the event of the generators A_3 and B_3 failing, these two compounds are both shifted versions of $A_1 + A_2$. They are, therefore, perfectly correlated. Thus a necessary restriction is that each compound contains only one element in common with any other compound. The problem has been reduced to a combinatorial one and it is easily seen that a solution involving only two extra generators is only possible when the number of digits is of the form $3m + 1$.

In conclusion, I think the existence of ERNIE settles the controversy about the relative merits of random and pseudo-random numbers and reduces the argument for pseudo-random numbers to one of convenience. How many computer programmers who cheerfully use them would be happy if their fate in the state lottery was decided by the use of pseudo-random numbers?

Dr. J. M. HAMMERSLEY: Some speakers have expressed doubts about pseudo-random numbers. Admittedly, one has to be careful in generating these. Methods of the "mid-squaring" type are usually inadequate, and additive congruential methods such as

$$u_n \equiv u_{n-1} + u_{n-2} \pmod m$$

have their dangers. But the multiplicative congruential methods such as

$$u_n \equiv au_{n-1} \pmod m$$

are perfectly satisfactory when the constants a and m are properly chosen. One must regard the residue u_n as a pseudo-random number, and *not* as a collection of pseudo-random digits: indeed, the terminal digits have short periods. For high-speed calculations, requiring many thousands or millions of random numbers, the multiplicative congruential methods are almost indispensible.

Mr. THOMSON subsequently replied in writing as follows:

Mr. Stuart and Dr. Good comment on the parameter v of equation (5). Dr. Good gives an interesting alternative approach to obtaining a criterion for the magnitude of $\lambda = Nnv$, leading to a suggested critical value $\lambda = 2$. With my approach this value of λ would make the probability of rejecting the null hypothesis about 2 per cent., and the corresponding value of N would be about 10^{12}, which is still exceedingly large. Mr. Stuart, although agreeing that the estimates of v in (7) are satisfactorily small, thinks that the associated discussion of a hypothetical test is irrelevant. Starting from a point of agreement, namely that it would take a prohibitively large number of years to carry out such a test, I might put my argument as follows: it is impossible, in practice, to distinguish

between $v = 0$ and $v = 10^{-13}$ and hence v is, in practice, satisfactorily small.

I must confess to having been somewhat overcautious in thinking it preferable to say something about the low chi-squared for the grand Gap Test, and can only agree with Mr. Stuart that this is not a matter for suspicion since there are no forms of malfunctioning in ERNIE which would lead to "too good" a fit. Mr. Stuart goes on to say that the deviation of the 30 Gap Test probabilities from the expected uniform distribution may be suspicious. I think the two points are essentially the same. The numbers, in probability ranges 0–10 per cent., 10–20 per cent., etc., are 0, 1, 4, 3, 1, 4, 4, 0, 7, 6 respectively, so that there are on the whole more high probabilities than low. One would expect this to be associated with a low grand chi-squared, both arising from the fact that, for some reason or other, the observed Gap distributions are much closer to the expected distributions than would be obtained on the average.

Mr. Stuart asks whether it would have been "technically feasible" to have given new bond numbers a sixfold chance for their first draw. This is not an easy question to answer, since a consideration of feasibility, as distinct from possibility, requires an appreciation of many technical and administrative problems. It would undoubtedly be possible to arrange this, with the basic principle of associating five extra eligible bond numbers with each new bond number; there are several ways in which this might be done. One way would be to add an extra character, with six possibilities, all six being eligible for new bonds but only one being eligible for old bonds. The elimination of these ineligible bond numbers, however, would have to be part of the manual stage, which would consequently involve nearly six times as much work, since only a few per cent. of the numbers in a draw are new numbers. An alternative scheme would be to use the 40 million bond numbers associated with the hypothetical 24th denomination (at present always rejected within ERNIE) and to assign them five at a time to new bond numbers, of which there are at the moment about six million in each draw; this scheme would also involve extra work in recording, each month, the correspondence between these extra numbers and the new numbers for that month.

The conclusion is that any scheme to introduce this feature would considerably increase the cost of making the draw; this would have to be balanced against the formal increase in fairness. With the present scheme, although bonds in a sense lose on the first month, they gain in subsequent months, and in the long run chances are evened out.

Mr. Stuart's alternative proposal, namely to have a sixfold prize rather than a sixfold chance, would introduce the novel feature of making the outcome of the draw a matter of chance not only for bondholders but also for the Government, since the total prize money would be a random variable; there would be some interesting problems in ensuring that the "bank" was not broken.

The question of compounding random digits has been touched on by several speakers. I am extremely grateful to Dr. Good for his proof of the validity of my conjecture about a method for testing a given compounding matrix. In the course of the proof, an alternative criterion is deduced, namely that $\mathbf{s} \,.\, A \equiv 0 \pmod{n}$ only when $\mathbf{s} = 0$, where $\mathbf{s}$ is a row-vector and A is either the full compounding matrix, or a reduced matrix formed by eliminating columns associated with potential non-random inputs. This criterion is in most ways much more convenient for analysis and synthesis of A-matrices than mine. With its help, I have been able to improve on previous efforts to the extent of finding schemes with three extra generators, any two generators being allowed to fail, this holding good for any n, e.g.

$$A = \begin{Vmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{Vmatrix}.$$

Dr. Tocher suggests some necessary conditions when two generators are allowed to become faulty. I cannot agree with his initial contention that there is no loss of generality in assuming that the elements of A are 0 or 1. In practice, however, one would want to restrict the elements to 0, 1, -1, so the results are helpful.

Mr. Mayne asks for some information on random noise as a stochastic process. Two recent textbooks (W. B. Davenport and W. L. Root (1958), *An Introduction to the theory of random signals and noise*. New York: McGraw-Hill; J. S. Bendat (1958), *Principles and application of random noise theory*. New York: John Wiley) give a comprehensive review of the subject from the point of view of the telecommunication engineer. Random noise in neon tubes is a particular case of "shot noise", in which the noise is the sum of a very large number of individual components; the central-limit theorem is invoked to show that the resulting process is effectively normally distributed in amplitude. The variance of this distribution and the autocovariance (or its Fourier transform, the power spectrum) can sometimes be determined theoretically, or else by experiment. I do not know of any comprehensive investigation for neon tubes. For ERNIE, the choice of neon tubes, as distinct from other noise sources with equally satisfactory stochastic characteristics, was a matter of technical convenience.

Mr. Mayne produces some interesting results relating to random inspection of randomly advanced cyclic counters; I am glad he emphasizes the fact, for it is rather surprising, that random inspection does not necessarily improve things. Random inspection is suggested as a safeguard against non-random disturbances superposed on the random noise, so I take the opportunity to mention that the neon tubes in ERNIE are specially shielded against both electrical and mechanical interference.

The reason for not eliminating all ineligible bond numbers within ERNIE is, as Dr. Tocher supposes, that the job is prohibitively large with current techniques, particularly because of the amount of information to be stored. He suggests that a great many more numbers might be eliminated by examining the trailing characters of the numbers generated, and assumes that this would only double the present storage. This is a natural assumption, but in fact the redundancy table in ERNIE has a much smaller capacity than might be expected.

A store to record the eligibility or ineligibility of all potentially eligible numbers would have a capacity, in the language of information theory, of $4 \times 23 \times 10^7$ bits; one for the first four characters would need $4 \times 23 \times 10^2$ bits, and one for the last four characters 10^4 bits. The redundancy table in ERNIE, however, is based on the assumption that, since bonds are issued to selling points in numerical order, the eligible blocks in any one denomination will be the first so many in numerical order. A store catering only for this possibility has a capacity of $23 \log_2 400 \approx 200$ bits. In practice, there are some ineligible blocks within the span of eligible blocks and the storage arrangements can deal with this to a limited extent; the actual capacity is thus more than 200 bits but very much less than the full capacity of $4 \times 23 \times 10^2 \approx 10^4$ bits. It would not be possible to have this kind of simplification in a store dealing with the last four characters.

Several speakers have discussed ERNIE as a potential general-purpose random-digit generator and have compared genuine with pseudo-random generation. I shall not say anything about this except to point out that ERNIE was not designed as a general-purpose machine, although many of its features would be suitable. The internal speeds and time-constants were greatly influenced by the particular output device used, namely a teleprinter, and the time taken by it (about $2\frac{1}{2}$ seconds) to print a bond number with its associated serial number.

Finally, I should like to thank all the speakers, including those whose contributions have not been explicitly mentioned, for their kind remarks and helpful comments.

As a result of the ballot taken during the meeting the candidates named below were elected Fellows of the Society:

Joan Margaret Garforth
Anders Hald
Che Shing Hui
Edwin Mark Kidd
Robert Leonard Lewis
Martin Bedingfeld Macdonald
Sidney Ivor Matthews
Mervin E. Muller
Richard Raymond Newell
James Robson
John Bridgford Robson
Gwilym Edffrwd Roberts
Ronald McGregor Shields
Phoebus Victor Zachariades

Private Motoring and the Demand for Petrol

By J. S. Cramer

Department of Applied Economics, Cambridge

This paper is concerned with an analysis of private expenditure on petrol and oil in two household budget surveys held in 1953–54. It provides new estimates of the extent of private motoring and of consumers' expenditure on motor spirit, and also yields some income elasticities. The results are used to interpret the post-war rise in vehicle registrations, and they suggest that the further rise of private petrol consumption will depend on an increasing number of vehicles rather than on their more intensive use.

In the theoretical model households that use motor vehicles (or motorist households for short) are considered as a distinct and separate category. The structure of demand is determined by the fact that non-motorists cannot consume petrol, while motorists are to some extent compelled to do so. Formally, this resembles the situation recently discussed by Tobin (1958); but the present paper differs from his in that it is mainly concerned with the application of an economic model, and pays little attention to statistical procedure.

The model is discussed in section 1. There follows an application to the data of the Ministry of Labour Enquiry (Ministry of Labour, 1957) and the Social Accounts of Cambridgeshire Survey (Utting, 1954; Utting and Cole, 1954; Cole and Utting, 1956). Section 2 deals with the prevalence of motorist households in different income-groups, and section 3 with their expenditure on petrol and oil. In section 4 we attempt an interpretation of the recent trend of vehicle registrations and draw some conclusions from our results.

1. The model is based on the distinction between motorist and non-motorist households and on the assumption that motoring, however modest its scale, requires a definite minimum expenditure on petrol and oil. We make three separate assumptions.

The *first assumption* refers to the individual preferences for motoring and hence to the frequency of motorist households at different levels of income. In the present context a motorist household does not necessarily own a motor vehicle: it may hire or borrow one, or use a business car for occasional private motoring. Car owners who have not taken out a licence for the current period, on the other hand, are regarded as non-motorists. We describe the demand for private motoring in this sense by the assumption that each household has a definite tolerance income level for it, according to its preferences, and that these tolerance levels follow a specific taste distribution. Let y_i^* be the tolerance income of the i^{th} household, so that it is motoring only if its actual income y_i exceeds y_i^*; then the distribution of the y_i^* is assumed to be lognormal with mean μ and variance σ^2, or

$$F(y^*) = \Lambda(y^* \mid \mu, \sigma^2), \tag{1}$$

where we use the notation

$$\Lambda(y^* \mid \mu, \sigma^2) = \int_0^{y^*} \left\{ \frac{1}{t\sigma\sqrt{2\pi}} \exp - \frac{1}{2\sigma^2} (\log t - \mu)^2 \right\} dt \tag{2}$$

for the lognormal distribution function. It follows at once that the expected proportion of motorist households at a given income level y is given by the fraction of households that have a tolerance income y^* lower than y, or

$$P_m(y) = \int_0^y dF(y^*) = \Lambda(y \mid \mu, \sigma^2). \qquad (3)$$

The fraction of motorists therefore follows a sigmoid curve, and approaches unity asymptotically as income increases.

The same model was used by Farrell (1954) for the ownership of distinct types of cars, and it has been discussed in more general terms by Aitchison and Brown (1957). In the present application we ignore variations in the type of vehicle and in the terms on which business cars are available for private use, so that the effect of these factors cannot be distinguished from the assumed dispersion of preferences. Moreover, even if private motoring were homogeneous it probably would not depend on current income alone. But these seem justifiable approximations.

The *second assumption* is that the Engel curve for petrol and oil of the i^{th} household is given by

$$x_i = \phi(y_i, y_i^*), \qquad (4)$$

where

$$y_i < y_i^* \qquad \phi(y_i, y_i^*) = 0, \qquad (5a)$$

$$y_i \geqslant y_i^* \qquad \phi(y_i, y_i^*) = \gamma_i + \beta(y_i - y_i^*). \qquad (5b)$$

Here x_i is the quantity of petrol and oil consumed, y_i is current income and y_i^* the tolerance income defined before. Equation (5a) merely states that consumption is zero whenever current income is less than the tolerance income, that is when we have a non-motorist household; equation (5b) specifies the Engel curve for the income range where the household does use motor vehicles. The consumption consists of a minimum requirement γ_i, which becomes necessary on the transition from non-motorist to motorist, and a variable part that is proportional to the excess of actual income over the tolerance income. The minimum requirement may vary between households, and thus provide another indication of their individual preferences; but while the individual Engel curves may thus have different starting-points, they all have the same slope β. This slope reflects both the more intensive use of vehicles and, equally important, the change to larger cars as income increases.

The *third assumption* connects the two indicators of individual preferences: the individual minimum consumption γ_i is inversely related to the tolerance income level y_i^* by the equation

$$\gamma_i = k\, y_i^{*-\pi} \qquad (6)$$

for all i; k, $\pi \geqslant 0$. Keen motorists, or households that have a real need for private transport, will acquire a vehicle at a relatively low level of income, and use it intensively; at the other end of the scale are individuals for whom motoring represents an inessential luxury, and when they are so rich as to buy a car they use it relatively little. It may be objected that these arguments disregard all variation in the quality and economy of vehicles; in fact, however, a keen motorist with a low income may equally well be using an old and uneconomical car as the latest bubble. The assumption refers to preferences as

reflected by behaviour at hypothetical income levels, and it bears only indirectly on the behaviour of the rich and the poor. Its plausibility must be judged by the general considerations given; its exact form is a matter of convenience, the exponential in (6) preventing awkward negative values of γ_i from arising.

We are not committed to a particular slope for the inverse relation assumed, and in extreme cases π may become equal to zero, so that

$$\gamma_i = k \tag{7}$$

for all i. The minimum consumption is then a constant, and the variation of tastes is reflected in the tolerance income levels alone.

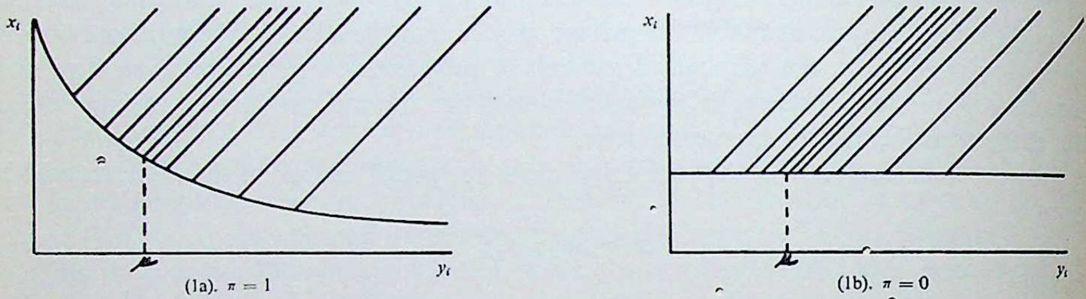

FIG. 1.—Two versions of the model.
(a) $\pi = 1$. (b) $\pi = 0$.

The three assumptions together determine the model shown in Fig. 1. It consists of a family of individual Engel curves that start off from the minimum consumption γ_i at the point on the income scale where the individual becomes a motorist. In the first version (1a) the minimum consumption declines with increasing tolerance level; in the simpler case (1b) it is a constant. The spread of the individual curves is determined by the lognormal distribution of their starting-points on the income scale.

In this form, however, the model is not readily applicable, and some further equations that correspond to the observed statistics must therefore be derived. They all follow from the properties of the lognormal distribution; although we shall indicate their nature, we refer the reader to Aitchison and Brown (1957) for the methods by which they are derived.

The proportion of motorist households at a given income level has been given in equation (3). The average petrol consumption at a given income level, for motorists and non-motorists alike, is, from (4) and (1):

$$\bar{x}(y) = \int_0^\infty \phi(y, y^*)\, d\Lambda(y^* \mid \mu, \sigma^2). \tag{8}$$

We may substitute (5a) and (5b) and integrate over the two parts of the range $(0, y)$ and (y, ∞) separately by using the formula for moment distributions of the lognormal. This yields the explicit expression

$$\bar{x}(y) = k \exp\left(-\pi\mu + \tfrac{1}{2}\pi^2\sigma^2\right) \Lambda(y \mid \mu - \pi\sigma^2, \sigma^2) + \beta y \Lambda(y \mid \mu, \sigma^2) \\ - \beta \exp\left(\mu + \tfrac{1}{2}\sigma^2\right) \Lambda(y \mid \mu + \sigma^2, \sigma^2). \tag{9}$$

The average petrol consumption among motorist households alone is, of course, the ratio of overall average consumption to the proportion of motorists, or

$$\bar{x}_m(y) = \bar{x}(y)/P_m(y). \tag{10}$$

The behaviour of this variable is of some interest, but as the result of substituting equations (9) and (3) in (10) admits of no simplification we cannot study it by inspection. Fig. 2

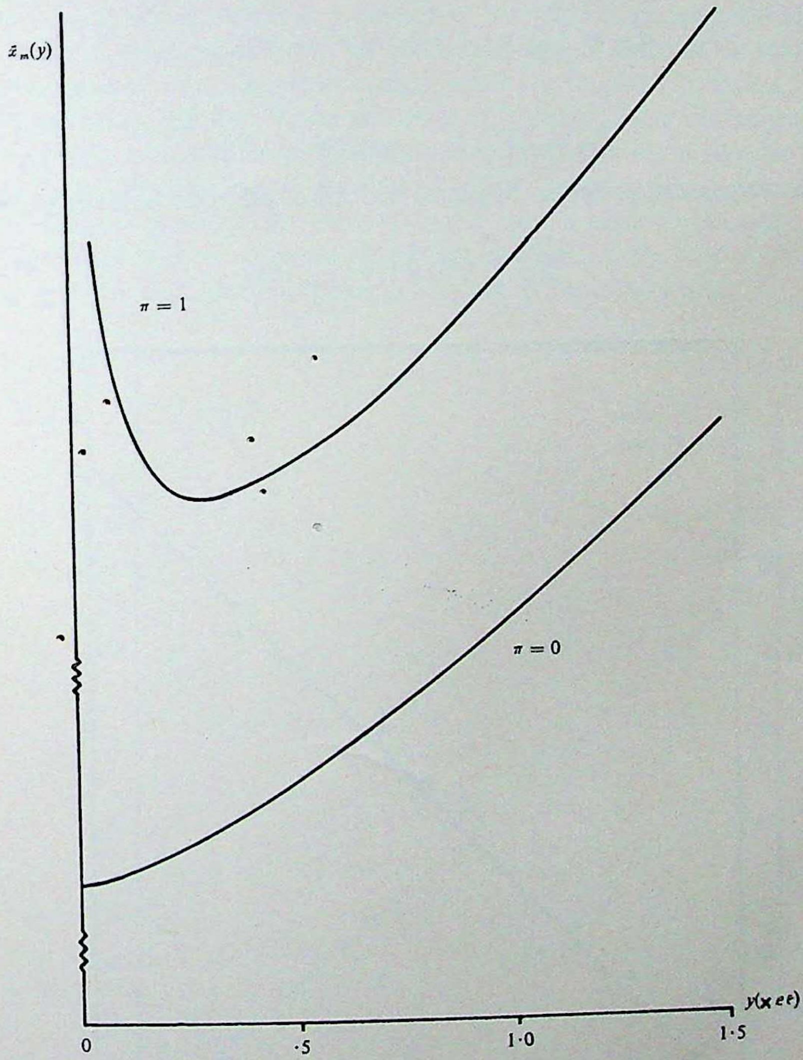

FIG. 2.—Engel curve of motorists' petrol consumption.

shows curves of $\bar{x}_m(y)$ for two values of π. If π is zero, i.e. minimum petrol consumption a constant, the average expenditure of motorists increases monotonically with income. But if π is positive the decline in the minimum requirement partly offsets the increase due to the slope of the individual Engel curves. For sufficiently large values of π this effect may dominate among the lower income groups and lead to a fall in average consumption, so that the curve is U-shaped. Although in the end we reject this form we shall see that it is not altogether as fanciful as it may seem.

We shall also require expressions for aggregate statistics of all United Kingdom households. These are obtained by aggregation of the model over an assumed lognormal income distribution. It is a convenient property of lognormal expressions that they readily permit this, as has been shown by Aitchison and Brown (1957). For a lognormal income distribution with parameters (μ_0, σ_0^2), the overall proportion of motorist households in the population is, from (3)

$$\bar{P}_m = \int_0^\infty \Lambda(y \mid \mu, \sigma^2)\, d\Lambda(y \mid \mu_0, \sigma_0^2), \tag{11}$$

which yields

$$\bar{P}_m = \Lambda\{1 \mid (\mu - \mu_0), (\sigma^2 + \sigma_0^2)\}. \tag{12}$$

Similarly, the average consumption of petrol and oil of all households is given by

$$\bar{\bar{x}} = \int_0^\infty \bar{x}(y)\, d\Lambda(y \mid \mu_0, \sigma_0^2). \tag{13}$$

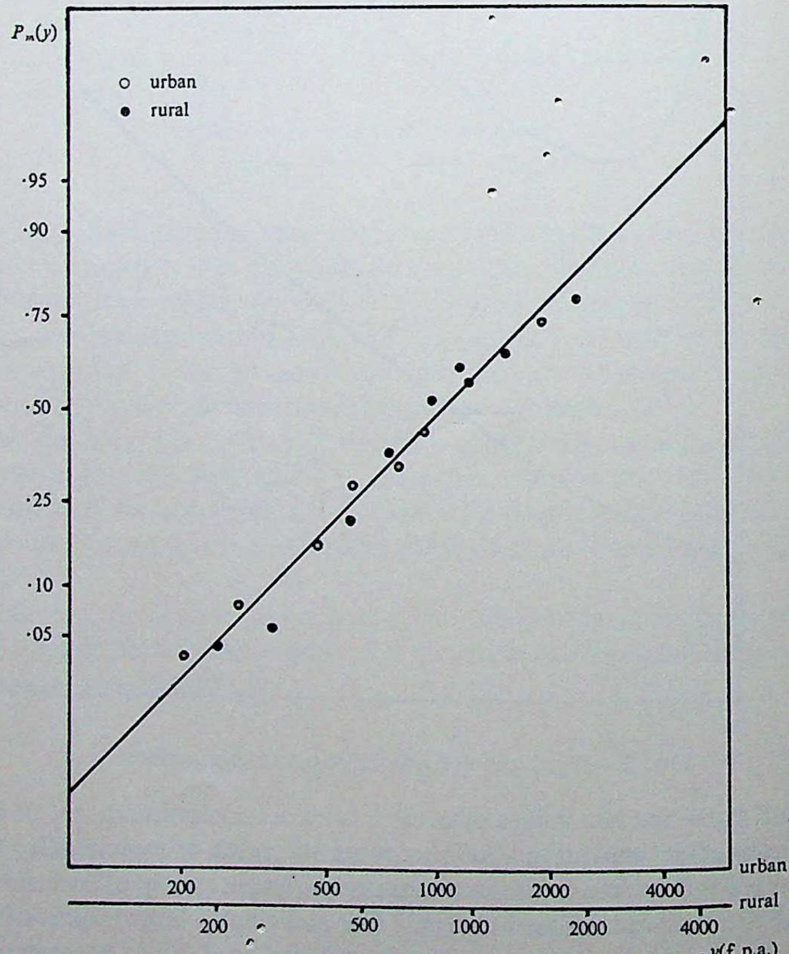

FIG. 3.—Engel curves for private motoring (lognormal transformation).
(a) Cambridgeshire Survey.

Substitution of (9), integration and rearrangement lead to

$$\bar{\bar{x}} = k \exp\left(-\pi\mu + \tfrac{1}{2}\pi^2\sigma^2\right) \Lambda\{1 \mid (\mu - \mu_0 - \pi\sigma^2), (\sigma^2 + \sigma_0^2)\}$$
$$+ \beta \exp\left(\mu_0 + \tfrac{1}{2}\sigma_0^2\right) \Lambda\{1 \mid (\mu - \mu_0 - \sigma_0^2), (\sigma^2 + \sigma_0^2)\}$$
$$- \beta \exp\left(\mu + \tfrac{1}{2}\sigma^2\right) \Lambda\{1 \mid (\mu - \mu_0 + \sigma^2), (\sigma^2 + \sigma_0^2)\}. \quad (14)$$

We shall return to these equations after estimates of the parameters have been obtained. The estimation procedure we adopt is expedient but not altogether justified on theoretical grounds. Two distinct implications of the model are considered in turn. The first is that the proportion of motorists at an income level y is given by equation (3); we shall fit this form to the data, and thus obtain estimates of μ and σ. The second implication of the model is that the expected petrol consumption of motorists at an income level y is given by equation (10). We can fit this expression to the observed consumption figures for any selected value of π, provided the estimates of μ and σ earlier obtained are accepted as given. This course will be adopted: estimates of β and k are thus obtained for various values of π, and the best set of all three is selected by repeated trials.

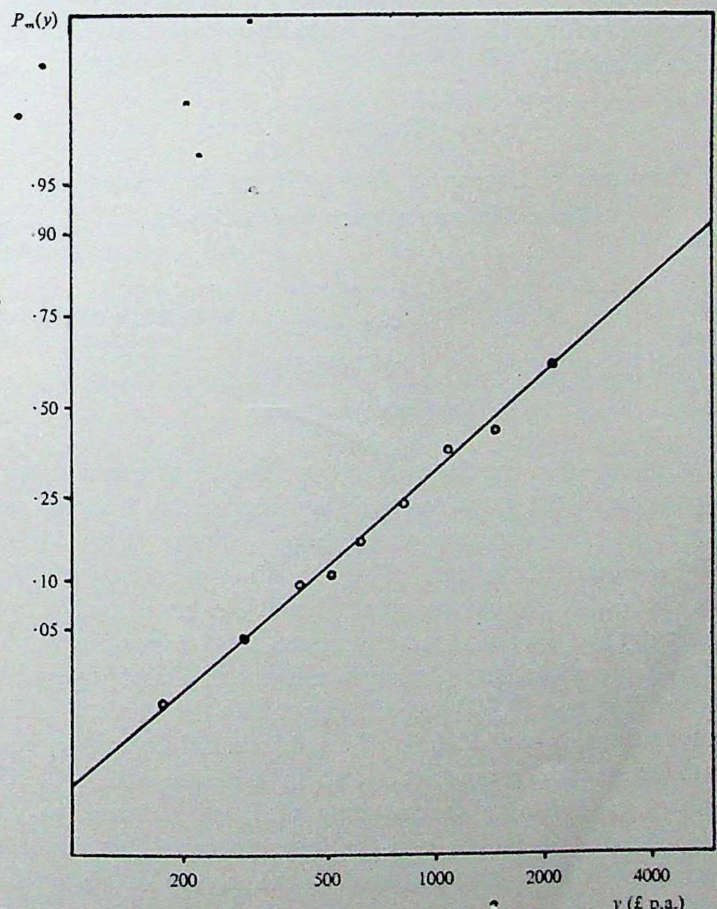

Fig. 3.—Engel curves for private motoring (lognormal transformation).
(b) Ministry of Labour Survey.

2. We begin, then, by fitting equation (3) to the proportion of motorists as a function of income. As neither survey provides direct evidence of motoring, we have accepted the expenditure on specific items as a sufficient indication instead. In the Ministry of Labour Enquiry this is spending on petrol and oil (a single item) over a period of three weeks; for the Cambridgeshire Survey, households are considered to be motorists if they either spend on petrol and oil in a week, or on motor licenses and insurance in three months. The data are given in an appendix.

Some preliminary experiments were tried out on the Cambridgeshire data alone. At first households of different size were considered separately, but contrary to expectation this factor was found to have no appreciable effect. The distinction between rural and urban districts, on the other hand, is of some importance, and separate curves have therefore been fitted to the two strata. The Ministry of Labour data have been used from the beginning without any classification other than by income groups.

The estimation follows the standard iterative method of probit analysis. In both surveys, total expenditure has been used as the independent variable in lieu of income; and in the Cambridgeshire survey a common value of the variance parameter was imposed on the two strata. The estimates are given in Table 1; the fit can be judged from Fig. 3. Because of their common variance the two Cambridgeshire curves could be made to coincide by a shift of the income scale.

TABLE 1

Estimates of Lognormal Engel Curves for Motoring

(Figures in brackets denote standard errors)

	σ	μ	*Median Tolerance Income in £ p.a.** ($M = e^{\mu}$)
Cambridgeshire, rural	0·839 (0·039)	6·727 (0·034)	835 (30)
Cambridgeshire, urban		6·946 (0·034)	1,040 (35)
United Kingdom (Min. of Labour Survey)	0·982 (0·034)	7·395 (0·034)	1,630 (55)

* Rounded to nearest £5.

The different estimates of σ for the two surveys need not cause much concern. As we shall see, the position of the Engel curve varies appreciably between urban and rural areas, and such shifts may well account for the larger variance of the Ministry of Labour Enquiry that covers a variety of regions. There is, however, a major difference between the two surveys in the level of the curves. This is best illustrated by the median tolerance income, M, that indicates the level of income (or, here, total expenditure) at which exactly half of all households are motorists. In rural and urban Cambridgeshire this situation obtains at a little over £800 and £1,000 respectively; but in the Ministry of Labour Enquiry the median tolerance income is over £1,600. This substantial difference reflects the greater preference for motoring in rural areas that is also in evidence within the Cambridgeshire survey. The Ministry of Labour's sample may well, on balance, be more urban in character than the City of Cambridge, and the importance of this factor should not be underrated; it has recently been demonstrated in the survey of regional vehicle registrations by Sleeman (1958).

The Engel curves we have obtained are consistent with this regional variation. Since the samples are not representative of the actual income distribution, and contain relatively

too few of the richer households, the unadjusted sample averages cannot be compared with aggregate statistics; we shall therefore aggregate the Engel curves over the income distribution of the population. If we assume that this distribution of income, or rather of total family expenditure, is lognormal, equation (12) can be used.

This assumption of a lognormal distribution of household expenditure is no more than an approximation, justified for illustrative purposes only. For its parameters we have assumed the values $\mu_0 = 6 \cdot 392$ and $\sigma_0 = 0 \cdot 585$. The latter figure was suggested by tabulations of net income from the Oxford Surveys of Savings and Income, in part published by Hill (1955); the value of μ_0 corresponds to the total consumer expenditure per household in 1953, calculated from total consumer expenditure (Central Statistical Office, 1959) at the average household size of 3·18 persons of the Ministry of Labour survey. Inserting these values in equation (12) we can obtain estimates of the overall proportion of motorist households implied by each of our Engel curves. In Table 2 these estimates are compared with actual vehicle registrations.

TABLE 2

Comparison of Engel Curves and Aggregate Statistics

	Motorist Households per 1,000, according to Engel Curves	*Motor Vehicle Registrations per 1,000 Households*		
		All	*Motorcycles*	*Cars*
Cambridgeshire, rural	372	383	89	294
Cambridgeshire, urban	294			
United Kingdom (Ministry of Labour)	190	244	66	178

Sources.—Population figures from the *Registrar-General's Statistical Review*: family size 3·18; vehicle registrations from *Ministry of Transport Return* (*Mechanically Propelled Vehicles*) nr. 164*A*.

The correspondence between the figures in each row confirms that the difference between the Engel curves represents a genuine regional difference; since a common income distribution is used in the aggregation, the regional variation shown is independent of differences in income. The margin of registration figures over private motoring is due to various causes. First, the registration figures have been taken from the annual census in the third quarter, whereas the surveys cover a whole year. Annual registration averages are 3 per cent. below the census peak for cars, and 12 per cent. for motorcycles. Secondly, a household may own more than one vehicle, if not more than one car, and such cases are not so rare that they can be neglected: in the Oxford Savings Survey of 1953 they account for an excess of 10 per cent. of vehicles over motor-owning households. Finally, part of the registered vehicles are used exclusively for business purposes. If this does not apply to motor cycles, we arrive at an estimate of 15 per cent. for the proportion of cars in the United Kingdom that is never privately used; in Cambridgeshire the proportion is 9 per cent. These figures give a fair indication of the order of magnitude, but they are subject to a substantial margin of error, mainly because the income distribution that we have used is not very well established.

3. We now proceed to the other parameters of the model. For this purpose, we consider the expected petrol consumption of a motorist household with income y, given

in equation (10); after substitution of equations (3) and (9), it can be written as

$$\bar{x}_m(y) = \beta Y + kZ(\pi) \tag{15}$$

where

$$Y = y + \exp\left(\mu + \tfrac{1}{2}\sigma^2\right) \frac{\Lambda(y \mid \mu + \sigma^2, \sigma^2)}{\Lambda(y \mid \mu, \sigma^2)} \tag{16}$$

and

$$Z(\pi) = \exp\left(-\pi\mu + \tfrac{1}{2}\pi^2\sigma^2\right) \frac{\Lambda(y \mid \mu - \pi\sigma^2, \sigma^2)}{\Lambda(y \mid \mu, \sigma^2)}. \tag{17}$$

For the purpose in hand we accept the estimates of μ and σ obtained earlier as given constants, and construct series of Y and several series of Z (for various values of π) on this basis for each set of data. Equation (15) can then be fitted to the observed average petrol and oil consumption of motorists in each income group by least squares regression on Y and $Z(\pi)$, and this provides estimates of β and k for each value of π. In these regressions the observations have been weighted by the number of (motorist) households that contribute to the observed average consumption in each income group.

In Table 3 a single equation has been fitted to the two Cambridgeshire strata after preliminary estimates had given very much the same results for each. No standard errors of estimates are given, as the conditioning assumptions—the saturation level, the values of μ, σ and π—on which the calculations depend would render them practically meaningless. The estimate of k is indicated by its logarithm; this should put its variation with π in proper perspective. Equation (6) can be rewritten as

$$\log \gamma_i = \log k - \pi \log y_i^*, \tag{18}$$

and the linear relation between π and $\log k$ which this leads us to expect is in fact to a large extent reflected in the estimates. Finally, we recall that the parameters refer to the system of linear individual Engel curves of equations (5); β expresses the slope of consumption in shillings per week to total expenditure in £ per annum, and k indicates the level of the individual minimum petrol requirement in shillings per week.

TABLE 3

Estimates for Equation (15) *for Various Values of* π

	β	$\log_e k$	R^2	s, *Residual s.e. (sh. p. week)*
Cambridgeshire Survey				
$\pi = 0$	0·0205	2·007	0·8688	2·38
$\pi = 0{\cdot}5$	0·0233	4·823	0·8806	2·27
$\pi = 0{\cdot}75$	0·0243	6·205	0·8830	2·25
$\pi = 1{\cdot}0$	0·0252	7·563	0·8821	2·26
$\pi = 1{\cdot}25$	0·0259	8·900	0·8789	2·29
$\pi = 2{\cdot}0$	0·0277	12·723	0·8500	2·54
Ministry of Labour Survey				
$\pi = 0$	0·0221	2·042	0·9203	1·32
$\pi = 0{\cdot}5$	0·0270	4·790	0·8838	1·59
$\pi = 0{\cdot}75$	0·0288	6·140	0·8575	1·76
$\pi = 1{\cdot}0$	0·0303	7·468	0·8268	1·94
$\pi = 1{\cdot}25$	0·0316	8·748	0·7914	2·13
$\pi = 2{\cdot}0$	0·0345	12·473	0·6702	2·68

The first question is what value of π to adopt. For the Ministry of Labour data the best fit is obtained for $\pi = 0$, which corresponds to the simpler version of the model with the minimum consumption a constant. For the Cambridgeshire Survey, however, unity is the better value by strict least squares standards. By themselves these data would therefore suggest a U-shaped Engel curve of the type shown earlier in Fig. 2. The improvement in fit relative to $\pi = 0$ is, however, slight, and the difference in the value of β of little account. (The variation in the estimate of k can be accounted for by equation (18) and is of no particular importance.) We conclude therefore in favour of the simpler model.

On this choice the estimates of β and k for the two surveys are sufficiently similar to justify the pooling of the data. The common estimates that result are:

$\pi = 0$ (by assumption)	$R^2 = 0 \cdot 8875$
$\beta = 0 \cdot 0209$	residual s.e. $= 1 \cdot 53$ sh. per week
$\log_e k = 2 \cdot 058$	$k = 7 \cdot 84$ sh. per week.

Again we may compare our results with what is known of the total consumption of petrol. For the aggregation of the system of individual Engel curves we use the same lognormal income distribution as before, and for the taste distribution of motoring preferences we take the estimates obtained from the Ministry of Labour Enquiry; for β and k we accept the joint estimates for the two surveys just given. With these values we evaluate the expression for the average household expenditure on petrol and oil given in equation (14). This works out at a total United Kingdom consumption of £137 mln. per annum in 1953–54, or just over 17 shillings per week per motorist. In view of what follows it should be noted that this estimate depends only to a limited extent on the particular model and income distribution that we have adopted. Without any model, and without allowance for the bias of the sample, the sample would give a figure of £99 mln. per annum for the whole population. This is no doubt an underestimate; but it sets a limit to the over-estimation that may be due to our particular assumptions, which raise both the proportion of motorist households and their average petrol and oil consumption by about 15 per cent. over the unadjusted sample statistics.

There exist no very firm aggregate statistics with which our estimate of £137 mln. can be compared. The total supply of motor spirit in 1953 was worth some £390 mln., of which about £130 mln. consisted of commercial deliveries. Of the remaining £260 mln., about £170 mln. of petrol was sold for use in cars and motorcycles, as apart from commercial transport; and of this it is estimated that £64 mln. was used for private motoring. This latter figure, which has been supplied to me from a trade source, is the one which is also given to the Central Statistical Office for use in building up the estimate of consumer expenditure on the running costs of private motoring given in *National Income and Expenditure* (Central Statistical Office, 1959).*

We may add £6 mln. to this figure for oil, and perhaps one or two million pounds to allow for the shift from calendar year to survey period. The uncomfortable fact remains that our estimate is nearly twice the figure of the consumer expenditure series. It may be felt that not too much trust should be put in the particular assumptions that we have made, but, as we have shown, these at most account for half the discrepancy. Another explanation

* The other figures in this paragraph are rough estimates based on statistics published in the *Monthly Digest of Statistics* (e.g. nr. 153, T.50), the *Ministry of Power Statistical Digest* and the *United Kingdom Petroleum Industry Statistics*.

of overestimation by the Engel curves would be that the surveys record expenditure that has not been incurred for strictly private motoring; but the questionnaire is found to be very careful and explicit on this account. In the Ministry of Labour survey, the editing procedure of ignoring the expenditure altogether if a car is used mainly for business would, in fact, appear to underestimate private expenditure.

We conclude, then, that while our figure may be slightly too high this cannot explain the whole of the observed discrepancy. The consumer expenditure series for running costs must be regarded as an underestimate in so far as petrol is concerned. It has been obtained broadly speaking by extrapolation of the division of business and private use during petrol rationing, allowance being made for the relative increases in commercial deliveries and dealer supplies since derationing. (cf. *Central Statistical Office*, 1956, pp. 124–6). A comparison of the series with the course of vehicle registrations suggests that it is the initial estimate of private consumption during rationing rather than the subsequent year-to-year changes that is too low.

4. We have established a model of individual demand and obtained estimates of its parameters. What are the implications for the market demand for motoring and motor spirit? To answer this question, we shall consider the effect of shifts in the income distribution on aggregate demand. Such shifts would for example result from a proportionate variation in all incomes; they are equivalent to changes in μ_0, the mean of the income distribution, while its variance remains constant. As μ_0 is a logarithm of income, the elasticity of, say, x with respect to such an income change is given by

$$\frac{\partial \log x}{\partial \log y} = \frac{1}{x}\frac{\partial x}{\partial \mu_0}. \tag{19}$$

We first apply this method to the increase of private motoring, or rather of vehicle registrations, that has occurred over the last decade. In section 2 we compared private motoring and vehicle registrations and indicated how seasonal factors, two-vehicle households and business cars contribute to the difference between the two. We now make the same percentage corrections in reverse to estimate the fraction of motorist households in the years 1948–1957 from vehicle registrations. On the other hand, estimates of μ_0 for each year can equally be obtained from total consumer expenditure (revalued at 1953–54 prices) in the same manner as before, and these estimates can be inserted in equation (12) with all other parameters at the values obtained for 1953–54 to give "model estimates" of private motoring. The agreement between the two series will indicate to what extent income changes account for the increase in vehicle registrations (and by implication in private motoring) since 1948.

It turns out that rising incomes explain only a minor part of the observed increase in motoring, and that other powerful factors making for its expansion must have been at work. Over the years these show little relation with price variations or supply conditions like delivery periods and hire-purchase requirements. We are therefore led to describe (if not to explain) their effect generally by a continuing shift of the taste distribution. The variance of this distribution is again assumed to remain constant at the value obtained for 1953–54; we may then indicate the position of the distribution in each year by calculating the value of μ which corresponds to the observed vehicle population. In Fig. 4 these estimates are shown in the equivalent values of M, the median income level at which the

expected fraction of motorist households equals one half. The lower M, the stronger is of course the public's preference for motoring.

As Fig. 4 shows, there has been a steady strengthening of these preferences which corresponds closely to a linear trend in M. While this may be a convenient description of the development so far, no great predictive importance can be attached to this particular linear trend, even if it shows no sign of slowing down. Since 1948 the value of M (expenditure per annum at 1953–54 prices) has declined from £2,050 to £1,300; clearly, it cannot continue to decline at this rate for more than a few years.

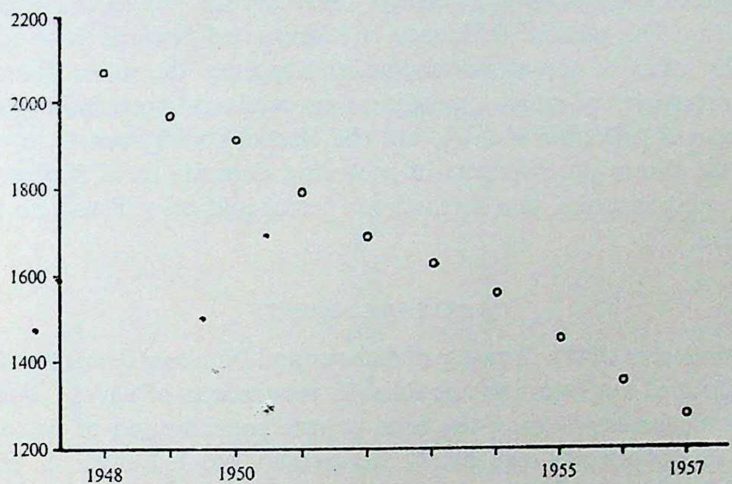

FIG. 4.—Calulated values of M.

The effects of further changes in tastes are easily assessed. Inspection of equation (12) shows that the overall fraction of motorist households is identically affected by opposite changes in μ and μ_0, and the elasticity with respect to (upward) shifts of the income distribution is therefore equally applicable to (downward) shifts of the taste distribution. We may evaluate this elasticity for the conditions obtaining in 1957, and find a value of 1·06. As an estimate of the long-run income elasticity of private motoring this is rather less than one would expect, and it seems definitely too low in comparison with the income elasticity of automobile ownership in the United States estimated by Chow (1957) at about 2. The difference can however in part be attributed to the fact that our elasticity (as opposed to Chow's) refers to all vehicles, not just cars, and that it moreover makes no allowance for the substitution of newer cars for older, as Chow's stock series implicitly does.

We now turn to the motorists' expenditure on petrol and oil. For this variable the best we can do by way of a comparison with recent time-series is to adopt the income and taste distributions that explain the extent of private motoring, and to insert these parameters with the other coefficients held constant in equation (10). The calculated expenditure per motorist which we obtain can be compared with corresponding figures from the National Income series, discussed before, and vehicle registrations. After allowing for price differences and rationing in the Suez crisis, both series show an increase of the same

order from derationing in 1950 to 1957, namely by 5 and 8 per cent. (in constant prices) respectively; they move of course on an entirely different level.

The main point of interest is that the mean expenditure per motorist should have risen by so little. This is related to the fact that the rise in motoring generally was ascribed to a shift in tastes rather than to rising incomes. For petrol consumption per motorist these two factors have different effects, and the elasticity with respect to a shift of the taste distribution is far smaller than the elasticity with respect to a shift in incomes. For those who are already motorists before the shift occurs, petrol demand is given by (equation 5)

$$x = k + \beta(y - y^*),$$

and as y must exceed y^*, a percentage change in the former will have a much larger effect than in the latter. The relative difference is accentuated because both increases are in part offset by the influx of new motorists, which depresses the mean figure per motorist. As a result, the elasticity of mean expenditure per motorist household with respect to a shift of the income distribution is 0·61, but the elasticity with respect to changing tastes only 0·03. If the future development of motoring depends on a further shift in tastes rather than on rising incomes, the demand for petrol and oil will rise no faster than the vehicle population.

Acknowledgments

I am greatly indebted to the Ministry of Labour and National Service and to the Oxford University Institute of Statistics for unpublished tabulations of survey data, and to Shell Mex and B.P. Ltd. for estimates of the total private consumption of petrol and oil. An earlier draft of this paper was read by W. Beckerman, M. J. Farrell, S. F. Kaliski, S. J. Prais and J. Wallander, and I have adopted many (though not all) of their suggestions for improving it.

Note added in proof. Mr. D. S. Ironmonger has drawn my attention to the analysis of petrol consumption by V. Perlo and C. F. Roos in the third chapter of the latter's *Dynamic Economics* (Roos, 1934). In this study petrol consumption per car is the dependent variable in a regression on various short time series for four American States. As far as income is concerned, the equation is of the form

$$x = \alpha + \beta y + \frac{\gamma}{y},$$

where x is petrol consumption per car and y income or purchasing power, in fact represented by new car registrations as a proxy variable. For the coefficients obtained this leads to a U-shaped curve, and some of the graphs shown are remarkably similar to the curve for $\pi = 1$ in Fig. 2 above. In explaining this phenomenon the authors state the essence of my model, viz. " . . . when purchasing power declines to abnormally low levels small consumers are driven out of the market, so that the *average* consumption of those remaining increases . . . " (Roos, 1934, p. 41). I should not have overlooked this interesting article.

Appendix. The Data

Income groups: by total household income, £ per week.
y, mean total expenditure per household in £ per annum.
N, number of households.
n, number of motorist households.
x, mean expenditure on petrol and oil per motorist household in shillings per week.

Social Accounts of Cambridgeshire Survey (1953–54)

Income Group	*y*	Rural Stratum *N*	Rural Stratum *n*	Rural Stratum *x*	Urban Stratum *N*	Urban Stratum *n*	Urban Stratum *x*
<3 . . .	202	178	8	20·32	103	4	11·40
3–5 . . .	288	139	8	12·67	118	9	14·90
5–8 . . .	471	364	76	9·87	296	48	10·97
8–12 . . .	595	328	124	11·23	438	125	10·02
12–16 . . .	795	183	97	13·15	273	92	14·90
16–20 . . .	935	95	59	17·63	115	50	11·89
20–30 . . .	1,243	112	74	22·57	136	79	22·09
>30 . . .	1,902	49	39	31·67	104	77	25·78
Total . .		1,448	485		1,583	484	

Ministry of Labour Survey

Obtained from the Ministry of Labour Household Expenditure Enquiry (1953–4) by permission of the Controller of H.M. Stationery Office.

Income Group	*y*	*N*	*n*	*x*
<3 . . .	176	747	11	10·34
3–6 . . .	291	1,279	54	10·55
6–8 . . .	416	1,437	134	10·20
8–10 . . .	509	2,031	212	10·05
10–14 . . .	618	3,425	543	12·41
14–20 . . .	803	2,578	604	14·41
20–30 . . .	1,081	1,065	392	18·74
30–50 . . .	1,456	271	112	24·69
>50 . . .	2,113*	78	49	26·52
		12,911	2,111	

* After eliminating one expenditure item of £1,903.

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A Survey of Fellows—II

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Contents

1. Introduction

In an earlier paper (Gales, 1958) some of the results of an Enquiry on the Utilization of Statistical Methods were discussed. In this paper further data obtained from the analysis of answers to Section II of the questionnaire are presented. This section was completed only by those respondents currently using statistical techniques, and information obtained concerned frequency of use of techniques, fields of application, statistical books and journals read and referred to, and attendance at meetings of statistical societies.

1.1. *Development of the Statistical Method*

The major part of this paper is devoted to the presentation and discussion of the results of a survey of the present utilization of statistical methods.

The spread in the use of statistical methods of experimentation and analysis has been so rapid in recent years that it seemed of value to try to assess the present situation.

Kendall (1942) looked forward to the time when statistics would be an "honourable and populous profession", when there would be a statistician in every large firm and as many professors of statistics as of divinity. Development of the use of the statistical methods has indeed been rapid since then and, although the third of these happy states has not yet arrived, Fellowship of the Royal Statistical Society has more than doubled. As well as statisticians in most large firms there is a Statistician grade in the Civil Service, at the same level as the Administrative grade.

Techniques, now of frequent use in many fields, were first devised to meet the specific conditions, such as scarcity of data, non-normal distributions and multiple measurements, met with in biological and agricultural research. The number of statisticians in this field, or biologists with statistical knowledge, has grown steadily, but no longer can it claim to be the principal field in which new statistical techniques are being developed.

With the vastly increased amount of medical, sociological and psychological research that has been carried out since World War II in research units and hospitals, there has come in these fields also a fuller acceptance of the importance of a knowledge of statistical methods. In the planning of surveys, the design of experiments and the analysis of data, the statistician is called upon.

Rapid as has been the expansion of the use of the statistical method in the Civil Service, medicine, the natural sciences and sociology, perhaps even more remarkable have been its applications in commerce and industry. Manufacturers of consumer goods rely increasingly on the results of surveys in designing new products, and marketing them. The larger retailing firms have found the need for statisticians among their staff, while there are few actuaries, accountants and stockbrokers who do not realize the importance of statistical techniques, not only in understanding what is at present happening but in predicting what may happen in future months.

In the manufacture of metal, glass, brick, pottery, and electrical equipment as well as in ordinary machine processes, methods of quality control have been successfully applied, spreading rapidly since the first control charts were used in 1924. As well as techniques used in testing the efficiency of the production process, industrial research has been greatly facilitated by the statistical design of experiments and analysis of results.

It is also in manufacturing industry that the operational research worker is now most frequently to be found, trying with a battery of statistical techniques, mathematical and scientific knowledge to increase the efficiency of both administratively and mechanically complex processes. In dealing with problems encountered in this field some of the newest of the statistical techniques, such as queueing theory, have met with frequent application.

That the demand for statisticians in all fields of activity has increased phenomenally since the war cannot be doubted, and that the supply of persons with any qualification in statistics has failed to keep pace with this demand is equally plain. Steps were taken by the statistical societies to add to the number of qualified statisticians by holding examinations at various levels but the flow of statistics specialists from the universities is still small and in only four universities is there a chair in statistics. In a survey made for UNESCO in 1955 there were found to be only 69 full-time teachers of statistics in the 54 universities and university colleges throughout the United Kingdom. Of these only 28 belonged to statistics departments, 15 to mathematics and 15 to economics departments. The rest were in departments of psychology, sociology, engineering, biology, agriculture and medicine. Courses in statistics offered were frequently only at a very elementary level, and although a total of 504 students each took at least one paper in statistics in their final degree, only in London University was it possible for students to specialize in statistics for a full year, during study for a first degree. Postgraduate diplomas in statistics were awarded by six institutions but the total output of students with either a diploma or a higher degree in statistics in 1955 was only 26.

Since 1955 there has been little change in the situation and until there are more statistics specialists the research units of universities must continue to compete with industry and the civil service for the services of an all too short supply of trained statisticians.

1.2. *The Present Situation—Aims of the Survey*

I have discussed somewhat briefly the way in which the use of statistical techniques has spread from one field to another, and accounts of the most recent of the developments in each field are to be found in books and journals, but no quantitative picture of the uses to which techniques are being put has so far been available. The Enquiry on the Utilization of Statistical Methods was made in an attempt to fill this gap, if not completely, at least partially.

The principal questions to which answers were sought were:

(1) Where are the people who use statistical techniques working, and what are their qualifications and salary?

(2) Which techniques do they use, and in which fields?

(3) Which statistical books and journals do they read?

(4) How many of them develop new techniques or find new applications of known techniques?

(5) Do they publish their results or attend meetings of the statistical societies?

In Gales (1958) the planning of the survey was discussed and a detailed analysis of answers by all 1,211 respondents to Section I of the questionnaire was presented. These results related to some 300 persons who never used statistical techniques in their current employment or who were no longer employed, as well as to over 900 users of techniques, and they were in fact a description of the employment, salary and qualifications of Fellows of the statistical societies. The questionnaire was given in an appendix to that paper.

It is in this second paper however that the answers to the questions formulated above may be sought. The data presented relate only to the 910 respondents to the questionnaire who said, in answer to Question 10, that a knowledge of statistical techniques was necessary to them in their work or research.

2. THE USERS OF STATISTICAL TECHNIQUES

2.1. *Age, Sex, Occupation and Industry*

Answers to Question 3 of the questionnaire were classified into 22 industrial and professional groups and eight occupational classes. Table 1 shows the distribution of the 910 respondents to Section II of the questionnaire, by industrial groups, while in Table 2 the distribution over seven broader categories formed by adding together some of the groups of Table 1, is given.

Persons engaged in university teaching and research formed the largest of the 22 groups, the non-scientific civil service being the second largest in size with 109 persons. From Table 2 however the 242 persons employed in manufacturing industry can be seen to form the largest of the seven broad categories there shown, and 26·6 per cent. of all respondents. Of the manufacturing groups, that producing chemicals, rubber, plastics etc. is the largest.

A classification was made of the nature of the work being performed by respondents in their present jobs. Although all had some knowledge of statistics less than half were employed as statisticians. The work was classified as administrative only where no specialized knowledge of economics, statistics etc. seemed to be required. In some cases a respondent may have described his work in this way for reasons of prestige. The category Operational Research was used only when respondents themselves specified this to be the nature of their work. A basic definition of this type of work is hard to find, but with a society and journal of its own, a separate category was felt to be merited.

In Table 3 the distribution of respondents by age at next birthday, within occupational categories, is shown. The largest category is that of the 435 persons engaged on statistical work who comprise 47·8 per cent. of all respondents. 129 scientists form the group second in size and 113 persons engaged primarily as administrators the third largest group, with the highest median age.

TABLE 1

Profession or Industry of Users of Techniques

No. of Group	Industry or Profession	No. of Women	Total No. Persons	% of All Users
1	Civil Service—scientific	3	49	5·4
2	Civil Service—non-scientific	12	109	12·0
3	Local Government Service	5	21	2·3
4	Public Utilities	1	53	5·8
5	Public Corporations	3	39	4·3
6	Social and Economic Research or Administration	0	37	4·1
7	Psychological and Medical Research or Administration	4	33	3·6
8	Other Scientific Research	3	40	4·4
9	Universities	15	134	14·7
10	Non-university Teaching	1	24	2·6
11	Advertising and Market Research	2	36	4·0
12	Banking, Insurance, Accountancy, Computing	0	64	7·0
13	Wholesale and Retail Distributive Trades	2	11	1·2
	Manufacturing Industry:			
14	Electrical and electronic goods	3	28	3·1
15	Engineering tools, machines	0	22	2·4
16	Chemicals, alcohol, rubber, plastics, etc.	3	79	8·7
17	Soap, detergents, food, tobacco	1	37	4·1
18	Paper, cotton, and woollen goods	2	26	2·9
19	Iron, steel and other metals	0	29	3·2
20	Oil Extracting, Refining, Distribution	1	21	2·3
21	Miscellaneous	1	12	1·3
22	None	0	6	0·7
	Total	62	910	100·1

TABLE 2

Employment Categories

No.	Employment Category	Groups of Table 3 Included	No. of Women	Total No. Persons	% of All Users
I	Civil Service	1, 2	15	158	17·4
II	Local Govt. Public Utilities and Corpns.	3, 4, 5	9	113	12·4
III	Research Bodies	6, 7, 8	7	110	12·1
IV	Universities	9	15	134	14·7
V	Finance and Commerce	11, 12, 13	4	111	12·2
VI	Manufacturing Industry	14–20	10	242	26·6
VII	Miscellaneous and None	10, 21, 22	2	42	4·6
	Total		62	910	100·0

TABLE 3

Age and Type of Work

Ages in Years	Type of Work: Stat. %	Sci. %	Econ. %	Admin. %	Act. %	Op. Res. %	Teach. %	Misc. %	Total No.	%
20–	20·0	15·6	15·1	2·7	5·2	25·4	16·7	—	145	16·0
30–	44·8	41·9	45·2	29·2	32·8	56·4	37·5	33·3	384	42·2
40–	24·1	27·1	33·3	37·2	37·9	16·4	37·5	33·3	254	27·9
50–	8·5	11·6	4·3	22·1	20·7	—	8·3	—	95	10·4
60–	2·5	3·1	2·2	8·8	3·4	1·8	—	33·3	31	3·4
No reply	—	0·8	—	—	—	—	—	—	1	0·1
Total	99·9	100·1	100·1	100·0	100·0	100·0	100·0	99·9		100·0
Total No.	435	129	93	113	58	55	24	3	910	—
Median Age	36·2	37·8	37·4	44·5	42·9	34·0	38·9	—		37·6

2.2. *Qualifications*

Respondents gave information concerning their qualifications, in answer to Questions 8 and 9 of the questionnaire. Information about the special subject, class attained, and university attended, for any first degree was cross classified by age, and results are shown in Tables 4, 5 and 6.

TABLE 4

Age and University of First Degree

Age in Years	No Degree or No Reply %	Cam. %	Oxf. %	Lond. %	Other Eng. %	Scottish %	Irish or Welsh %	Other %
		University						
20–	7.9	17·0	18·7	16·4	27·1	8·5	15·4	4·3
30–	38·4	41·2	42·7	44·4	40·7	55·3	46·2	17·4
40–	30·5	26·7	25·3	27·6	24·6	29·8	38·5	34·8
50–	16·6	10·3	13·3	9·2	7·6	—	—	26·1
60–	6·6	4·8	—	2·0	—	6·4	—	17·4
No reply	—	—	—	0·3	—	—	—	—
Total	100·0	100·0	100·0	99·9	100·0	100·0	100·1	100·0
Total No.	151	165	75	305	118	47	26	23

TABLE 5

Age and Subject of First Degree

Age in Years	No Degree or No Reply %	Maths. %	Stats. %	Econ. %	Sci. and Med. %	All Arts %	Tech. %	Genl. %
20–	7·8	23·0	22·2	14·7	8·5	12·1	—	15·4
30–	39·2	41·1	49·5	45·2	38·3	36·4	40·9	44·6
40–	30·1	27·9	22·2	27·4	28·7	36·4	22·7	29·2
50–	15·7	6·6	6·1	11·5	17·0	12·1	18·2	6·2
60–	7·2	1·4	—	1·3	6·4	3·0	18·2	4·6
No reply	—	—	—	—	1·1	—	—	—
Total	100·0	100·0	100·0	100·1	100·0	100·0	100·0	100·0
Total No.	153	287	99	157	94	33	22	65

TABLE 6

Age and Class of First Degree

Age in Years	No Degree or No Reply %	1st %	2nd %	3rd %	Pass %
		Class			
20–	8·6	12·1	20·5	27·8	16·1
30–	35·4	40·7	44·3	50·0	47·5
40–	31·8	32·7	25·8	16·7	23·2
50–	17·7	9·3	8·1	5·6	9·1
60–	6·6	4·7	1·1	—	4·0
No reply	—	0·5	—	—	—
Total	100·1	100·0	99·8	100·1	99·9
Total No.	198	214	345	54	99

In answer to Question 9 it was found that 56 per cent. of respondents had some higher degree or diploma. 14·9 per cent. held a M.A. degree, 12·0 per cent. a Ph.D., 4·5 per cent. the Oxford or Cambridge diploma in statistics, 8·4 per cent. the diploma or certificate of the Royal Statistical Society or a qualification of the Association of Incorporated Statisticians, and 22·7 per cent. other qualifications. Twelve respondents held a D.Sc.

2.3. *Salary*

Answers to Question 7 of the questionnaire related to present salary and are shown in Table 7 cross-classified by age. Table 8 gives the distribution within age groups. Median salaries within age groups were also calculated and are seen to increase fairly regularly with age.

TABLE 7

Salary and Age

Salary (£)	*Age* 20–29	30–39	40–49	50–59	60 *and Over*	*No Reply*	*Total*
600–	45	15	3	2	2	—	67
750–	56	45	9	—	2	—	112
950–	26	81	25	3	—	—	135
1,150–	10	118	48	15	1	—	192
1,450–	6	93	82	26	4	—	211
Over 2,000	—	27	85	47	21	1	181
No reply	2	5	2	2	1	—	12
Total	145	384	254	95	31	1	910

TABLE 8

Percentage Distribution of Salary within Age Ranges

Salary (£)	*Age* 20–29	30–39	40–49	50–59	60 *and Over*	*No Reply*	*Total*
600–	31·0	3·9	1·2	2·1	6·5	—	7·4
750–	38·6	11·7	3·5	—	6·5	—	12·3
950–	17·9	21·1	9·8	3·2	—	—	14·8
1,150–	6·9	30·7	18·9	15·8	3·2	—	21·1
1,450–	4·1	24·2	32·3	27·4	12·9	—	23·2
Over 2,000	—	7·0	33·5	49·5	67·7	100	19·9
No reply	1·4	1·3	0·8	2·1	3·2	—	1·3
Total %	99·9	99·9	100·0	100·1	100·0	100·0	100·0
Median	£846·4	£1,274·6	£1,425	Over £2,000	Over £2,000		£1,360·9

In Table 9 the distribution of salary within occupation groups is shown, together with the median salary in each group. In comparing the median salaries of the different groups their median ages must also be taken into account and these are also therefore included in the table.

TABLE 9

Salary and Type of Work

Salary (£)	Stat.	Sci.	Econ.	Admin.	Act.	Op. Res.	Teach.	Misc.	Total
0–	8·3	10·8	7·5	1·8	1·7	5·5	12·5	33·3	7·4
750–	18·2	7·7	8·6	5·3	—	12·7	8·3	—	12·3
950–	17·1	13·1	9·7	6·2	13·8	23·6	29·2	—	14·8
1,150–	23·3	25·4	20·4	9·7	12·1	27·3	20·8	33·3	21·1
1,450–	21·2	22·3	30·1	25·7	27·6	21·8	20·8	—	23·2
Over 2,000	11·1	19·2	22·6	50·4	39·7	7·3	8·3	33·3	19·9
No reply	0·9	1·5	1·1	0·9	5·2	1·8	—	—	1·3
Total %	100·1	100·0	100·0	100·0	100·1	100·0	99·9	99·9	100·0
Total No.	435	129	93	113	58	55	24	3	910
Median salary	£1,229	£1,364	£1,509	£2,000+	£1,863	£1,250	£1,180	—	£1,360·9
Median Age	36·2	37·8	37·4	44·5	42·9	34·0	38·9	—	37·6

The administrative and actuarial groups, while having the highest median salary, have also the highest median ages. The only difference in salary which is not associated with an age difference is that between the median salary £1,509 of economists who have a lower median age than the group of scientific workers whose median salary is £1,364.

For each age group a cross classification of salary by class of degree was made, showing a marked association between the two. A higher proportion of persons with firsts were in the top salary categories in each age group, and a lower proportion were in the lowest categories of salary. Persons with seconds were noticeably less well paid than those with firsts although those with no degree were, in the highest age categories, better paid than those with seconds.

2.4. *Time in Current Job and New Posts*

In Question 11 the respondent was asked how long he had held his present position and whether or not he was the first person to hold this position. Although this question no doubt raises problems of definition, a surprisingly high proportion of persons, 50·1 per cent., stated that their positions were indeed new ones, and an analysis of these new posts by type of work and industry is given in Tables 10 and 11. The distributions of time in present job, and total time spent on work of a statistical nature, are given in Tables 12 and 13. In connection with the term "of a statistical nature" it might be pointed out that this was meant to refer back to Question 10 and to imply "involving knowledge of some statistical techniques".

TABLE 10

Percentage of Persons Engaged in Each Type of Work in New Posts

	Stat.	Sci.	Econ.	Admin.	Act.	Op. Res.	Teach.	Misc.	Total
Percentage	49·7	52·7	49·5	61·1	31·0	54·5	37·5	0	50·1
Number of persons	435	129	93	113	58	55	24	3	910

TABLE 11

Percentage of Persons in Each Industrial Group in New Posts

	Industrial Categories							
	I Civil Service	*II Local Govt.*	*III Re-search*	*IV Univ.*	*V Finance Commerce*	*VI Mfg.*	*VII Misc.*	*Total*
Percentage	30·4	67·3	55·4	43·7	47·3	60·8	33·3	50·1
Number of persons	158	113	110	134	111	242	42	910

The percentage of new posts is seen from Table 10 to be highest (61·1 per cent.) among those doing administrative work, with operational research second (54·5 per cent.) and scientific workers third (52·7 per cent.).

An examination by industry of the 69 new administrative posts showed 21 to be in manufacturing, 12 in market research, 9 in banking and 8 in public utilities and corporations. The remaining 19 were distributed among the other industrial groups. The proportion is lowest among the actuaries and teachers. Classifying the new positions by industrial groups (Table 11) showed that the proportion of new posts was highest (67·3 per cent.) in Group II, comprising Local Government, public utilties, and public corporations, with manufacturing industry second with 60·8 per cent., and the Civil Service lowest with 30·4 per cent.

TABLE 12

Time in Present Job

Time	*Number of Persons*	*Percentage*
0–	17	1·9
3 months–	115	12·6
1 year–	362	39·8
5 years–	267	29·3
10 years–	137	15·1
No reply	12	1·3
Total	910	100·0

TABLE 13

Total Time on Statistical Work

Number of Years	*Persons*	*Percentage*
0–1	32	3·5
2–5	217	23·8
6–10	284	31·2
11–15	172	18·9
16–20	51	5·6
21–	127	14·0
No reply	27	3·0
Total	910	100·0

The question on time in present job was asked in order to obtain some idea of the periods of time to which the information on use of techniques would relate. Thus the information given by the 14·5 per cent. of persons shown to have been in their present jobs for less than one year would probably not present as complete a picture as that given

by the others, as the range of statistical techniques used in any job will constantly increase.

From Table 13 it is seen that although 52·2 per cent. of respondents were not working primarily as statisticians many of them had been doing work requiring some knowledge of statistics for many years, 9·6 per cent. of all respondents having been engaged on such work for over 5 years, and only 3·5 per cent. for 1 year or less.

3. Techniques Used

3.1. *Total Usage of Specified Techniques*

It would have been difficult to formulate a question on the use made of specific statistical techniques which would have elicited answers from which an estimate of total frequency of use could be made. Eventually no attempt was made to obtain any absolute figures and in Question 13 respondents were required to state merely whether they considered they used the techniques "frequently", "quite often", "occasionally" or "never". The exact interpretation of these terms was left to the respondents, and assuming consistency of use within each questionnaire it was felt that a meaningful picture of the relative frequencies of use of the various techniques would emerge.

In Table 14 the total number of persons using each of the techniques listed in Question 13 is shown in the "total usage" column, the techniques being arranged in rank order according to "total usage". In the other columns the breakdown according to frequency of use is shown.

Although correlation is seen to be used, at some time, by 84·0 per cent. of all respondents, it is used only occasionally by a large number of these persons; whilst the standard error test of significance although used by slightly fewer persons (77·4 per cent.) is used frequently by 29·8 per cent. of all respondents, and seems, from Table 14, to be the technique of which most use is made.

TABLE 14

Total Usage of Techniques

Techniques	*Number of Persons Using Techniques* Frequently	*Quite Often*	*Occasionally*	*Total Usage*	%
Correlation	186	201	377	764	84·0
S.e. test	271	162	271	704	77·4
Regression	229	192	255	676	74·3
Sampling design	139	120	398	657	72·2
Analysis of variance	215	121	297	633	69·6
"t" test	242	153	237	632	69·5
χ^2 (goodness of fit)	115	122	364	601	66·0
Moving average	177	141	275	593	65·2
Curve fitting	89	148	331	568	62·4
Seasonal variation	147	127	262	536	59·9
Design of experiments	154	109	262	525	57·7
χ^2 (independence)	133	123	252	508	55·8
"F" test	212	97	192	501	55·1
Ranking methods	42	81	339	462	50·8
Index numbers	142	90	208	440	48·4
Quality control	72	65	173	310	34·1
Non-parametric methods	27	55	215	297	32·6
Sequential methods	22	41	220	283	31·1
Factor analysis	18	25	157	200	22·0
Correlogram analysis	29	26	125	180	19·8
Discriminant analysis	9	27	119	155	17·0

Techniques used by between 25 per cent. and 50 per cent. of respondents were index numbers, quality control methods, non-parametric methods and sequential methods. Factor analysis, correlogram analysis and discriminant analysis were used by 22·0 per cent., 19·8 per cent. and 17·0 per cent. of respondents, although usually only infrequently.

The relative frequencies of usage of the techniques seem to be very much what one would expect. Those techniques most easily applied are made use of by the largest number of persons, and those such as factor analysis and discriminant analysis, which are most cumbersome to apply, are used less frequently. Indeed, the number of persons who had at some time made use of these techniques is surprisingly high, and may result from response bias.

Other techniques used frequently or quite often were asked for in Question 14 and the techniques quoted are shown in Table 15.

TABLE 15

Other Techniques

Techniques	*Number of Persons*
Stochastic processes, queueing theory	33
Monte Carlo methods	40
Unspecified applications of probability theory	20
Estimation and confidence limits	16
Analysis of covariance	30
Linear programming	28
Programming for electronic computer	3
Demographic and actuarial techniques	41
Population projections	6
Graphical methods	29
Probit analysis	18

3.2. *Fields of Application*

In Question 15 respondents were asked to list the principal fields in which they applied those techniques listed in Question 13 or specified in Question 14. In the form of the question and in the space left for answers, sufficient allowance had not been made for those persons who used a large number of techniques and found the required information rather lengthy and laborious to write out. This caused in some cases omissions and apparent inaccuracies.

The categories into which fields of application were divided were limited to seven for reasons concerned with the coding. These seven categories were:

1. Actuarial, demographic and sociological.
2. Economic (including market research).
3. Industrial (production).
4. Industrial (research, operational research).
5. Medical and psychological.
6. Biological and agricultural.
7. Other Scientific (including engineering, chemistry).

It should be noted that two types of industrial applications were distinguished, while the last category of "other scientific" applications included all applications in the fields of chemistry, physics, engineering and geology by persons not employed by any firm or trade organization. No informant specified applications in any of the artistic fields. No

attempt was made to elicit information concerning techniques taught or used in theoretical research.

In Table 16 the number of persons using the three techniques used "frequently" by the greatest number of persons in each field of application is shown. Similarly, Table 16 shows the total numbers using the three techniques used, however infrequently, by the greatest number of persons in each field.

TABLE 16

Number of Persons in each Field of Application using the Three Techniques with Highest Frequency of "Frequent Use" in that Field

Techniques	*Fields of Application* Act. and Soc.	*Econ.*	*Prodn.*	*Ind. Res.*	*Med.*	*Biol.*	*Phys.*
Standard error test	30	40	—	49	31	—	32
χ^2 test (independence)	22	—	—	—	27	—	—
"t" test	—	—	—	50	30	44	34
"F" test	—	—	—	46	—	42	31
Sampling design	—	46	—	—	—	—	—
Design of experiments	—	—	20	—	—	—	—
Analysis of variance	—	—	19	—	—	45	—
Ranking methods	—	—	22	—	—	—	—
Correlation	23	37	—	—	—	—	—

TABLE 17

Number of Persons in each Field of Application using the Three Techniques with Highest Frequency of "Total Use" in that Field

Techniques	*Fields of Application* Act. and Soc.	*Econ.*	*Prodn.*	*Ind. Res.*	*Med.*	*Biol.*	*Phys.*
Standard error test	100	—	—	—	52	—	88
"t" test	—	—	—	117	54	—	—
"F" test	—	—	—	—	—	58	—
Sampling design	101	—	53	—	—	—	—
Design of experiments	—	—	—	—	—	58	—
Analysis of variance	—	—	53	—	—	—	—
Correlation	92	194	53	117	53	56	88
Regression	—	—	—	115	—	56	85
Index numbers	—	199	—	—	—	—	—
Moving averages	—	209	—	—	—	—	—

The relative importance of the different fields of application of statistical techniques can be measured by the numbers of respondents using techniques in each field. The rank order of the different applications is then economic, industrial research, sociological, scientific, biological, medical, and industrial production. The frequency of use in the various fields varies considerably, however, and taking this into consideration a ranking might better be made in terms of the number of persons using techniques frequently. The following order then results: industrial research, biological, economic, scientific, medical, sociological, and industrial production.

If the ratio of use "frequently" to "total use" is taken as a measure of relative frequency of use of techniques, then this is highest among those working in biological and medical fields and lowest among those in economic and sociological work.

More persons apply techniques in the fields of economics and market research than in any other although comparatively few of the workers in these fields are applying any one technique frequently. Techniques with greatest frequency of application are sampling design, standard error tests and correlation, while those used by the greatest number of persons are moving averages, index numbers and correlation. Correlogram analysis has its main application in this field.

Applications of statistics in industrial and operational research, although made by fewer persons than in the commercial field, are made more frequently by these persons, and this field would appear to be the most extensive of the seven separately identified. The "t", "F" and standard error tests of significance are used more frequently than other techniques and it is in this field that most use is made of factor analysis and non-parametric methods.

The related field of industrial production showed very different characteristics; design of experiments, ranking methods, and analysis of variance being the techniques most frequently applied by the comparatively small number of persons in this field.

In biological and agricultural work the frequency with which statistical techniques are applied by the small number of respondents working in these fields seems to be at its highest, most use being made of "t", "F" and standard error tests, design of experiments, analysis of variance and regression. Discriminatory analysis was used "occasionally" by 26 persons, about the same number as in economic and industrial applications of this technique.

Applications of techniques in various scientific fields other than those separately identified were grouped together showing "t", "F" and standard error tests as techniques most frequently used. In medical (and psychological) work the number of persons using tests frequently was about the same, although there were fewer workers in the field. In the category of actuarial, demographic and sociological applications there were, however, a large number of persons using techniques "occasonally" but only a small number using any frequently. Those techniques used most frequently in these fields were standard error and χ^2 tests and correlation.

4. Books and Journals

Respondents were asked in Question 16 of the questionnaire to which three books on statistical theory they referred most frequently. In Question 22 they were asked to indicate the frequency with which they read articles in 9 of the most well known statistical journals, and in Question 23 to list other journals connected with work or research read frequently. In this section the results of the analysis of answers to these three questions are presented.

4.1, *Books Quoted*

Answers to Question 16 were coded according to whether or not any of the books quoted were of an advanced mathematical nature. Although the borderline between the class of "advanced" books and the elementary "cookery books" was difficult to decide upon, in practice any borderline book, such as Yule and Kendall's *Introduction to the Theory of Statistics*, was usually quoted with others either more or less mathematical, from which the code could be determined.

43·3 per cent. of all respondents did quote at least one book on mathematical statistics while 38·0 per cent. quoted only elementary books and 18·7 per cent. either gave no answer or the answer "none". Several of those who listed no books made comments to the effect that they had persons working under them to whom they left such things.

The number of different books quoted was very large, and in the analysis of actual books quoted a detailed inspection of a sample of 200 questionnaires was made. A total of 77 books were mentioned on these questionnaires, the twenty mentioned most frequently being listed in Table 18 in rank order according to number of mentions.

TABLE 18

Books Referred to Most Frequently

Author	*Book*	*Frequency*
Kendall	*Advanced Theory of Statistics*	65
Yule and Kendall	*Introduction to the Theory of Statistics*	48
Brownlee	*Industrial Experimentation*	23
Weatherburn	*A first course in Mathematical Statistics*	22
Croxton and Crowden	*Applied General Statistics*	19
Yates	*Sampling Methods for Censuses and Surveys*	18
Fisher	*Statistical Methods for Research Workers*	18
Davies	*Design and Analysis of Industrial Experiments*	17
Moroney	*Facts from Figures*	15
Davies	*Statistical Methods in Research and Production*	14
Snecedor	*Statistical Methods Applied to Experiments in Agriculture and Biology*	13
Tippett	*Methods of Statistics*	11
Mood	*Introduction to the Theory of Statistics*	9
Cramér	*Mathematical Methods of Statistics*	9
Cochran and Cox	*Experimental Designs*	8
Feller	*Introduction to Probability Theory and its Applications*	8
Connor	*Statistics in Theory and Practice*	7
Allen	*Statistics for Economists*	7
Hoel	*Introduction to Mathematical Statistics*	7
Ferber	*Statistical Techniques in Market Research*	6

Of the other 57 books, three were quoted five times, two quoted four times, five quoted thrice, thirteen twice, and thirty-four once. Although small differences in the number of times a book was quoted are obviously meaningless the particularly high frequencies of the books by Kendall and Yule and Kendall cannot be ignored, although a bias was present as the questionnaires were, when sent out, accompanied by a letter from Professor Kendall in his capacity as Director of the Research Techniques Unit.

4.2. *Differences According to Age, Occupation and Special Subject*

Do people refer to books more or less frequently as they get older? A classification of type of book quoted, according to age, could provide at least a partial answer to this question.

From Table 19 it can been seen that among respondents the percentage of persons within an age group referring to no books rose steadily with age, reaching a maximum of 55 per cent. among persons over sixty. Correspondingly, the percentage referring to some advanced book declined steadily with age from 67·0 per cent. of the 20–25 group to 19·0 per cent. of the over sixties. The percentage of persons referring to elementary books reached a maximum of 41 per cent. in the 40–50 age group and then dropped to 26 per cent. for the over sixties. As the percentage of persons doing statistical work within each age category declined from 72 per cent. of those under 25, to 34 per cent. of

those over 60, these figures concerning books referred to do not necessarily show any association with age. In fact, when a cross classification of age and books referred to was made for the group of 435 statisticians it was found that within the first four age groups the percentages of persons referring to no books and to advanced books did not vary to any great extent, changing noticeably only in the 60 and over class.

TABLE 19

Age and Statistical Books Referred To

	Age					
Type of Book	20–29 %	30–39 %	40–49 %	50–59 %	*60 and Over* %	*Total* %
Advanced	60·7	48·2	35·0	26·3	19·4	43·3
Elementary	35·2	38·0	41·3	37·9	25·8	38·0
"None"	4·1	13·8	23·7	35·8	54·8	18·7
Total %	100·0	100·0	100·0	100·0	100·0	100·0
Total No.	145	384	254	95	31	909

TABLE 20

Occupation and Statistical Books Referred To

Type of Book	*Stat.* %	*Sci.* %	*Econ.* %	*Admin.* %	*Act.* %	*Op. Res.* %	*Teach.* %	*Total* %
Advanced	56·8	41·1	21·5	16·8	12·1	69·2	33·3	43·3
Elementary	31·5	50·4	48·4	43·4	41·4	27·3	41·7	38·0
"None"	11·7	8·5	30·1	39·8	46·6	3·6	25·0	18·7
Total %	100·0	100·0	100·0	100·0	100·1	100·1	100·0	100·0
Total No.	435	129	93	113	58	55	24	907

In Table 20 the type of book to which reference is most frequently made is cross classified by occupation showing considerable differences between occupational categories. Whilst 69·0 per cent. of the 55 persons in operational research and 56·8 per cent. of the statisticians refer to advanced books, only 12·1 per cent. of the actuaries and accountants come into this category and 46·6 per cent. of them refer to no statistical books. These proportions give a fair picture of the extent to which the workers in the various fields require to use advanced techniques, and the proportion 31·5 per cent. of statisticians who refer most frequently to elementary books is of interest.

TABLE 21

Special Subject and Statistical Books Referred To

		Special Subject							
Type of Book	*No Degree No Reply* %	*Maths.* %	*Stats.* %	*Econ.* %	*Sci. and Med.* %	*Arts* %	*Techn.* %	*Genl.* %	*Total* %
Advanced	19·0	75·3	54·5	14·6	34·0	12·1	40·9	41·5	43·3
Elementary	51·6	15·3	32·3	56·7	54·3	45·5	40·9	41·5	38·0
"None"	29·4	9·4	13·2	28·7	11·7	42·4	18·2	16·9	18·7
Total %	100·0	100·0	100·0	100·0	100·0	100·0	100·0	99·9	100·0
Total No.	153	287	99	157	94	33	22	65	910

By cross classifying by special subject the results shown in Table 21 were obtained. Of the 287 graduates in mathematics 75·3 per cent. refer to advanced books and only 9·4 to no books. 54·5 per cent. of specialists in statistics refer to advanced books and 13·2 per cent. to no books, whilst 12·1 per cent. of arts graduates refer to advanced books and 42·4 per cent. to no books.

4.3. *Readership of Journals*

The percentage of total respondents to see regularly and read papers from nine specified statistical journals is shown in Table 22. It was intended that respondents should indicate whether or not they saw each journal regularly, as well as the frequency with which they read articles in it. In answer to Question 23, only 1 per cent. of respondents did not specify some other journal read regularly, 19·6 per cent. quoting various government publications, and 19·3 per cent. commercial or trade journals.

TABLE 22

Percentage of Respondents Viewing and Reading Journals

		Read		
Journals	*See Regularly*	*One Article or More per Issue*	*About One a Year*	*Less than One per year*
J.R. Statist. Soc., Ser. A	88·8	63·2	20·5	4·3
J.R. Statist. Soc., Ser. B	64·7	42·6	17·6	7·0
Appl. Statist.	52·3	44·2	10·5	4·5
Inc. Statistician	24·1	18·0	6·5	11·0
J. Amer. Statist. Ass..	20·5	14·7	8·8	9·2
Biometrics	20·2	14·9	7·4	9·8
Econometrica	11·2	5·4	8·0	14·5
Ann. Math. Statist.	18·0	8·5	10·5	12·5
Biometrika	28·1	19·2	10·9	9·8

5. RESEARCH

5.1. *New Techniques, New Applications, and Research Projects*

In Questions 18, and 19, information concerning research in statistics was sought. The questions asked were:

1. Do your official duties involve the development of new techniques?
2. Do your official duties involve the application of known techniques in new fields?
3. Are you at present engaged in any specific research project in statistics?

The third question was not sufficiently clear in its meaning and had a higher percentage of non-response than the other two questions. A more precise form of the question would have been, "Are you at present engaged in any research project involving the development of new statistical techniques or new applications of known techniques". The definition of a "research project" was not given and may also have led to some bias in response.

In Tables 23–26 the percentage of persons within each of the age, industrial, occupational and special subject categories answering in the affirmative to each question, is shown.

Of the total 910 respondents 32·6 per cent. replied in the affirmative to the first question, 52·5 per cent. to the second question and 33·6 per cent. to the third question.

TABLE 23

Percentage Affirmative Response within Age Categories

Question	20–29 %	30–39 %	40–49 %	50–59 %	60 and Over %	Total %
New techniques?	37·9	31·5	31·1	32·6	35·4	32·6
New applications?	68·3	52·9	45·7	47·4	48·4	52·5
Project now?	36·6	33·6	33·1	33·7	25·8	33·6
No. in category	145	384	254	95	31	909

TABLE 24

Percentage Affirmative Response within Industrial Categories

Questions	I Civil Service %	II Local Govt. %	III Research %	IV Univ. %	V Finance Commerce %	VI Manfg. %	VII Misc. %	Total %
New techniques?	22·8	24·8	35·5	51·5	30·6	35·1	14·3	32·6
New applications?	60·1	52·2	54·5	54·5	39·3	56·2	26·2	52·5
Project now?	24·1	27·4	40·0	65·7	23·4	26·4	35·7	33·6
No. in category	158	113	110	134	111	242	42	910

TABLE 25

Percentage Affirmative Response within Occupational Categories

Question	Stat. %	Sci. %	Econ. %	Admin. %	Act. %	Op. Res. %	Misc. %	Total %
New techniques?	37·0	34·1	28·0	23·9	27·6	36·4	11·1	32·6
New applications?	58·6	62·0	43·0	33·6	24·1	80·0	25·9	52·5
Project now?	43·0	23·3	26·9	23·0	15·5	38·2	29·6	33·6
No. in cateogry	435	129	93	113	58	55	27	910

TABLE 26

Percentage Affirmative Response in Special Subject Categories

Question	Maths. %	Stats. %	Econ. %	Sci. and Med. %	Arts %	Tech. %	Genl. %	No Degree %	Total %
New techniques?	46·7	35·4	20·4	26·6	27·3	27·3	29·2	24·2	32·6
New applications?	62·0	64·6	38·2	58·5	27·3	40·9	56·9	43·1	52·5
Project now?	46·0	51·5	27·4	25·5	27·3	31·8	21·5	17·0	33·6
No. in category	287	99	157	94	33	22	65	153	910

From Table 23 the proportions of persons whose work involves the development of new techniques and of those at present in a project are seen to be highest among those of age 20–29, but do not vary much with age. The proportion applying known techniques in new fields is highest (68·3 per cent.) again among the youngest age group of respondents, dropping to 45·7 per cent. of those in the 40–49 years category.

The proportion of persons developing new techniques and participating in research projects is at its highest among those at universities but the proportion of persons whose work involves new applications of techniques is at its highest in the Civil Service (60·1 per cent.) and is over 30 per cent. in all industrial categories, as is shown in Table 24.

In the occupational classification of Table 25 the development of new techniques is shown to be undertaken by 37·0 per cent. of statisticians, 36·4 per cent. of operational research workers and 34·1 per cent. of the medical and scientific workers. Among the economists and actuaries the figure is slightly lower but not considerably. The percentage of persons whose work involves new applications, however, varies considerably with the occupational group, being at its highest, 80·0 per cent., among the operational research workers, and at its lowest, 24·1 per cent., among the actuaries.

43 per cent. of the statisticians, 38·2 per cent. of the operational research workers and 15·5 per cent. of the actuaries were at present engaged in research projects.

The cross classification by special subject of degree showed the proportion developing new techniques to vary between 46·7 per cent. of mathematics specialists to 20·4 per cent. of economics specialists. The proportions of persons making new applications of techniques and currently involved in a research project were at their highest among the statistics specialists with mathematics a close second.

The persons with no degree or a general degree can be seen to be in work where only infrequently do they develop new techniques or take part in specific research projects, but nearly half of them indicated that they applied known techniques in new fields.

In answer to Question 20, of those 306 people currently engaged in one or more research project, 42 per cent. indicated that they were working alone on their projects, 33 per cent. were working as members of teams, and 25 per cent. were working on projects of their own as well as team projects.

Question 21 was somewhat vague, asking in what fields of applied statistics or statistical

TABLE 27

Research Interests

Techniques or Field	*Numbers*	%
Stochastic processes, Monte Carlo methods	56	6·2
Time-series analysis	20	2·2
Prediction, estimation, tests of significance	18	2·0
Other fields of mathematical statistics	94	10·3
Survey techniques	30	3·3
Economic statistics	87	9·6
Industrial techniques	46	5·1
Operational research	46	5·1
Actuarial techniques, social statistics	107	11·8
Psychological testing	8	0·8
Miscellaneous other	40	4·4
None or no reply	358	39·3
Total	910	100·1

theory respondents were most interested in doing research. Answers to this question indicated interests rather than fields in which research was actually being carried out. 39·3 per cent. of respondents gave either no reply or the answer "None". Other answers are shown in Table 27 for which as detailed as possible a breakdown of the data is given.

The largest categories can be seen to be formed of those interested in actuarial techniques and social statistics. It should be noted that this class does not include those who specified survey techniques as their field of interest. The other large class was formed of those interested primarily in various fields of mathematical statistics, no one of which was specified by more than four persons.

6. Publications

6.1. *Total Publications and Journals of Publication*

In answer to Question 24(*a*) respondents gave information concerning their last three books or articles published or accepted for publication. It was found that 50·8 per cent. of respondents listed no publications and of the 49·2 per cent. who had published books and articles 29·7 per cent. had published 3 or more articles and 5·3 per cent. had published a book. Of the persons with publications, however, 32·9 per cent. had published only in non-statistical journals and only 16·3 per cent. had published any of their last three articles in a statistical journal.

The number of persons who had published one, two or three of their last three articles in one of the nine journals listed in Question 22 is shown in Table 28.

TABLE 28

Journals of Publication of Last 3 Articles

Journal	Number of Articles 1	2	3
J.R. Statist. Soc., Ser. A	30	4	0
J.R. Statist. Soc., Ser. B	27	6	1
Appl. Statist.	25	1	1
Inc. Statistician	10	0	0
J. Amer. Statist. Ass.	2	0	0
Biometrics	12	3	0
Econometrica	3	0	0
Ann. Math. Statist.	10	2	0
Biometrika	31	11	2

6.2. *Rate of Publication*

A measure of frequency of publication rather than total output was obtained from answers to Question 24(*b*) which asked (in April 1957) for the total number of books and articles published since January, 1955. Answers to this question were cross classified by age, industry, occupation and class and subject of degree. Results are shown in Tables 29–33 in percentage form. 61·4 per cent. of respondents had published nothing since 1955, 27·9 per cent. had published up to 4 articles, 6·3 per cent. 5 or more articles and 4·4 per cent. one or more books.

TABLE 29

Publications by Age

Publications	Age: 20–29 %	30–39 %	40–49 %	50–59 %	60 and Over %	Total Persons
None	69·0	59·2	54·3	56·8	71·0	541
1 article	13·1	11·7	11·4	8·4	6·5	103
2 articles	5·5	7·8	6·7	7·4	3·2	63
3 articles	4·8	5·2	7·1	5·3	3·2	51
4 articles	1·4	3·4	3·1	2·1	—	25
5 or more	5·5	8·3	9·4	11·6	9·7	78
1 or more books	0·7	4·4	7·9	8·4	6·5	48
Total %	100·0	100·0	99·9	100·0	100·1	
Total No.	145	384	254	95	31	909

TABLE 30

Publications by Category of Employment

Publications	Industrial Category: I Civil Service %	II Local Govt. %	III Research %	IV Univ. %	V Finance %	VI Mfg. %	VII Misc. %	Total %
None	61·4	69·9	43·6	23·1	72·3	73·8	66·7	59·6
1 article	13·9	9·7	7·3	15·7	12·5	9·2	11·9	11·3
2 articles	7·6	8·8	9·1	9·7	5·4	4·2	4·8	6·9
3 articles	3·2	1·8	9·1	11·1	0·9	6·3	7·1	5·6
4 articles	3·2	1·8	4·5	5·2	0·9	1·7	2·4	2·7
5 or more	6·3	3·5	24·5	19·3	2·7	2·9	2·4	8·6
1 or more books	4·4	4·4	1·8	15·7	5·4	2·1	4·8	5·3
Total %	100·0	99·9	99·9	99·8	100·1	100·2	100·1	100·0
Total No.	158	113	110	134	111	242	42	910

TABLE 31

Publications by Type of Work

Publications	Type of Work: Stat. %	Sci. %	Econ. %	Admin. %	Act. %	Op. Res. %	Teach. %	Misc. %
None	55·6	50·4	61·3	73·5	75·9	63·6	54·2	100·0
1 article	12·6	14·7	14·0	3·5	12·1	9·1	—	—
2 articles	6·4	7·0	7·5	6·2	6·8	7·3	16·7	—
3 or more articles	18·8	24·0	14·0	9·9	1·7	20·0	20·8	—
1 or more books	6·4	3·9	3·2	7·1	3·4	—	8·3	—
Total %	99·8	100·0	100·0	100·2	99·9	100·0	100·0	100·0
Total No.	435	129	93	113	58	55	24	3

TABLE 32

Publications and Subject of Degree

Publications	Special Subject: Maths. %	Stats. %	Econ. %	Sci. %	Arts %	Tech. %	Genl. %	No Degree %
None	48·8	61·6	65·6	48·9	57·6	40·9	72·3	76·5
1 articles	12·2	13·1	8·3	10·6	12·1	18·2	7·7	11·8
2 articles	11·5	5·1	3·8	4·2	15·2	9·1	3·1	4·6
3 or more articles	22·0	15·2	14·7	34·1	6·0	27·3	13·9	2·7
1 or more books	5·6	5·1	7·6	2·1	9·1	4·5	3·1	4·6
Total %	100·1	100·1	100·0	99·9	100·0	100·0	100·1	100·2
Total No.	287	99	157	94	33	22	65	153

TABLE 33

Publications and Class of Degree

Publications	Class of Degree: 1st %	2nd %	3rd %	Pass %	None or No Degree %
None	39·3	60·9	70·4	70·7	70·7
1 article	15·0	9·3	11·1	8·1	12·1
2 articles	13·1	5·8	7·4	4·0	4·0
3 or more article	23·8	19·7	7·4	15·1	8·0
1 or more books	8·9	4·3	3·7	2·0	5·1
Total %	100·1	100·0	100·0	99·9	99·9
Total No.	214	345	54	99	198

The proportion of each age group with no publications in the period considered can be seen from Table 29 to show comparatively little variation with age, although at its lowest in the 40–47 age group. The percentage of persons having published 5 or more articles in the period rose steadily with age to 11·6 per cent. of the 50–59 age group, falling slightly in the over 60 group. A similar pattern was shown by the proportion of each group to have published a book in the period, which reached a maximum of 8·4 per cent. in the 50–59 age group.

The cross classification by industrial categories, of Table 30, shows much more marked differences, the percentage of persons in a category with no publications in the period varying from a minimum of 23·7 per cent. of those in universities (IV), to a maximum of 73·8 per cent. of those in manufacturing industry (VI). The percentage having published a book showed a corresponding maximum of 15·6 in the university group, and was 2·1 per cent. in the manufacturing group, although its minimum was in the Research group (III). Although few persons in this group had published books in the period covered it must be noted that the proportion of persons having published 5 or more articles was at a maximum of 24·5 per cent. in this group.

There were also differences, though less noticeable, in volume of publication by type of work, as can be seen in Table 31. "No publication" varied from 50·4 per cent. of the scientific group to 75·9 per cent. of the actuarial group. 8·3 per cent. of those doing non-statistical teaching and 7·1 per cent. of the administrative group had published a

book, this proportion being at a minimum of 3·2 per cent. in the economic group. Small differences in the percentage of a group having published a book are obviously meaningless, however, in view of the small numbers involved.

The classification of Table 32 shows the percentage of persons with no publications to vary from 40·9 per cent. of those with some technological subject as special subject of their degree, to 76·5 per cent. of those with no degree; the percentage for mathematics being 48·8, for science and medicine 48·9, and for arts subjects 57·6. The percentage of persons having published 5 or more articles is at its highest, 18·1, for science and medicine, and is 11·5 for mathematics. Book publication is highest among the arts specialists and lowest in the science and medicine group.

The variation of number of publications with class and type of degree is very marked, the percentage of persons with no publications falling from over 70 per cent. of those with third class degrees, pass degrees and no degrees, to 39 per cent. of those with firsts. Similarly, the percentage having published 5 or more articles or a book, is at a maximum among those with first class degrees and at a minimum among the thirds.

In interpreting the results of Tables 32 and 33 it must be remembered that as they refer only to persons in a limited range of work and belonging to a statistical society, inference about graduates in general cannot justifiably be made.

In answer to Question 25, as factors seriously discouraging or impeding their research, 52·1 per cent. of persons indicated "lack of time", 14·6 per cent. "difficulty of discussing with colleagues", 9·0 per cent. "lack of money", 6·5 per cent. "lack of references" and 3·5 per cent. "time taken to publish".

7. ATTENDANCE AT MEETINGS

Respondents were asked in Question 26 to list which meetings of the Royal Statistical Society and the Association of Incorporated Statisticians they had attended in the past year. In many cases the actual meetings were not listed but only the total number was given. It was obvious that the numbers were sometimes mere guesses. 44·9 per cent. of persons had been to no R.S.S. meetings, 30·7 per cent. to up to 3 meetings and 17·9 per cent. to 4 or more meetings. 6·5 per cent. had been to an A.I.S. meeting.

The number of meetings attended was cross classified by age, industry, occupation, subject of degree and class of degree. The results are shown in Tables 34–38.

TABLE 34

Age and Meetings Attended

	Age					
Number of Meetings	20–29 %	30–39 %	40–49 %	50–59 %	60 *and Over* %	*Total* %
None	37·3	43·0	46·0	53·7	67·8	44·9
1 R.S.S.	15·9	14·3	16·1	16·8	19·4	15·5
2 R.S.S.	15·2	8·1	7·1	5·3	3·2	8·5
3 R.S.S.	6·9	7·8	5·9	6·3	—	6·7
4 or more R.S.S.	22·8	18·5	16·9	14·7	6·5	17·9
Some A.I.S. no R.S.S.	0·7	4·2	2·0	1·1	—	2·5
A.I.S. and R.S.S.	1·4	4·2	5·9	2·1	3·2	4·0
Total %	100·2	100·1	99·9	100·0	100·1	100·0
Total No.	145	384	254	95	31	909

TABLE 35

Industrial Category and Meetings Attended

Number of Meetings	Industrial Categories: I Civil Service %	II Local Govt. %	III Re-search %	IV Univ. %	V Finance %	VI Mfg. %	VII Misc. %
None	48·7	53·1	42·7	36·3	45·5	43·3	50·0
1 R.S.S.	14·6	11·5	20·0	18·5	18·8	12·9	14·3
2 R.S.S.	9·5	8·8	7·3	4·4	10·7	9·2	9·5
3 R.S.S.	4·4	3·5	6·4	11·9	2·7	9·6	2·4
4 or more R.S.S.	15·2	15·0	20·0	26·7	12·5	17·1	21·5
Some A.I.S., no R.S.S.	3·2	1·8	1·8	0·7	1·8	4·2	2·4
A.I.S. and R.S.S.	4·4	6·2	1·8	1·5	8·0	3·8	—
Total %	100·0	99·9	100·0	100·0	100·0	100·1	100·1
Total No.	158	113	110	134	111	242	42

TABLE 36

Type of Work and Meetings Attended

Number of Meetings	Type of Work: Stat. %	Sci. %	Econ. %	Admin. %	Act. %	Op. Res. %	Teach. %	Misc. %
None	37·0	50·4	47·3	64·6	60·3	30·9	50·0	66·7
1 R.S.S.	15·0	17·1	21·5	14·2	10·3	12·7	16·6	33·3
2 R.S.S.	9·7	4·7	5·4	7·1	15·5	9·1	8·3	—
3 R.S.S.	7·4	8·5	7·5	2·7	1·7	10·9	4·2	—
4 or more R.S.S.	22·2	16·3	12·9	6·2	5·2	34·5	16·6	—
Some A.I.S., no R.S.S.	3·2	1·6	3·2	0·9	3·4	1·8	—	—
A.I.S. and R.S.S.	5·5	1·6	2·2	4·4	3·4	—	4·2	—
Total %	100·0	100·2	100·0	100·1	99·8	99·9	99·9	100·0
Total No.	435	129	93	113	58	55	24	3

TABLE 37

Subject of Degree and Meetings Attended

Number of Meetings	Special Subject: Maths. %	Stats. %	Econ. %	Sci. %	Arts %	Techn. %	Genl. %	None %
None	37·6	31·3	45·9	47·9	63·6	59·1	50·8	56·2
1 R.S.S.	15·7	17·2	19·7	16·0	9·1	18·2	12·3	11·8
2 R.S.S.	10·5	15·2	8·3	2·1	6·1	4·5	7·7	5·9
3 R.S.S.	9·8	1·0	9·6	7·4	6·1	—	4·6	3·3
4 or more R.S.S.	22·3	30·3	10·2	19·1	12·1	18·2	18·5	9·8
Some A.I.S., no R.S.S.	1·4	—	5·1	4·3	—	—	1·5	3·9
A.I.S. and R.S.S.	2·8	5·1	1·3	3·2	3·0	—	4·6	9·2
Total %	100·1	100·1	100·1	100·0	100·0	100·0	100·0	100·1
Total No.	287	99	157	94	33	22	65	153

TABLE 38

Class of Degree and Meetings Attended

Number of Meetings	Class of Degree: *1st* %	*2nd* %	*3rd* %	*Pass* %	*None* %
None	38·3	45·5	44·4	40·4	53·5
1 R.S.S.	17·3	12·5	16·7	24·2	14·1
2 R.S.S.	10·7	9·9	5·6	5·1	6·0
3 R.S.S.	9·3	6·4	—	10·1	4·5
4 or more R.S.S.	19·6	20·0	24·1	16·2	11·6
Some A.I.S., no R.S.S.	1·9	2·3	3·7	2·0	3·5
A.I.S. and R.S.S.	2·8	3·5	5·6	2·0	6·6
Total %	99·9	100·1	100·1	100·0	99·8
Total No.	214	345	54	99	198

From Table 34 attendance at R.S.S. meetings can be seen to be decidedly more frequent among the younger age-groups, only 37·3 per cent. of those aged 20–29 having attended no meetings, and 22·8 per cent. 4 or more meetings, whereas 67·8 per cent. of those aged 60 and over had attended no meetings, and only 6·5 per cent. of them had attended 4 or more meetings.

In the industrial cross classification of Table 35, attendance is seen to be highest among persons at universities, only 36·3 per cent. of whom had been to no meetings, whilst 26·7 per cent. had been to 4 or more meetings. The industrial category showing lowest figures of attendance at meetings is that of those employed in local government, public utilities and corporations.

Only 6·3 per cent. of those doing administrative work (Table 36) went to 4 or more meetings and 65·2 per cent. went to no meetings, whilst of the statisticians 22·2 per cent. went to 4 or more meetings and 37 per cent. to none. The operational research group showed highest attendance, 34·5 per cent. of persons going to 4 or more meetings and only 30·9 per cent. to none.

Statistics specialists showed highest attendance (Table 37), 31·3 per cent. having attended no meetings and 30·3 per cent. 4 or more meetings. Variation of attendance with class of degree (Table 38) shows less marked differences and although those with firsts show the lowest percentage (38·3) for no attendance, the highest proportion (24·1 per cent.) in the 4 or more group is of those with third class degrees.

Figures for attendance at A.I.S. meetings are not sufficiently large for cross classification to be meaningful.

SUMMARY

1. A postal survey of all Fellows of the Royal Statistical Society and Fellows and Associates of the Association of Incorporated Statisticians, in Great Britain, was carried out in April 1957. The purpose of the survey was to obtain information about those persons using statistical techniques and about the techniques they use. More specifically, information sought included place of employment, salary, qualifications, techniques used and fields in which used, research engaged in, books and journals read, meetings attended and publications.

2. Of the 1,211 persons who completed the questionnaire 910 were using statistical

techniques in their current work or research. These people completed Section II of the questionnaire and in this paper only that information provided by these 910 users of techniques has been discussed. In an earlier paper the data provided by all 1,211 respondents to Section I of the questionnaire were presented.

It was found that 26·6 per cent. of the 910 users of techniques were working in manufacturing industry, 17·4 per cent. in the scientific or non-scientific Civil Service, and 14·7 per cent. in universities. 47·8 per cent. were working as statisticians, about 14 per cent. as scientists and 13 per cent. as managers or administrators. The statisticians and operational research workers, of whom there were 55, were the youngest of the groups.

3. With respect to qualifications, 33·5 per cent. of respondents were graduates of London University, 18·1 per cent. of Cambridge and 8·2 per cent. of Oxford. 16·6 per cent. had no degree, 31·5 per cent. of persons had degrees in mathematics, 17·3 per cent. in economics, 10·9 per cent. in statistics and 10·5 per cent. in science or medicine. Nearly one quarter of all persons had obtained first class honours degrees whilst 37·9 per cent. had seconds and 16·8 per cent. thirds. 12 per cent. had a Ph.D. and 14·9 per cent. an M.A. or M.Sc.

4. 43 per cent. of persons had a salary over £1,450. Of the occupational groups the youngest groups had the lowest median salaries and the oldest group, the administrators, had the highest median. Those statisticians with first class degrees within each age group were better paid than those with seconds, thirds or no degrees.

Just over 50 per cent. of all persons were in new posts.

5. The statistical technique most often used proved to be the standard error test, which 30 per cent. of persons said they used frequently. Correlation was used by slightly more people but with a lower frequency. The technique used least, of those specified, was discriminatory analysis, although 17 per cent. of persons had used this technique.

6. The largest field of application, from the standpoint of the number of persons sometimes applying statistical techniques, was economic. The fields in which those working in them applied techniques with the greatest frequency, were the biological and medical fields.

7. With respect to books referred to, 43 per cent. of respondents quoted some book of advanced mathematical statistics and 19 per cent. quoted no books. The proportion of those referring to no books did not seem to depend on age. Journals most read were *J.R. Statist.. Soc.*, Ser. A, *Applied Statistics* and *J.R. Statist. Soc.*, Ser. B.

8. The development of new statistical techniques seemed to have little association with age. Of the industrial groups, manufacturing industry contained the largest number of persons who indicated that their work did involve the development of new techniques, yet they represented the smallest proportion of their group. The proportion was at its highest among those with a degree in mathematics and the overall percentage was 32·6.

With regard to new applications of known techniques 52·5 per cent. of respondents said that their work did involve this. The proportion was greatest among those with a degree in statistics, those in the Civil Service and those whose work was operational research or of a scientific nature. As in the case of new techniques there was little association with age, although the youngest age group 20–29 gave the proportionately largest affirmative response to both questions.

Answers to the question on present participation in a research project showed a similar pattern to that of the answers to the question on development of new techniques, the only

difference being that the percentage was at its highest among graduates in statistics rather than in mathematics.

Mathematical, economic and social or actuarial statistics were the fields in which the largest numbers of respondents expressed interest in doing research.

9. 49·2 per cent. of persons indicated that they had published either articles or books but of these persons only 16·3 per cent. had published any of their last three articles in a statistical journal. In the 27 months since January 1955, 48 respondents had published books, and 16·9 per cent. had published three or more articles. 59·6 per cent. had published neither books nor articles. Publication was highest for the 50–59 age group, for those in universities or research bodies, for those in scientific work with degrees in science or technology and for those with first class degrees.

10. Attendance at meetings of the R.S.S. or the A.I.S. was highest for the youngest age group, only 37·3 per cent. of whom had attended no meetings in the past year. 17·9 per cent. of all respondents had attended 4 or more R.S.S. meetings and this percentage was highest for those at universities, and for those in statistical work or operational research, those with degrees in statistics and those with third class degrees.

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A Note on the Paper by D. D. Kosambi and U. V. Ramamohana Rao on "The Efficiency of Randomization by Card Shuffling"

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Kosambi and Rao (1958) criticize the statistical methods used in testing extra-sensory perception by arguing that a percipient who is trying to guess by telepathy cards which he knows are dealt one by one from a pack consisting of 5 varieties of 5 like cards is likely to guess each variety about one-fifth of the time for each sequence of 25 cards. They claim, quite rightly, that under such conditions the distribution of the "number of hits" will not be binomial with $p = \frac{1}{5}$, $q = \frac{4}{5}$.

Unfortunately they leave the reader with the false impression that all investigators of *ESP* used this technique. This is all the more strange because the bibliography at the end of their paper refers to the work of Dr. S. G. Soal and to G. N. M. Tyrrell's book *The Personality of Man*, in Chapter 14 of which Dr. Soal's experiments are described as follows:

"Each sheet used at the agent's table contained 50 randomized numbers (1 to 5 inclusive) prepared by Dr. Soal beforehand, either from logarithmic tables, or from published tables of random numbers. Care was taken that no one but Dr. Soal should see this sheet until it was handed to the agent's experimenter. The latter, seated at the opposite side of the table, separated from the agent by the screen, had five cards in front of her bearing the numbers 1 to 5. Looking at the sheet of random numbers, she held up to the hole in the screen the card bearing the number next on the list. The agent, seeing this number through the hole in the screen, picked up and looked at the corresponding card in the row of five face downward in front of him. He then replaced the card, again face downwards in the row . . . . The percipient in the other room recorded his choice each time he heard the call, by writing down on a blank score-sheet the initial letter of the animal he selected."

Surely the binomial is the correct model for these experiments.

But even if the experiments were conducted in the way supposed by the authors the distribution of number of hits would not be that shown in Table 1 of their paper. Table 1 was calculated "on the basis of 5 independent stochastic variables added together where each of the variables has equal probability of every one of its 5! arrangements", and the total frequency was $(5!)^5$. The true total frequency is, of course, $25!/(5!)^5$ and the correct distribution is as shown in Table 1 of this note, which also shows the frequency percentages to four places of decimals of the true distribution, the authors' distribution, and the binomial distribution $(\frac{1}{5} + \frac{4}{5})^{25}$. It will be seen that the binomial distribution is much closer to the true than the author's distribution.

The above comments do not detract from the valuable experiments in card shuffling made by the authors, but it is suggested that there is little weight left in their criticism of the *ESP* investigations.

TABLE 1

Frequency Distribution of Hits

Hits	Distribution for 5 varieties of 5 like cards	Percentages: From previous Column	Kosambi and Rao	Binomial $(\frac{1}{5}+\frac{4}{5})^{25}$
0	2,671,644,472,544	0·4286	0·6628	0·3778
1	15,865,811,944,500	2·5452	3·3891	2·3612
2	45,731,782,136,000	7·3363	8·4386	7·0835
3	85,133,419,148,000	13·6572	14·0050	13·5768
4	114,899,955,351,500	18·4323	17·5290	18·6681
5	119,649,702,080,045	19·1943	17·5595	19·6015
6	99,898,095,752,500	16·0257	14·6417	16·3346
7	68,610,815,305,250	11·0066	10·4524	11·0842
8	39,465,495,670,000	6·3311	6·5190	6·2349
9	19,259,638,091,625	3·0897	3·6185	2·9442
10	8,049,523,797,520	1·2913	1·8109	1·1777
11	2,900,903,187,000	0·4654	0·8253	0·4015
12	905,725,731,000	0·1453	0·3455	0·1171
13	245,747,376,750	0·0394	0·1329	0·0293
14	58,035,729,000	0·0093	0·0475	0·0063
15	11,932,686,260	0·0019	0·0157	0·0011
16	2,133,510,000	0·0003	0·0048	0·0002
17	331,120,250	0·0001	0·0014	—
18	44,350,000	—	0·0003	—
19	5,141,000	—	0·0001	—
20	497,500	—	—	—
21	44,125	—	—	—
22	2,500	—	—	—
23	250	—	—	—
24	0	—	—	—
25	1	—	—	—
Total	623,360,743,125,120 $= (25!)/(5!)^5$	100·0000	100·0000	100·0000

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WHOLESALE PRICES IN 1958

BY THE EDITOR OF "THE STATIST"

Method of Construction of the Index Numbers

The following table illustrates the method of construction of the index numbers. The index numbers here given are based on the average prices for the eleven years 1867–1877. Take, for instance, the *Gazette* price of English wheat:

	s.	*d.*	
Average, 1867–77	54	6,	average point.
,, 1914	35	0,	64 or 36 per cent. *below* the average point.
,, 1930	80	7,	148 ,, 48 ,, *above* ,, ,,
,, 1936	53	3,	98 ,, 2 ,, *below* ,, ,,

The individual index numbers, therefore, represent simple percentages of the average point. The 45 articles are grouped in six categories. They are shown in detail in Table 5.

The general average is drawn from all 45 descriptions, which are treated as of equal value, and is the simple arithmetic mean.

TABLE 1

Construction of Index Number for 1958

1867–77 = 100

	Number of Commodities in Index	Total Numbers 1867–77	Total Numbers 1958	Index for 1958
General average	45	4,500	15,992	355
Food	19	1,900	5,945	313
Vegetable food	8	800	2,515	314
Animal food	7	700	2,428	347
Sugar, coffee and tea . . .	4	400	1,002	251
Materials	26	2,600	10,047	386
Minerals	7	700	3,775	539
Textiles	8	800	2,764	346
Sundry materials . . .	11	1,100	3,508	319

TABLE 2

*Annual and Monthly Index Numbers**

1867–77 = 100

	Year	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1902–1911 .	74	73·9	74·0	74·2	74·5	74·9	74·5	74·5	74·3	74·3	74·5	74·4	74·8
1912–1921 .	148	145·2	146·3	146·9	147·7	147·9	147·2	147·9	149·2	149·0	149·3	149·1	149·2
1922–1931 .	120	122·7	123·0	122·5	122·0	121·0	119·3	119·0	118·0	118·0	118·6	119·2	118·5
1932–1941 .	97	96·0	96·5	96·5	96·8	97·3	96·9	97·5	97·8	98·7	98·8	99·4	101·4
1942–1951 .	231	221·4	224·1	224·6	227·3	228·0	227·1	226·7	227·3	231·2	235·3	238·5	241·2
1943 . .	155	153·3	153·2	154·0	154·9	155·6	155·4	156·0	154·4	154·6	153·7	153·9	153·9
1944 . .	160	154·9	155·1	155·7	157·9	159·5	160·0	161·9	161·5	160·1	159·6	160·3	160·7
1945 . .	164	161·4	161·5	162·1	162·4	164·0	166·7	165·2	163·2	162·7	162·7	162·7	163·2
1946 . .	186	165·6	166·2	166·6	168·4	169·3	169·7	174·4	175·7	180·5	196·2	198·1	200·5
1947 . .	230	206·7	209·6	212·1	215·9	216·5	218·1	223·4	225·9	228·9	236·0	240·8	246·5
1948 . .	260	250·5	253·4	256·0	256·4	260·0	263·9	260·4	260·1	258·1	261·5	263·8	266·0
1949 . .	274	267·6	266·8	265·4	272·0	269·2	267·2	263·3	262·5	285·7	291·5	293·7	297·4
1950 . .	324	300·2	301·0	301·7	305·2	308·3	310·3	317·7	332·1	335·7	349·2	363·1	372·3
1951 . .	402	404·8	420·5	419·3	425·7	420·6	405·2	394·5	387·9	395·9	392·0	396·9	399·7
1952 . .	380	403·5	394·8	384·4	377·5	372·2	379·5	374·4	370·4	369·9	368·9	369·4	372·9
1953 . .	366	374·5	375·2	374·9	370·3	368·1	367·5	361·3	357·9	357·0	357·3	357·3	360·0
1954 . .	361	359·8	361·2	366·1	366·5	363·9	361·2	355·5	353·5	353·7	356·8	362·9	371·7
1955 . .	370	378·0	376·5	375·8	370·2	370·1	370·1	373·1	363·9	366·8	372·4	370·9	374·1
1956 . .	384	372·6	379·2	379·8	385·9	383·8	379·7	374·2	373·4	376·2	384·1	393·1	402·9
1957 . .	376	398·0	392·7	389·1	390·7	381·8	377·3	375·1	364·6	364·3	361·4	364·0	364·7
1958 . .	355	360·0	356·1	358·8	361·7	362·9	363·3	357·8	350·3	353·5	356·2	359·6	357·4

* The average of the twelve monthly figures of each year does not necessarily coincide with the annual figures, as the latter are calculated mostly from the average of 52 weekly quotations, while the former are based on end-of-the-month prices.

TABLE 3

Annual and Quarterly Index Numbers for Groups

1867–1877 = 100

	All Groups	Food				Materials			
		Total	Vegetable Food	Animal Food	Sugar, Coffee and tea	Total	Minerals	Textiles	Sundry Materials
1902–1911 . .	74	71	66	88	49	77	90	70	74
1912–1921 . .	148	146	136	167	100	154	166	151	148
1922–1931 . .	120	115	101	150	80	122	137	128	110
1932–1941 . .	97	88	84	116	49	103	131	94	91
1942–1951 . .	231	172	201	169	121	273	300	306	232
1943 . . .	155	138	156	156	72	167	187	166	156
1944 . . .	160	137	152	156	73	178	197	182	161
1945 . . .	164	139	155	156	78	182	209	189	159
1946 . . .	186	140	155	154	88	219	239	231	198
1947 . . .	230	156	191	149	100	284	304	295	263
1948 . . .	260	171	217	155	107	324	368	348	279
1949 . . .	274	201	240	185	151	328	382	356	273
1950 . . .	324	235	268	204	224	390	416	491	299
1951 . . .	402	265	305	227	249	502	516	639	394
1952 . . .	380	294	336	285	227	443	532	475	363
1953 . . .	366	325	365	317	261	399	473	430	329
1954 . . .	361	326	330	325	329	386	476	429	298
1955 . . .	370	329	313	367	281	400	529	410	310
1956 . . .	384	326	319	351	282	428	581	400	352
1957 . . .	376	313	295	343	298	422	570	408	338
1958 . . .	355	313	314	347	251	386	539	346	319
1957 1st Quarter .	393·3	328·9	304·3	351·2	339·1	440·4	594·9	423·3	354·4
2nd ,, .	383·3	321·2	285·9	372·6	302·4	428·5	571·2	429·3	337.2
3rd ,, .	368·0	304·0	292·3	336·8	270·7	414·7	560·5	404·8	329.0
4th ,, .	363·5	304·4	298·7	326·7	276·8	406·5	549·2	387·4	329·6
1958 1st Quarter .	358·3	303·6	312·5	320·9	255·4	398·1	537·2	372·4	328·1
2nd ,, .	362·6	321·7	336·1	348·0	249·1	392·4	532·5	364·8	322·9
3rd ,, .	353·9	311·5	295·5	364·2	251·3	384·9	532·0	354·1	313·8
4th ,, .	357·7	316·6	318·4	354·4	246·9	387·8	539·5	351·1	317·8

TABLE 4

Monthly Index Numbers for Groups

1867–1877 = 100

	Year	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1957													
Food:													
Total . . .	313	336·2	331·7	318·9	326·6	319·6	317·6	313·0	299·7	299·4	296·5	310·4	306·4
Vegetable food .	295	313·3	305·7	293·9	288·9	277·9	290·9	302·6	288·3	286·0	293·1	301·9	301·2
Animal food . .	343	357·1	353·7	342·8	378·1	377·7	362·1	348·1	332·3	329·9	315·6	337·5	327·0
Sugar, tea and coffee .	298	345·3	344·7	327·2	312·3	301·7	293·3	272·5	266·5	273·0	270·2	279·7	280·5
Materials:													
Total . . .	422	443·3	437·3	440·5	437·5	427·1	421·0	420·3	412·0	411·7	408·8	403·3	407·4
Minerals . . .	570	593·5	594·1	597·2	588·5	567·3	557·8	565·8	553·0	562·6	558·1	542·3	547·2
Textiles . . .	408	428·3	419·7	421·8	435·6	429·0	423·4	415·1	403·0	396·3	391·8	385·7	384·7
Sundry . . .	338	358·5	350·4	354·3	342·8	336·8	332·1	331·5	328·7	326·8	326·2	327·6	334·9
1958													
Food:													
Total . . .	313	301·3	300·2	309·2	315·6	321·7	328·3	314·0	305·8	314·8	313·5	318·7	317·6
Vegetable food . .	314	304·1	307·7	325·8	335·0	340·8	332·4	293·6	291·0	301·9	310·4	319·0	325·9
Animal food . .	347	318·5	316·6	327·6	334·4	343·2	366·4	373·2	358·1	361·4	351·7	359·7	351·8
Sugar, tea and coffee .	251	265·7	256·4	244·2	247·8	245·9	253·7	251·1	243·8	259·0	252·9	246·6	241·1
Materials:													
Total . . .	386	402·6	396·7	395·0	394·9	393·1	389·1	389·8	382·8	382·1	387·4	389·4	386·5
Minerals . . .	539	542·5	539·0	530·1	532·7	526·8	538·0	538·7	525·7	531·6	542·3	543·5	532·[illegible]
Textiles . . .	346	379·9	369·0	369·3	369·2	367·9	357·2	359·9	354·5	347·9	353·7	352·0	348·[illegible]
Sundry . . .	319	330·2	326·6	327·6	325·5	326·2	317·0	316·9	312·5	311·9	313·4	318·6	321·[illegible]

TABLE 5

Average Prices of Each of the 45 Commodities

1954 to 1958

			Average Prices					Index Numbers (1866–77 = 100)				
			1954	1955	1956	1957	1958	1954	1955	1956	1957	1958
Vegetable Food												
1	Wheat, English Gazette	*s.* and *d.* per qr.	121 0	98 7	109 4	92 6	93 2	222	181	201	170	171
2	,, Manitoba	,, ,, ,,	124 7	127 6	130 9	122 0	117 8	222	227	233	218	210
3	Flour	*s.* per sack (280 lb.)	$95\frac{1}{4}$	$97\frac{1}{4}$	$103\frac{1}{3}$	$102\frac{5}{16}$	$98\frac{5}{8}$	207	211	225	223	214
4	Barley, English Gazette	*s.* and *d.* per qr.	91 11	92 11	91 8	82 8	82 0	236	238	235	212	210
5	Oats, ,, ,,	,, ,, ,,	62 10	73 1	68 ·9	63 8	66 10	242	281	264	245	257
6	Maize, American mixed (*a*)	*s.* per qr.	$114\frac{9}{16}$	$114\frac{3}{8}$	$119\frac{5}{12}$	$100\frac{3}{4}$	$86\frac{3}{4}$	353	352	368	310	267
7	Potatoes, good English†	,, ton	304	$374\frac{5}{8}$	$454\frac{1}{2}$	$397\frac{1}{2}$	$635\frac{3}{4}$	260	320	388	340	543
8	Rice, Burma. No. 2	*s.* and *d.* per cwt.	90 0	74 3	63 6	63 9	64 3	900	743	636	638	643
Animal Food												
9	Beef, prime	*d.* per 8 lb.	187	234	200	204	222	317	397	340	346	376
10	,, middling	,, ,,	170	222	184	190	211	340	444	368	380	422
11	Mutton, prime	,, ,,	$196\frac{1}{2}$	214	$205\frac{1}{4}$	201	205	312	340	326	320	325
12	,, middling	,, ,,	139	182	181	191	187	253	331	330	347	340
13	Pork, large and small, average	,, ,,	196	206	$218\frac{1}{2}$	208	200	377	396	420	400	485
14	Bacon, Home Produced, Grade 1	*s.* per cwt.	296	$270\frac{5}{8}$	298	$276\frac{1}{2}$	$283\frac{3}{4}$	400	366	403	374	383
15	Butter§	,, ,,	346	364	339	294	$246\frac{1}{2}$	277	291	271	235	197
Sugar, Coffee and Tea												
16	Sugar, Java floating cargoes (*b*)	,, ,,	$26\frac{1}{8}$	26	$28\frac{3}{4}$	41	$28\frac{5}{8}$	92	91	101	144	100
17	,, Continental crystals	,, ,,	$35\frac{3}{16}$	$37\frac{1}{16}$	$42\frac{1}{2}$	$54\frac{3}{4}$	$36\frac{1}{4}$	153	161	185	238	158
18	Coffee, Ceylon plantation, low middling (*c*)	,, ,,	609	$438\frac{1}{2}$	491	439	392	700	504	564	505	450
	Rio, good	,, ,,	481	312	$321\frac{1}{3}$	313	278	752	488	502	489	434
19	Tea, average import price	*d.* per lb.	54·65	64·61	53·62	54·21	52·00	310	375	311	314	302
Minerals												
20	Iron, Scottish pig	*s.* and *d.* per ton	344 8	372 4	428·2	464 6	464 6	} 500	540	603	671	671
	Cleveland (Middlesborough) pig	,, ,, ,,	296 3	324 7	350·5	401 0	401 0					
21	bars, common	£ per ton	$31\frac{1}{4}$	$38\frac{7}{8}$	$43\frac{3}{8}$	45	45	379	471	526	545	545
22	Copper, Standard	,, ,,	$248\frac{7}{8}$	$351\frac{1}{5}$	$328\frac{3}{4}$	$219\frac{2}{5}$	$197\frac{11}{16}$	332	468	438	293	264
23	Tin, Straits	,, ,,	$707\frac{15}{16}$	$740\frac{3}{8}$	$787\frac{7}{10}$	$754\frac{1}{4}$	$734\frac{9}{16}$	670	705	750	719	700
24	Lead, English pig	,, ,,	$96\frac{4}{5}$	$106\frac{1}{16}$	$116\frac{1}{4}$	$96\frac{5}{8}$	$72\frac{3}{4}$	472	520	567	471	355
25	Coal, Best Yorkshire House*	*s.* per ton	$62\frac{1}{2}$	$66\frac{5}{8}$	$74\frac{1}{4}$	$79\frac{3}{4}$	$82\frac{7}{8}$	284	303	337	362	377
26	average export price	,, ,,	86·75	86·96	105·41	116·23	105·31	694	696	843	930	843

TABLE 5—*cont.*

		Average Prices					Index Numbers (1866–77 = 100)				
		1954	1955	1956	1957	1958	1954	1955	1956	1957	1958
Textiles											
27 Cotton, Middling American . .	*d.* per lb.	32·72	31·70	26·95	25·35	24·21	363	352	299	282	270
28 East Indian . . .	,, ,,	32·81	31·61	27·58	27·55	27·12	486	468	409	408	402
29 Flax, Medium grade continental retted . . .	£ per ton	$318\frac{5}{6}$	$312\frac{1}{2}$	$282\frac{1}{3}$	$270\frac{2}{3}$	260	635	592	543	511	463
Belgian import price . .	,, ,,	279·4	243·7	228·4	210·1	174·9					
30 Hemp, Manila for roping . .	,, ,,	92	$92\frac{3}{8}$	96	113	116	396	470	474	465	401
Italian S.B. . .	,, ,,	$216\frac{3}{4}$	$274\frac{1}{8}$	$273\frac{1}{2}$	250	$232\frac{3}{4}$					
31 Jute, L.J.A. Firsts . . .	,, ,,	94	$84\frac{5}{8}$	$89\frac{1}{2}$	$104\frac{1}{4}$	93	495	445	471	549	490
32 Wool, Merino, Port Philip, average fleece . . .	*d.* per lb.	$111\frac{1}{4}$	$99\frac{3}{8}$	$102\frac{7}{12}$	$100\frac{3}{4}$	$70\frac{3}{4}$	579	493	546	550	336
Merino, Adelaide, average greasy . . .	,, ,,	$69\frac{1}{8}$	54	$67\frac{1}{3}$	$70\frac{7}{12}$	$51\frac{1}{8}$					
33 ,, English Lincoln half hogs .	,, ,,	$63\frac{1}{4}$	$61\frac{3}{4}$	$61\frac{7}{8}$	$68\frac{1}{2}$	$53\frac{1}{2}$	320	313	313	347	271
34 Silk, Japanese	*s.* per lb.	$36\frac{2}{3}$	$34\frac{1}{4}$	$32\frac{7}{12}$	$33\frac{11}{12}$	$30\frac{1}{8}$	160	149	142	148	131
Sundry Materials											
35 Hides, River Plate, dry‡ . .	*d.* per lb.	$19\frac{3}{8}$	$17\frac{5}{16}$	$21\frac{1}{4}$	24	$22\frac{3}{8}$	281	245	253	272	260
River Plate, salted . .	,, ,,	19	$16\frac{1}{16}$	$15\frac{3}{4}$	$15\frac{7}{8}$	$15\frac{1}{4}$					
average import price . .	,, ,,	25·02	21·71	20·95	22·40	21·79					
36 Leather, dressing hides . .	,, ,,	$55\frac{7}{8}$	$49\frac{1}{4}$	49	49	47	390	387	373	378	400
average import price .	,, ,,	$79\frac{3}{4}$	$85\frac{1}{4}$	$80\frac{1}{2}$	$82\frac{1}{2}$	$92\frac{1}{8}$					
Tallow, Town	*s.* per cwt.	$68\frac{1}{4}$	$73\frac{3}{8}$	$73\frac{3}{8}$	$74\frac{3}{4}$	$75\frac{3}{4}$	152	163	163	166	168
Oil, palm	£ per ton	$71\frac{1}{4}$	$84\frac{1}{8}$	97	$92\frac{1}{8}$	83	183	216	249	236	213
olive	,, ,,	$263\frac{13}{16}$	260	416	345	279	528	520	832	690	558
linseed	,, ,,	$80\frac{1}{4}$	$102\frac{5}{8}$	$137\frac{1}{2}$	$119\frac{1}{8}$	$111\frac{3}{4}$	292	345	411	341	340
Seeds, linseed	*s.* per qr.	$182\frac{15}{16}$	$207\frac{1}{2}$	$232\frac{1}{2}$	$188\frac{1}{8}$	194					
41 Petroleum, motor spirit (c.i.f.) .	*d.* per imp. gall.	12·08	11·53	11·81	12·96	11·31	76	73	74	82	71
Kerosene (burning oil) c.i.f. . . .	,, ,, ,,	10·87	10·86	11·74	12·82	11·39	172	172	186	203	180
gas oil, c.i.f. . .	,, ,, ,,	10·32	10·70	11·44	13·03	11·09	259	269	288	327	279
42 Soda, crystals	*s.* per ton	190	$212\frac{1}{2}$	$222\frac{1}{2}$	$222\frac{1}{2}$	$222\frac{1}{2}$	206	231	242	242	242
43 Sodium nitrate	*s.* per cwt.	27	$27\frac{1}{4}$	28	30	30	193	195	200	214	214
44 Indigo, Bengal, good consuming .	*s.* per lb.	$12\frac{1}{2}$	$12\frac{1}{2}$	$13\frac{3}{4}$	15	15	172	172	190	207	207
45 Timber, hewn, average import price	*s.* per load	338·12	338·44	342·64	340·62	336·50	717	762	777	768	729
sawn, or split, average import price . .	,, ,,	479·04	530·16	543·02	534·72	494·33					

(*a*) La Plata from 1924–40; Argentine (Feeding Stuffs) from 1941–52; Yellow American subsequently. (*b*) Raw centrifugals. 96 per cent. Pol., from 1924. (*c*) East India, good middling from 1908–1947; Kenya from 1948. * From 1954 the quotations relate to the average pit price of all grades of Yorkshire coal. † Average of weekly quotations during the eight months, January to April and September to December. ‡ Nigerian Butcher Dry from 1956; Dry Bahias not quoted. § Dutch butter to 1954; subsequently average of Australian, New Zealand and Danish.

TABLE 6

Index of Silver Prices

The base of the Index Numbers given below is 60·84*d.* per standard ounce = 100, this being a parity of 1 fine oz. of gold to $15\frac{1}{2}$ standard ozs. of silver.

	Price per oz. Standard (d.)		*Index Number*	
	Average for year	*At end of December*	*Year*	*At end of December*
1942 . . .	$23\frac{1}{2}$	$23\frac{1}{2}$	18·1	18·1
1943 . . .	$23\frac{1}{2}$	$23\frac{1}{2}$	18·1	18·1
1944 . . .	$23\frac{1}{2}$	$23\frac{1}{2}$	18·1	18·1
1945 . . .	$30\frac{1}{2}$	44	23·1	31·1
1946 . . .	$48\frac{5}{8}$	$55\frac{1}{2}$	36·7	41·6
1947 . . .	$44\frac{7}{16}$	45	33·3	33·8
1948 . . .	45	$42\frac{1}{2}$	34·1	31·9
1949 . . .	$49\frac{1}{5}$	64	32·8	33·3
1950 . . .	$64\frac{4}{5}$	70	33·7	36·4
1951 . . .	$77\frac{27}{32}$	77	40·5	40·1
1952 . . .	$74\frac{3}{8}$	$72\frac{3}{4}$	38·7	37·9
1953 . . .	$73\frac{15}{16}$	$73\frac{3}{4}$	38·5	38·5
1954 . . .	$73\frac{1}{2}$	$74\frac{1}{4}$	38·4	38·6
1955 . . .	$77\frac{1}{2}$	$78\frac{3}{4}$	40·0	40·8
1956 . . .	$79\frac{1}{8}$	$79\frac{1}{2}$	40·8	40·9
1957 . . .	$78\frac{9}{10}$	$77\frac{1}{8}$	40·7	40·0
1958 . . .	$76\frac{7}{32}$	$75\frac{7}{8}$	39·4	39·1

TABLE 7

Gold and Silver: Production

	Gold Value of output (£ thousand)	*Silver (million ounces)*				
		United States	*Mexico*	*Canada*	*Australia*	*Central and South America*
1942 . . .	289,153	54·5	84·9	21·8	14·2	31·8
1943 . . .	226,397	44·8	71·2	18·6	10·3	29·3
1944 . . .	209,031	37·4	63·0	14·8	9·4	29·7
1945 . . .	199,116	29·3	61·1	14·0	8·1	27·4
1946 . . .	202,435	21·4	48·3	13·6	9·0	26·1
1947 . . .	206,434	36·1	49·2	13·5	9·5	23·6
1948 . . .	214,647	36·1	45·8	17·0	10·1	23·0
1949 . . .	320,887	34·6	49·5	17·6	9·8	24·0
1950 . . .	329,580	42·0	49·0	23·2	10·7	28·0
1951 . . .	322,623	39·9	43·8	23·1	10·8	28·7
1952 . . .	329,381	39·8	50·4	25·2	11·4	32·6
1953 . . .	325,500	37·7	47·9	28·3	12·4	35·5
1954 . . .	344,865	35·6	39·9	31·1	13·8	33·2
1955 . . .	340,732	36·5	48·0	28·0	14·6	35·3
1956 . . .	351,600†	38·7	43·1	28·4	14·4	37·5
1957 . . .	366,543†	38·7	47·2	28·4	14·5	37·3
1958* . . .	377,500†	34·0	47·5	31·0	15·5	42·0

* Estimate. † Excluding U.S.S.R.

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1—*Information Theory and Statistics*. By Solomon Kullback. New York, Wiley; London, Chapman & Hall; 1959. xvii, 395 pp. 9″. 100*s*.

The author presents this book as a study of logarithmic measures of information and their application to the testing of statistical hypotheses, in an attempt to give a unified derivation of known results, with some extensions. Matters relating to communication channels receive only passing mention; the applications are limited to samples of fixed size. It is the author's hope that "the experienced statistician will see . . . familiar things in a unified, if unfamiliar, way, and that the student will find this approach instructive."

Considerable familiarity with statistical theory is assumed. The author writes at a pleasing mathematical level which combines generality with precision; measure-theoretic concepts are used with restraint, and are carefully explained when introduced. A glossary is included, and there are three numerical tables. There are copious references, many worked examples, and numerous problems at the ends of chapters. Obviously a great deal of work has gone into the book.

After a brief historical review, the quantities known as the "Kullback-Leibler information numbers" are defined; $I(1:2)$ is a measure of the directed divergence between two distributions $f_1(x)$ and $f_2(x)$. By suitable specialisation and re-interpretation this definition is shown to include all previous ones. Various attractive properties are derived, such as additivity for independent events, positivity, convexity, invariance, and suitable behaviour when sufficiency obtains. Some interesting inequalities are derived concerning the first and second kinds of error. A discussion of limiting properties for very large sample sizes completes the first part of the book (80 pages).

Passing over the fifth chapter for the moment, the applications (244 pages) are, firstly, to various "counting" situations in which χ^2 tests would normally be used. Here the (asymptotically equivalent) likelihood ratio tests are proposed. There is no detailed investigation into the distributions of the test statistics, nor a comparison of power with that of the usual procedures. The discussion is carried as far as to the consideration of various hypotheses of independence in multi-way contingency tables; for nested hypotheses the procedure can be presented as an "analysis of information".

The other applications are to normal populations. Topics treated include the multivariate linear hypothesis, where the procedures reduce immediately to analyses of variance; other multivariate hypotheses, where the likelihood ratio tests are derived; and finally there is an interesting chapter on linear discriminant functions.

We come now to the heart of the book—chapter five (28 pages), which constitutes the bridge between the "population" mathematics of the first part and the techniques of analysis used subsequently. It is here that judgment must be passed on the degree to which the author has attained his object of unification. In the reviewer's opinion he has not attained it; the development is highly arbitrary and unappealing. Consider for example the steps involved in the derivation of the critical region for testing a simple hypothesis against a simple alternative. These seem to be four in number. First a quantity $I(*:2)$ is defined to be the minimum value of $I(1:2)$ for variations in $f_1(x)$, subject to the expectation (over $f_1(x)$) of some statistic $T(x)$ being held constant, equal to θ, say. Secondly it is stated that when a sample is available, $I(*:2)$ will be estimated by taking θ to be the observed value of $T(x)$; this gives $\hat{I}(*:2)$, which measures the "distance" between the sample and the distribution $f_2(x)$, on the basis of the statistic $T(x)$. Next it is stated that the null hypothesis (which specifies $f_1(x)$) will be accepted rather than the alternative ($f_2(x)$) if

$$\hat{I}(*:2) - I(*:1) \geqslant c$$

for some constant c (some confusion is introduced since this rule is not correctly stated at its first appearance on page 85). Finally, halfway through a paragraph in small type, we find that we are to take $T(x)$ to be $\log f_1(x)/f_2(x)$; and the desired result follows. Now even if we accept the first three steps, though arbitrary, as being not too unreasonable, the crucial point in the argument is the choice of the statistic $T(x)$; and here the complete absence of discussion (or even, apparently, recognition by the author that this constitutes part of the problem), is a fundamental weakness in a book of avowedly pedagogic character.

Similar obscurities pervade the whole of this vital chapter. What the author has done is to derive the classical techniques from an extended sequence of *ad hoc* rules for which no independent justification is even attempted. It is difficult to see how a student's understanding could be enhanced by this process; any instructor who attempts to present this material will be forced to work out the arguments for himself.

The reviewer has considerable sympathy with the author's intention; it is the more disappointing to find that such an inadequate attempt has been made towards achieving it.

C. L. MALLOWS.

2.—*Studies in Linear and Non-Linear Programming*. By Kenneth J. Arrow, Leonid Hurwicz and Hirofumi Uzawa, with contributions by Hollis B. Chenery, Selmer M. Johnson, Samuel Karlin, Thomas Marschak and Robert M. Solow. Stanford, University Press, 1958. London, Oxford University Press, 1959. [ix], 229 pp. 10″. 60*s*.

This book, the second in a series of Stanford Mathematical Studies in the Social Sciences, consists of an introduction (19 pp.) and 15 research papers, grouped into 3 parts: Part 1, Existence theorems (91 pp.); Part 2, Gradient method (59 pp.); Part 3, Methods of linear and quadratic programming (51 pp.).

Part 1 is concerned with the mathematical structure of linear and non-linear programming, and the development of a general theory of programming in linear topological spaces. The longest paper, No. 4, " Programming in linear spaces " (by L. H., 65 pp.) is primarily concerned with extending the Kuhn-Tucker (1951) results to general linear topological spaces. The paper includes the required background information on linear systems and topological spaces.

Part 2 is concerned with what is, perhaps misleadingly, called the gradient method for concave programming. The problem is to maximize some concave function $f(\mathbf{x})$ subject to the constraints $\mathbf{x} \geqslant \mathbf{0}$, $g_j(\mathbf{x}) \geqslant \mathbf{0}$, $(j = 1 \ldots m)$, where the functions $g_j(\mathbf{x})$ are concave.

Kuhn and Tucker (1951) showed that the optimum value of $\mathbf{x}$, say $\bar{\mathbf{x}}$, must satisfy the following saddle-point problem:

Define $\phi(\mathbf{x}, \mathbf{u}) = f(\mathbf{x}) + \mathbf{u}'g(\mathbf{x})$, and find $\bar{\mathbf{x}}, \bar{\mathbf{u}}$ such that $\phi(\bar{\mathbf{x}}, \bar{\mathbf{u}}) = \max_{\mathbf{x}} \phi(\mathbf{x}, \bar{\mathbf{u}}) = \min_{\mathbf{u}} \phi(\bar{\mathbf{x}}, \mathbf{u})$, subject to the conditions $\mathbf{x} \geqslant \mathbf{0}, \mathbf{u} \geqslant \mathbf{0}$.

This result can be regarded as an extension of the concept of Lagrange multipliers to cover inequality conditions. It still seems strange to me that it should be profitable to

transform a maximizing problem into a saddle-point problem. But in fact many, though not all, of the useful methods that have been proposed for non-linear programming problems are based on this transformation. And the gradient method of this book is in fact based on the gradient of the function ϕ in the saddle-point problem. The authors observe on p. 138 that one variant of this method "supplies an algorithm which is applicable to any concave programming problem. It appears to be the only such algorithm known." I think that this claim was justified when the book was published. Now two other approaches are available, both based directly on the maximum problem. Methods based on the gradient of the function $f(\mathbf{x})$ have been developed, see Rosen (1960) and Zoutendijk (1959). Another approach, due to J. E. Kelley, was announced at the summer meeting of the Econometric Society in 1958, and at the RAND Symposium on Mathematical Programming in 1959. This essentially involves maximizing an auxiliary variable, say x_0, and introducing a sequence of linear inequalities on x_0 and $\mathbf{x}$ implied by the constraints $f(\mathbf{x}) \geqslant x_0$, $g_j(\mathbf{x}) \geqslant 0$, derived by taking a linear approximation to $f(\mathbf{x})$ or some $g_j(\mathbf{x})$ in the neighbourhood of each trial solution.

Computational experience with all these methods seems very meagre as yet. One apparent disadvantage of the gradient approach of this book is that in its original form the iteration does not converge for linear problems. One may therefore suspect that the slight modifications introduced to overcome this will still tend to give slow convergence on linear or nearly linear problems. Paper 9, "An example of a modified gradient method for linear programming" (by T.M.) discusses a linear problem with 5 inequalities in 9 variables in which 2,000 iterations were required to give 1 significant figure in the value of the objective function. But much more experience is needed to confirm or refute this suspicion.

There are 4 papers in Part 3. No. 12, "An elementary method for linear programming" (by H. U.) describes an elegant method of computing all basic feasible solutions and the corresponding values of the objective function. No. 13, "Price speculation under certainty" (by K. J. A. and S. K.) describes two simple dynamic programming problems that are solved in the usual way, working backwards from the final time period. No. 14 presents a type of linear programming problem whose solution is trivial (once one has seen it). The proof seems unduly long ; since if the proposed algorithm requires a machine to tackle a task that it cannot do, then it follows that there are more tasks of this difficulty or greater than there are machines capable of tackling such tasks. The final paper "Non-linear programming in economic development" (by H. B. C. and H. U.) describes some problems that are solvable by methods related to the authors' gradient methods: one guesses shadow prices for the 2 primary factors—labour and foreign exchange, and hence computes in turn shadow prices for the other activities, and levels for all activities. One ends up with net demands for labour and foreign exchange, and if these differ from the available supplies one revises the guessed shadow prices for these factors accordingly.

To sum up, the reader should be warned that this is not a textbook surveying the whole field of mathematical programming. But the material will be of interest to specialists in this field ; particulaly those who think of it as a branch of theoretical economics or pure mathematics, and not simply as a branch of numerical mathematics.

E. M. L. BEALE.

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3.—*Operations Research for Industrial Management*. By Dimitris N. Chorafas. New York, Reinhold; London, Chapman & Hall; [c]. 1958. ix, 303 pp. 9″. 70*s*.

It would seem that a book on operational research must be directed at one of three sets of readers. It is either written for the operational research specialist, to give him an

account of some new developments or techniques now available in this field, or to act as a text book to help in the training of new recruits in the field, or to be an explanation of the subject to the interested consumer. Whichever of these audiences the book is directed at, it should recognize that operational research is a *useful* subject and therefore consists of more than a series of mathematical techniques.

In reading this book, one is continually forced to ask for whom it has been written, and ultimately is reluctantly compelled to the conclusion that it is not being written for any of the three audiences named above. It deals in an elementary way with most of the mathematical techniques which have been developed, but could not contribute to an experienced operational research worker's understanding of the subject, and the matter dealt with has already been covered very thoroughly by other general books on operational research and by books on the specialized techniques.

If it is directed to the eager student it will do him a very grave disservice in that it will give the appearance that operational research is nothing but a collection of mathematical techniques, and all the difficulties of discovering what is the proper technique for a given problem are quite lost sight of.

If it is aimed at explaining to management what operational research is and how it can help them, it will not be very successful. A business man will surely want to see some examples of how operational research is helping a real situation. Almost without exception, the examples in this book are imaginary ones which have been simplified to such an extent that they are readily recognized to be unrealistic and therefore are unlikely to impress any layman.

The book starts, as do most books of this kind, with a philosophizing type of account of where operational research fits into science and it must be said that this is rather better done than usual.

This high promise is continued into the next chapters where some simple examples are converted into mathematical problems in a quite sensible way and they are then followed by a chapter on the dangers involved in abstracting a model from the real situation with which no-one can quarrel. From this point on, one might almost be reading a different book, because all the lip service paid in the first chapters is entirely forgotten in an enthusiasm for the untrammelled application of the author's favourite mathematical topics.

The theory of games is dealt with in a very simple way as a mathematical theory and its relation to the real life is entirely misrepresented. Since this theory aims at maximizing expectation of gain, it is clearly only relevant in cases where there are likely to be a series of repetitions of the play of the game. Notwithstanding, Dr. Chorafas proceeds to give illustrations of its application in situations where the game can only possibly be played once, as for example in a military situation.

A chapter is then devoted to business games which rather uncritically accepts these games as being useful.

However, it is in the chapter on research on management analysis that all caution is thrown to the winds and the author's own strictures on realism disappear in a welter of mathematical symbols. This chapter describes a complex situation where a system is described by a set of variables which are related by a series of equations so that each variable is a function of the others. The reader is cautioned that these functions will have certain natural upper and lower bounds, but then, when enough pages have elapsed for us to forget this, the assumption that all these functions are linear is introduced with some delightful simplifications which I leave to the reader's imagination.

The most interesting chapter is the case study of the AMA top management decision game which five mythical companies with business men controlling them played. The chapter gives for each mythical company in turn a diary of events in the course of the game, explaining the motives reported by the participants for what they did and the arguments they used for justifying their actions. These arguments are the strongest case made in the book for the need for operational research, but unfortunately by this time there have been so many artificial examples that the book fails to convince one of the reality of this case study.

Naturally, linear programming must find an important place in a book of this kind and one has to search hard to find any criticisms of the value of this technique.

The method of regarding a system as a linear transformation from input to output, and dividing the system into stages with each transformation represented by a matrix so that the total transformation of the system is given by the product of the individual matrices, forms a further chapter which leads on to a discussion of transportation problems.

The last chapter deals with flow problems and this manages to get across the very simple ideas evolved in this technique which have been rather hidden in a mass of abstract algebra in some of the earlier American writings on this subject. K. D. TOCHER.

4.—*Sampled-Data Control Systems*. By John R. Ragazzini & Gene F. Franklin. New York & London, McGraw Hill, 1958. ix, 331 pp. 9″. 74*s*. (McGraw-Hill Series in Control Systems Engineering).

The advent of commercially available electronic computers has led to the consideration that these might be used to control industrial processes. This field has long been the prerogative of servo-mechanists but, as part of the process of assimilating digital techniques into the control field, methods have been devised which replace the continuous signals, handled by servo-techniques, by discrete signals occurring only at fixed intervals of time whose magnitude represents the instantaneous value of the continuous input.

This book considers how such sampled data can be handled, develops the necessary mathematical tools and shows how to apply those tools to the design of control systems with desirable characteristics.

The simplest way to adapt servo-mechanism theory is to reconstruct the data from the sampled input and this is done by a mechanical version of one of the Newton-Gregory interpolation formulae. In practice, a linear interpolation is found to be adequate. Although such interpolatory functions seem adequate when viewed as an approximation to the input, the book demonstrates that there are startling differences between the spectrums of the original signal and the reconstructed one. It is shown theoretically how complete recovery of the original signal is theoretically possible only if the input signal has a finite band width and the sample repetition rate is high enough.

Just as in ordinary servo-theory the Laplace transform is a powerful theoretical tool, so the generating function of the sequences of values taken by the input can be used extensively. The conventional use of z as a carrier variable has led to this generating function being known as the Z transform of the sequence. An analogous theory to that of the Laplace transform is, of course, available for these generating functions. In the same way that the behaviour of some network can be described by the ratio of the Laplace transforms of its output and its input, so the pulse transfer function can be defined as a ratio of the two Z transforms.

The rules of combination of pulse transfer functions for circuits in series follows the usual pattern, but it is necessary to consider the effect of an inversion of the two processes of continuous transformation and sampling which the book brings out very clearly.

Once the method of writing down the transfer equations of complete systems is understood, it is necessary to consider the stability of the system. This can be done by analogues of the Routh-Hurwitz criterion and also by considering the frequency response.

There are severe difficulties in the design of sample data control systems if one attempts to follow the pattern of conventional servo-mechanism design which is caused essentially by the complex relation relating to a sample transform and a general continuous transform. The book describes the series of approximate methods for dealing with this problem and leaves one distinctly with the feeling that this is not a practical proposition.

However, the next chapter saves the situation by using a digital form of control in which the sample sequences are transformed to other sample sequences directly without the intervention of interpolatory transforms. The mathematical element then regains all the elegance which had been lost and it is possible to design circuits to meet the usual criteria of minimum settling time and so on.

A complication arises for sample data systems in that digital control can only prescribe

the behaviour of the values at the sampling times and it is quite possible for a ripple to occur at the sampling frequency which would not be detected by the ordinary analysis. To study such ripple, the analyst must insert fictitious sampling arrangements at twice the normal frequency to enable the output to be inspected between the ordinary sampling points, and this leads to the need for generalizations to deal with systems in which the sampling rates on different inputs are different.

So far, the book has paralleled the development of servo-theory and has been dealing as if the inputs were known. The extension of this to the case of random signals is carried out by the analogue of the Weiner analysis.

This subject has grown very rapidly in only a few years and it is interesting to see how much clearer and more connected an account can be given of such a young subject when it is possible to base the developments on the long history of servo-theory.

The book is beautifully written and can either be read very seriously, following the multitude of carefully worked examples through in detail, or lightly skimmed by those less directly concerned but with the wish to understand the scope of this new technique. For the really busy man there is a summary at the end of each chapter.

For those who have an interest in automatic computers, process control or industrial applications of statistics, this book is quite valuable as helping to knit these topics into a coherent whole.

K. D. TOCHER.

5.—*Statistical Quality Control: An Introduction for Management.* By D. H. W. Allan. New York, Reinhold; London, Chapman & Hall; 1959. [ix], 129 pp. $7\frac{1}{4}''$. 28*s*.

This little book is the third to appear in a series called the "Reinhold Management Science Series".

In reviewing an American book on statistical quality control it is a pleasant change to be able to write of "this little book", for in addition to the small area of the page and the small number of pages the print is fairly large, so that the whole book can be read in a few hours. This of course is just as it should be if the book is to serve its intended purpose as "an introduction for management".

Assuming a manager to start off with no statistical knowledge he may be either in the position of having no statistical staff and wishing to know whether he should engage any, or in the position of already having some and wishing to know what they think they are doing. In either case he will want answers to the questions: what is statistical quality control for? Why is it necessary? How does it work? What costs and organization are involved? In addition it would be helpful if he could obtain some slight knowlege of the statistician's jargon, but not too much, since in managing almost anything there will be other jargons requiring his attention.

Judged by these requirements, Mr. Allan's book should serve its purpose very well. The first sentence—"Variation is always present in the measured quality of manufactured products"—is an excellent beginning. The book might be compared with Beethoven's fifth symphony in starting straight off with the main subject for discussion, and no time wasted.

I think it was a mistake to include, in the second chapter, six pages quoting the report of the Special Committee on Professionalism of the American Society for Quality Control. This makes rather dull reading compared with the author's own style, and it would be a pity if a manager were to put the book among "Books I never finished reading" at this stage.

The example used for illustrating the main ideas of a frequency distribution (Table 2) seems a very odd one, and therefore ill-chosen for its purpose. From a total of 109 employees it seems difficult to believe that 12 would be aged 37 years, 21 aged 40, 13 aged 42, and yet none aged 38, 39 or 41 years.

The section on shapes of frequency curves is not very satisfactory. The peaked distribution shown in the picture is certainly leptokurtic, as it should be, but the description would seem to refer to a distribution with a small standard deviation rather than to the shape. The important point that has been missed is that a leptokurtic distribution has

more observations than a normal curve in the tails as well as more in the centre, and it is the intermediate ranges that have fewer observations.

The triangular distribution is described as being the worst case of a skewed distribution, but surely a "J-shaped" distribution, which receives no mention, is worse (in the sense in which "worst" is being used).

The picture of the rectangular distribution is not truly rectangular at all but a flat-topped distribution with curved tails. Now a rectangular distribution is a precise concept, which it is wrong to dilute in this way. If it is this shape that the author wishes to discuss, he should have found another name for it—perhaps the plateau distribution would be suitable.

The picture of the normal distribution seems to have been reflected about the mean, the positive scale appearing on the left, the negative scale on the right.

In the discussion on the interpretation of control charts it is stated that a shifting mean with constant variability will show on the average chart while the range chart indicates that the variation is unchanged; this is correct. It is next stated that a lack of control due to changing variability with a constant mean would show on the range chart, but no shift would appear on the average chart; this is not correct. An increase in variability will often put averages outside the limits calculated from a smaller variability.

The essentials of acceptance sampling are quite well covered in a short space, but some indication that it is conditional probabilities that are shown by an operating characteristic curve would be helpful, since this is a point on which the novice so frequently seems to go wrong.

The final chapter is concerned with statistical methods for investigation and experimentation, and this is a useful part of the book, since it would be a pity to leave management with the idea that control charts and acceptance sampling are the only uses of statistical methods in industry. A clearer discussion of the meaning of "not significant" would have been an improvement. It would be easy, after reading this chapter to conclude that "not significant" meant "undoubtedly due to chance alone".

The book finishes with a four-page bibliography divided into sections. Doubtless I shall not be the only person to be surprised at finding Barlow's Tables in the section headed "Handbooks on Quality Control Techniques". I. D. HILL.

6.—*Introduction to Difference Equations*. By Samuel Goldberg. New York, Wiley, 1958. London, Chapman & Hall. xii, 260 pp. 9". 54*s*.

In the preface the author writes: "Although I hope it will find favor with students and teachers of mathematics, this book is primarily intended for social scientists . . . . Wherever possible, the mathematical topics in the text are related to and illustrated by material drawn from the social sciences, especially economics, psychology, and sociology."

The book therefore has two aims. The impression given to the reader is that, whatever the author's intention, the ways of thought of the mathematician have tended to take precedence over those of the social scientist who wishes to learn some mathematics. The treatment tends to be formal throughout, and definitions (of, for instance, a linear difference equation) are set out in the language of the pure mathematician. The order in which the different topics are taken confirms this feeling.

Chapter 1 deals with the calculus of finite differences. The operators Δ and E are introduced, the equivalence of operators is defined, and the operator Δ^{-1} is then discussed. Chapter 2 starts by discussing what is meant by a difference equation and its solution. Then the author goes on to discuss sequences and limits and only in the second half of the chapter are applications of these equations introduced. Chapter 3 deals with linear difference equations and Chapter 4 with some special topics: equilibrium and stability, cobweb cycles, boundary value problems, generating functions, and matrices. Answers are given to the numerous problems.

The book does not seem to have avoided the risk of falling between two stools. The mathematical undergraduate who reads it will learn something about difference equations, but will have to skip a very considerable amount which he will find much too simple or

which he will already know. On the other hand, he may get interested in some of the examples from the social sciences. The social scientist will, unless he is exceptional, find the formal approach discouraging, especially in the first two chapters. He is likely to be put off a subject which should be both intuitively appealing and useful to him.

One feels that if the author had co-opted a social scientist to help him, the book might have read more like a work on *applied* mathematics, and have more nearly achieved what it primarily set out to do.

C. B. WINSTEN.

7.—*An Introduction to Mathematics for Students of Economics.* By J. Parry Lewis. London, Macmillan, 1959. x, 394 pp. $8\frac{3}{4}''$. 40*s*.

It is still true that the majority of people who set out to study economics have virtually no knowledge of mathematics. Such students, unless they take steps to remedy their ignorance, must find that a large body of articles and books on more advanced topics in economics is closed to them. The present introduction to mathematics adds one more to the growing collection of books intended to help such students in getting the relevant knowledge. It attempts to avoid the pitfall common to many of these books that in a desire to get on with some mathematics of direct economic application the pace is rapidly made too hot for most of the prospective readers. This book, although selecting topics most likely to be of use to economists, covers the ground much more in the manner of a school course in mathematics. It starts with a revision of elementary algebra and a discussion of summation and convergence of series, permutations and combinations, and the binomial theorem; there follows some trigonometry and co-ordinate geometry (including a discussion of conic sections), differentiation and integration, maxima and minima for functions of one and of several variables (including the use of Lagrange multipliers), homogeneous functions, and differentials. The final section deals with difference and differential equations; there are also several appendices on such topics as determinants, Newton's method for numerical solution, and the solutions of a cubic equation. The book contains a large number of exercises to which answers are given. Throughout the emphasis is mainly on the basic principles and on the practical applications of the mathematical techniques.

Within the scope which he has set himself, the author has been highly successful. The exposition is clear and the reader who works through the book carefully should have no difficulty in applying the techniques discussed in other similar cases. The economic examples are, perhaps, a little less satisfactory than the pure mathematics. At times they look too artificial, as when the relation $Y_t = C_t + I_t$ in the usual notation is used as a definition of I_t in a multiplier-accelerator difference equation model; at others, the way in which the analysis is terminated seems more the result of mathematical curiosity than economic importance. The term "volume of sales" is used both when quantity and when value is meant and there is one mild howler, due to a mathematical slip, in the statement that the marginal product is at a stationary value when it is equal to the average product of a factor. These points may deter somewhat a student who is initially rather sceptical about the value of using mathematics in economics, but are clearly not otherwise important.

The main test of the book is whether it gives the student what he needs. One potential reader is the student who hopes ultimately to be able to contribute to mathematical economics himself. He may be irritated by the reiterated "it can be shown that" and by the rather condescending tone of parts of the book; he would benefit much from the omitted proofs. Another potential reader wishes to be able to follow articles in journals but is without greater ambition. The detailed examples of differentiation hardly seem necessary for him. Possibly the book would have benefited both types of reader more if it had been pruned of some of the detail and of a few topics of minor importance to the economist (when has one met a surd in an economic argument?) and the space used to give proofs which need be read only by the first type of reader. However, the author has lectured on mathematics to students of economics and perhaps he has found that many students do benefit from his type of approach. These students have a textbook which is unlikely to be bettered.

W. J. CORLETT.

8.—*Modern Business Statistics.* By J. E. Freund and F. J. Williams. London, Pitman, 1959. xv, 539 pp. 9″. 50*s.*

This is the English edition of a book published in the United States in 1958. It is intended as a text book for students studying statistics as part of business administration courses in the United States, but a book of this type could also, *prima facie*, be useful to the very many students in Britain now taking such examinations as those leading to the B.I.M./Ministry of Education Diploma in Management. In this respect it suffers to some extent from the disadvantage of drawing its examples from American experience and dealing only with sources of American data.

The book follows the conventional pattern of the many available text books of business statistics. It begins with a brief discussion of frequency distributions and proceeds to a description of the various forms of mean and dispersion. Then follow chapters on sampling and estimation and the testing of hypotheses; pictorial presentation is dealt with very briefly. Towards the end the authors devote 13 pages to quality control.

Neither the coverage of the book nor its treatment of elementary methodology is such as to make it more suitable than existing texts; certainly its advantages, if any, are not sufficient to recommend it to students in preference to the currently used and well tried texts.

Its treatment of methodology is by no means novel but the book is written in an easy, chatty, fashion and this may appeal to the students for whom the book is intended.

How adequately a book of this type serves its readers will depend upon what the student is expected to learn about statistical method during his course and to what extent he will be expected to apply the methods taught on taking up a business appointment. If the aim is to acquaint readers with forms of statistical summarization and elementary analysis and the ways in which these computations can be done, then this book, like so many others, serves its purpose well; and one should add that, like many other similar American texts, it instructs clearly.

But if the aim is to help the business student to understand and interpret the results of the application of statistical method, then it is rather less successful. The authors are right to concentrate on "the logical foundations underlying the statistical solutions", but it is very doubtful whether many "beginning students in business economics" will understand these without much assistance. Certainly the chapter on probability theory needs to be much fuller and couched in terms which the student with a scanty mathematical background can understand.

The authors are right to attempt in a book of this sort to deal with "the newer topics of statistical inference". Practising business statisticians unable to devote sufficient time to reading the appropriate journals would doubtless welcome a book which describes new methodology, and how this may be applied to business statistics. But to do this within a book of this size requires much less concentration on methods of computation and much more discussion, in non-mathematical terms, of the problems of inference.

A book of this type might surely serve the useful function for the practising business statistician of bringing up to date his knowledge of methodology. The book under review is disappointing in this respect; its reference to newer methodology is largely confined to a brief discussion of the theory of runs and sequential sampling.

It is possible to be critical, too, about its choice of the older methodology. Thus when it says that power functions are rarely fitted to business data and for this reason are not illustrated with numerical examples, surely it overlooks the occasions, sometimes quite frequent, when the business statistician will use them to examine demand elasticity or sensitiveness as it might preferably be called in business circles?

One obvious misprint occurs on page 417 where a reference to the methods of section 18—3 is incorrectly given as section 18—2.

To summarize, this volume is competent but adds little to the selection of business statistics books already available from British and American authors.

E. SHANKLEMAN.

9.—*The Structure of British Industry: a Symposium.* Edited by Duncan Burn. Vol. 1, xvii, 403 pp., 45*s*. Vol. 2, xiii, 499 pp., 50*s*. Cambridge University Press, 1958. 9″. (Nat. Inst. of Econ. and Soc. Res., Economic and Social Studies, XV.)

These two volumes are among the most important products of the generous Conditional Aid Scheme. With this assistance a survey of British industry was undertaken under the sponsorship of the National Institute of Economic and Social Research. Apart from a concluding chapter by the editor, Duncan Burn, which consists of a general summary and discussion, the work comprises separate and independent chapters on different industries contributed mainly, but not exclusively, by academic economists. There are obvious dangers in such a plan of campaign but, by and large, the venture has been unexpectedly successful. It is difficult to fault any particular chapter in detail and as the complete work already runs to some 900 pages it would be uncharitable and churlish to ask for more. Some shortcomings must, however, be mentioned.

Firstly as regards the scope of the survey. In the non-manufacturing sector of the economy, Distribution and the Public Utilities group of industries are not included but there are chapters on Agriculture, Building, Coal, and Surface Transport. Within manufacturing industry, Machine Tools and Electronics are the only parts of the vital Engineering group included. Clothing, Food, Drink and Tobacco, Timber, Paper and Printing are not represented at all. Yet while a reviewer must beware of referring to any sector as a minor one, certainly the particular selection of industries included would be difficult to defend on any grounds other than availability of contributors. Then again, there is considerable duplication of material recently published elsewhere. Professor Hague contributes a chapter on man-made fibres more or less simultaneously with the publication of his book on the industry. Mr. G. W. Furness writes on the cotton industry just when Dr. R. Robson has published the authoritative modern work on the industry.

In a major study with this title it would have been helpful to include an introductory chapter in which an overall picture of British industry was painted. Information on the distribution of employment and net output by industry, on the geographical location of industry, capital investment, the pattern of foreign trade is, of course available elsewhere and the editor disclaims any intention of providing a statistical digest. Perhaps, too, such a summary is not so essential in a survey in which incompleteness is implicit in the method adopted. Nevertheless the title of the work does justifiably give rise to expectations which are only partly realized in the event.

With regard to the treatment adopted in the various chapters, generally there are excellent descriptions of productive processes and of industrial organization. There is much of value on marketing and price policy. But there are some disappointments. Industrial relations are not always adequately covered and matters relating to finance are treated somewhat unevenly. Yet each chapter is already a mine of information and it would be very difficult to suggest what should go if this were the cost of further inclusions.

It is claimed that this work is intended for the "intelligent layman" as well as the professional economist and, undoubtedly, it forms interesting, stimulating, and valuable reading for both classes of reader. The economist in industry will find here much which is of importance in his professional career. However, it seems that the real success of the venture will be judged on its suitability as the basis of a university course on industrial structure and organization. Thus, inevitably it is the emergence of generalizations and "laws" of industrial structure to which one looks as indicators of achievement. From this point of view Mr. Duncan Burn's retrospect stands out as an outstanding contribution. It would indeed be invidious to mention by name individual contributors but the editor can take credit and be complimented for the success of the whole undertaking and also, particularly, for his own chapters on the oil and steel industries and for his concluding essay. Perhaps the most significant features of the latter are the sections dealing in a most penetrating manner with the whole question of productivity and efficiency and the competitive (or otherwise) atmosphere in which industry operates. These matters, after all, form the main justification for allocating Conditional Aid funds to this project.

The task of the authors and editor was a most difficult one, and it would have been very

easy to criticize whatever had emerged as a result of their efforts. This is truly an achievement of which all concerned—editor, contributors, the National Institute of Economic and Social Research, and the Cambridge University Press—may be proud.

K. S. LOMAX.

10.—*The Industrial Efficiency of Rural Labour.* By C. D. Harbury and A. D. Smith, Cardiff, University of Wales Press, 1958. ix, 228 pp. $8\frac{1}{2}''$. 35*s.*

At the present time less than 20 per cent. of the population of England and Wales live in rural districts. Of this rural minority, the number of those directly engaged in agriculture is falling rapidly, the regular male working force employed on agricultural holdings in England and Wales having been reduced from its post-war peak of 523,153 in 1950 to 411,839 in 1957. Thus the rural population is at the same time falling and becoming industrialized.

In part this change is a result of the movement of agricultural labour from the land to the secondary supply and service industries needed to support modern mechanized farming. But it is also the result of the process of industrial dispersion that has taken place since the war. The government has encouraged industrialists to move out of the large industrial conurbations, and the shortage of labour, the higher building costs and the increasing problem of traffic congestion in the cities have all aided the government in its efforts.

Yet this policy of industrial dispersion has been carried out with little thought, and less information, as to its probable economic and social effects. The possibility that there might be a continuing loss arising from the employment of a less efficient rural labour force in place of the experienced city labour force is examined in this book. The authors have studied eleven similar pairs of factories, in each case one being situated in a large industrial area (mainly English) and the other in a rural area (mainly Welsh). The efficiency of a small group of workers in each factory employed on similar processes has been determined by the labour input/physical output relationships, measured in terms of unit time requirement, either weighted for differences in product mix, etc., or unweighted.

The authors disclaim any pretence of sophistication in the measurement used or exhaustiveness in the size of the sample. Yet it is clear that for the factories taken and the processes measured there is no evidence of a consistently higher level of efficiency in the urban factories. Again, when indirect measures of efficiency are considered such as the labour turnover or absenteeism rates, there is no evidence of overall rural inferiority.

This is a valuable book. Its aim and the language in which the results are stated are modest, but it throws a good deal of light on a problem of the greatest importance by showing that for mainly unskilled work at least, rural workers are capable of quite as high a level of efficiency as urban workers. However, it fulfils another function by suggesting a whole series of further questions. Is there, for example a difference in the ability of rural workers to acquire skills and are they as promotable? What is the effect of this industrial invasion on the surrounding agriculture? Clearly the provision of work for wives and daughters may help to stabilize agriculture's labour force. On the other hand the pressure on agricultural wage rates may be increased both directly by the provision of local alternative employment and indirectly by the creation of new consumption patterns in rural communities raising the economic "horizon".

It is a pity that the opportunity was not taken to discuss some of these problems in greater detail. Management opinion, for example, has been dealt with in three pages at the end of the volume, whereas it would have been most illuminating to have had the results of a fairly detailed questionnaire on the attitude of entrepreneurs to industrial dispersion. The social results of this dispersion may well be profound, and the attitude of entrepreneurs may greatly affect those results. Where will the managerial and supervisory groups for the rural factories be recruited? If they are imported from the towns, as would seem to be the case at present, what will be the relationship of these "foreigners" to the local community? Will they provide a new leadership group or, as was shown in the Glossop study, will their refusal to participate help to deprive the community of much of its previous cohesion?

If the present book stimulates further equally careful examination of these related topics, it will have placed us all in its authors' debt. M. MARKS.

11.—*Efficiency in Government Through Systems Analysis, with Emphasis on Water Resources Development.* By Roland N. McKean. New York, Wiley, 1958. London, Chapman & Hall. x, 336 pp. 9″. 64*s.* (Rand Corporation Research Study; Operations Research Society of America, Publications in Operations Research 3.)

The idea that it is possible consciously and systematically to make the most of available resources has an obvious appeal and forms the raison d'être of a group of sciences of management interest including applied economics and operational research. As a broad objective "the most" is all well and good; in application it is troublesome, most particularly in the field of public spending. This book is an attempt to meet this ideal by applying the principles of systems analysis, as developed at the Rand Corporation for military planning, to Government expenditure on non-military capital projects. The examples chosen for illustration are water resource projects in the United States, but the value of the book is not limited thereby since it consists largely of a careful analysis of the general problems of criteria and the evaluation and comparison of benefits and costs. The problem tackled is that of presenting to the decision maker, be he politician, administrator or business man, such information as will help him to arrive at the best selection of projects against some form of budgetary limitation, the magnitude of which might also have to be decided.

The scope of the treatment is defined in Part I. The need for quantitative analysis of Government projects is stated to be intensified by the absence of any built-in mechanism tending towards greater efficiency comparable with the profit motive. Firms can often use relatively poor criteria in a way that leads them to genuine economies or, at least, to profitable survival, whereas with Government agencies the objective is so complex that the selection of appropriate criteria is a prerequisite for useful analysis; indeed, the bulk of the book is a discussion of the pitfalls, and the means of avoiding them, in the analysis and selection of criteria. The standpoint of welfare economics which has to be taken may add to the complexity, and the mechanisms of price, profit, and survival in the face of natural selection may be believed to render analysis unnecessary in the private sector. Nevertheless, much of the analysis could have its application in the presumably more defined context of the individual firm needing only to consider the costs and benefits accruing to itself.

Part II deals in general with problems of analysis covering the choice of criteria, concepts of cost and gain, the specification of appropriate alternatives and the treatment of streams of cost and benefit accruing over time. This section of the book is admirably summed up by three pages of advice deploring, for instance, the use of ratios of gain to cost as being particularly treacherous.

The two examples to which Parts III and IV are devoted are two water resource projects having similar ratios of benefit to cost, despite which one shows an internal rate of return of 55%, and the other only 6%. The marginal internal rate of return is adopted as being the appropriate criterion where there is capital rationing and where no market exists in which the project could be sold as a going concern, by which to establish a rate of interest for discounting. The marginal internal rate of return and the correct set of investments can be found approximately from the sets of projects that yield positive present worths at various discount rates and choosing that which exhausts the given investment budget. Various simplifications of this near-ideal are discussed. This procedure seems sound enough and yet if it were followed too rigorously it would seem to have the effect that very few projects of a research nature or involving risk taking, would ever be undertaken. The period of net expenditure which has to precede the appearance of any net benefit is such, in the case of research projects, that with any reasonable rate of discount there would be little economic case for undertaking the kind of capital expenditure which has given us gas turbines or atomic energy. This point is not meant as criticism of McKean's book but to put some limit to the justification for this type of economic analysis.

The treatment of uncertainty is an aspect which will be of interest to the statistician

but little more can be done, apparently, than to recognise its existence and to present to the decision maker a variety of results to show the effect of relevant contingencies.

By page 100 the main purpose of the book is served, although the examples give some practical significance to the theoretical analysis. The book is completed by a discussion of the form of presentation of U.S. Federal budgets and seeks to demonstrate that useful quantitative indicators of performance could be devised to examine the effect of changes in the scale of expenditure under budget heads covering about 60% of the total expenditure. The detail is solely of United States interest. The principles might well be more generally explored.

This is the third publication in the series sponsored by the Operations Research Society of America. Such a sample is small enough for one to retain the hope that the apparent tendency for the size of volume to increase with each new appearance is not a statistically significant observation.

The book is well worth reading for the analysis contained in its first 100 pages. The further 200 or so rather tend to paint the lily.

J. STRINGER.

12.—*The Income of Nations: Theory, Measurement and Analysis, Past and Present; A Study in applied Economics and Statistics*. By Professor Paul Studenski. New York, University Press, 1958. xxii, 554 pp. 10″. $25.

Over the past twenty-five years, the teaching of economics has changed to a degree which would have been deemed inconceivable by the older generation—and on the whole very much for the better. A reasonably good knowledge of the use and methods of preparation of national income and national product statistics is now one of the necessary staples in all economics courses.

It would be going too far to suggest that undergraduates ought to buy this undoubtedly expensive book; but it is very desirable for all libraries, and certainly should be used by those who teach in this field. It is a labour of love on the part of the author, who has been engaged in collecting material for it for a very long period.

The book opens with some really fascinating chapters describing, and giving the results of, early national income studies. Most of us know a little about Petty and King; but this is a real international survey, giving us the full richness and ingenuity of statistical thought in France in the 18th century, and also work of which few English readers have ever heard, in Italy, Germany and Russia. The Bourbon restoration of 1814, the author sadly points out, temporarily put a stop to all statistical work; and 19th century Russia did not live up to the promise of the previous century.

The first official estimate of national income by a government, it is interesting to notice, was that officially prepared for New South Wales, in 1886, by that neglected genius of engineer-turned-statistician, Sir Timothy Coghlan. It is sad to record that this work was suspended in 1904, and that it was many years before interest in this subject revived in Australia.

After these fascinating historical chapters, there follow a good account of the theory of the subject, of debatable points of definition, and the manner in which they are treated in different countries; an interesting and stimulating account of some of the outstanding conclusions to be drawn from international comparison of the abundant national income and product statistics now available; and a detailed account of the statistics for thirteen leading countries with a summary account for the rest of the world.

In his theoretical discussion of the problems of comparing incomes in wealthy and in poor or primitive communities, the author follows Kuznetz in stating that it is quite impossible (with this proposition we should all agree) to value the goods and services consumed by the former at the prices prevailing in the latter—some of our goods are almost beyond the primitive man's range of comprehension. But the author also follows Kuznetz in the highly debatable converse proposition, that it is possible to restate the consumption of a *primitive* people at *our* prices. This reviewer, after many years of attempting to do so, felt compelled to reach the conclusion that this procedure often leads to arbitrary if not

nonsensical results. There are some things enjoyed by very poor communities—for instance an abundant supply of domestic help, and of fresh fruits and vegetables—which we would price very highly indeed. Revaluing their national incomes at our prices might value these minor amenities higher than their staple consumption.

COLIN CLARK.

13.—*Mental Illness in London*. By Vera Norris. London, Chapman & Hall, for the Institute of Psychiatry, 1959. 317 pp. $9\frac{3}{4}''$. 35*s*. (Maudsley Monograph, 6.)

Until recently mental health has been a much neglected branch of medicine. Comparatively little is known about the incidence of various mental disorders while hospital accommodation available for treatment is admittedly inadequate. Public interest in the problem, however, is increasing and the late Dr. Vera Norris's analysis appears at an opportune time when a clear assessment of the facts is required.

The object of this survey was to describe the populations admitted to observation units and mental hospitals in the London area. Case summaries were constructed for all patients admitted in the period 1947–49 to two observation units and three mental hospitals and these patients were followed up as far as possible to 31st December, 1951. Those patients discharged home and not re-admitted during the follow-up period were not approached.

The expected number of first admissions, E, was calculated from the formula :

$$E = \sum_{x=z}^{x=0} \tfrac{1}{2} a_x n (S + S')$$

where S is the number of survivors at age r and S' the number of survivors at age $(r + n)$ taken from a life-table population for England and Wales for the survey years 1947–9, a_x the average annual first admission rate in the age group r to $(r + n)$, z the age at which there are no survivors out of the original population of 10,000 infants and n the number of years exposed to risk ($n = 5$ when quinquennial age groups are used). This method of deriving the expected number of cases is compared with methods used by other research workers and the results discussed.

In determining the survival rate, a modified life table method was used. For each period the average number of persons exposed to risk of death or discharge is calculated by averaging the survivors at the beginning and end of the period. The probability of surviving in hospital until the next period is calculated as follows:

$$p_0 = \frac{2 - r}{2 + r}$$

where r is the combined discharge and death rate. Starting with 1,000 admissions, this number is reduced by multiplying by p_0, the resulting number is reduced by p_1 and so on until the limit of information of the population is reached.

Age, sex and diagnosis of the sample numbers are examined in detail to see how far they represent the general mental hospital population. An analysis of diagnosis showed that it was fairly consistent between the observation units and the mental hospitals. However, the assessment of prognosis tends to err on the side of optimism since all patients whose whereabouts were unknown during any part of their follow-up period were assumed to be alive and out of a mental hospital.

A consideration of the differences between the admission units and the mental hospitals leads to the conclusion that observation units in general hospitals fulfil a useful function in large cities where the demand for them is sufficient to justify large well-equipped units. In rural districts such units might prove unjustifiably expensive and in these areas it is suggested that an admission unit within the local mental hospital might be preferable.

The assessment of the incidence of mental disorders in a highly urbanized community like London is difficult for many reasons, but an attempt was made to relate the first

admissions to the estimated population at risk. The rates, in round numbers, per million persons in the general population were:

Age Group (years)	*Males*	*Females*
16–29	500 ± 60	500 ± 50
30–49	500 ± 40	700 ± 50
50–69	700 ± 70	900 ± 70
70+	800 ±130	1,200 ±130

Although these figures indicate that the prevalence of mental disorder is high, they almost certainly underestimate the true incidence. In both men and women under 30, schizophrenia is the major cause of admission. In men aged 39–49 years schizophrenia is still the chief cause, but in women of this age manic depressive psychosis takes the leading place. Between 50 and 69 years, most admissions for men and women are due to manic depressive psychosis, while at ages 70 and over senile and cerebrovascular disorders are commonest. The expectation at birth of being admitted at least once in a lifetime for a mental disorder is as high as 5%. Those entering mental hospitals for conditions other than schizophrenia, manic depressive psychosis, or psychosis of old age have a reasonably good prognosis but consitute a relatively small part of a mental hospital practice. On the whole, men have a higher mortality than women.

The report concludes by emphasizing the need for more hospital beds for mental patients and more efficacious methods for the treatment and prevention of the disease. Those particularly concerned with mental health and the related problems will find this book both interesting and instructive. A. BARR.

14.—*Economics of Mental Illness*. By Rashi Fein. New York, Basic Books Inc., 1958. xx, 164 pp. $8\frac{1}{4}''$. \$3.

This book is the second in a series of ten reports sponsored by the Joint Commission on Mental Illness of the United States. The author's primary purpose is to develop a sound methodology for computing the costs of mental illness which will be of general application. Dr. Fein regards the economic loss due to mental illness as the total costs involved, measured by the sum of direct and indirect costs. Direct costs include the actual expenditures by Government and other agencies and by individuals on the care, cure and prevention of mental illness. Indirect costs he defines as the economic loss to society due to some of its members being mentally sick, and they are equated to the loss in production for these people as measured by the loss in wages. One effect of this is that in the case of women the author deals only with those whose productive services enter the money market: this makes the money value assigned to the services of married women very low, but avoids the difficulty of assessing the monetary value of work not performed for a wage.

Dr. Fein stresses what is usually one of the chief difficulties of work on morbidity, that of having to use data primarily collected for other purposes, and throughout the book he is careful to point out the limitations of available data. Thus, in estimating direct costs no account is taken of capital expenditures for building new hospitals or extending old ones, since it would distort the final figure to allocate these to any one year, and there is no depreciation formula which would allow these expenses to be spread over a number of years. Similarly, in estimating indirect costs, the average wage and salary incomes earned by employed individuals are imputed to those who are unemployed because of mental illness, thus assuming that the mentally sick would be "average" if well.

It is well known that many visits to general practitioners are related to mental problems of one sort or another, and the author suggests (p. 45) how their costs may be estimated. He is, however, unable to include expenditures on drugs, lectures on mental health, training programmes, research in the basic sciences, research done by university teachers and similar items, and makes it clear that much more intensive study is needed to estimate the size of these cost components.

The problems to be faced are connected with data which are either incomplete or not comparable. Dr. Fein contends that a significant contribution is made by explaining why

something cannot be done at present with complete accuracy. We are then obliged to ask what it is we are trying to measure, and this will determine the nature of our definitions and the questions to be asked. Certainly some of the author's calculations are based on data which are far from complete. It was surprising to find that whereas the age distribution of 294,080 resident patients in state hospitals in 1952 was known, there were 213,725 for whom this distribution was not known, and this information was based on only 141 out of 204 state hospitals (Tables 4.1 and 4.2, pp. 62–63). This gives point to Dr. Fein's plea for improved data, without invalidating his methodology.

A favourable feature of the book is that worked examples are accompanied by directions for computation, for example, in the sections on losses due to prolonged illness-absenteeism and future losses due to first admissions (p. 67 *seq*.). Those readers who are acquainted with the variations in incidence of mental illness but know little of accountancy are warned against possible double counting, while for those who specialize in accountancy there are useful sections, for example, on direct costs as functions of age at admission and diagnosis. For indirect costs, in terms of expected working years lost, comparison is made between various diagnostic categories of mental illness and other disabilities which cause prolonged sickness absenteeism. These would be useful for internal comparison, but diagnosis in mental disease will have to be much more standardized before meaningful international comparisons can be made of the relative burdens imposed by different types of mental illness.

Some indication of the size of the problem is given by Dr. Fein's estimate that Americans spend at least \$2.4 billion a year on mental illness; allowing for items not included in his calculations, \$3.0 billion might be a more realistic estimate. What is to be done? Clearly total costs could be diminished by cutting down those services which are included in direct costs, but this would only transfer the burden to indirect costs. The community cannot escape the economic cost of mental sickness. Increases in direct costs are often viewed with disfavour, but such increases may result in lowering indirect costs, with a consequent decrease in total costs. Thus, whereas in a group of six states, 100 patients admitted to state hospitals for the first time with schizophrenia required 71 patient-years of care in their first year, the Massachusetts hospitals, by increasing direct expenditure aimed at treatment and discharge, have reduced this patient-year requirement to 53. We cannot estimate the probable savings from increasing our expenditure on research.

Dr. Fein does not suggest how much should be spent on mental illness, but contents himself with pointing out that an economy can afford to spend whatever it likes, since all that is necessary is to spend less on something else. It is to be hoped that this book will provoke a number of detailed studies on the economics of mental illness, so that accurate knowledge may replace the somewhat vague references often made to the intolerable burden of this disability. We shall then be in a better position to answer Dr. Fein's question: "What can society afford *not* to spend on this disease?" EILEEN M. BROOKE.

15.—*Social and Biological Factors in Infant Mortality*. By J. A. Heady and M. A. Heasman. London, H.M. Stationery Office. 1959. viii, 195 pp. $9\frac{1}{2}''$. 20*s*. (General Register Office, Studies in Population Subjects No. 15.)

Infant mortality in this country has reached a stage where improvement has been slow; in fact in recent years the early infant mortality and stillbirth rates have been most resistant to further improvement. This report contains the basic tables relating to an enquiry into the social and biological factors affecting the deaths of children born in the years 1949 and 1950. The method employed was unique—for the first time in this country particulars of deaths of infants were related to the details of their birth. Thus it has been possible to study the effects on infant mortality of age of mother, number of previous children (both live and dead), occupation and social class of father, area of residence, place of birth (hospital or home) and legitimacy of the infant.

Many of the data have been commented upon piecemeal in analyses published in earlier papers, all catalogued in the report, and only a brief commentary, together with diagrams, is included. This is a pity, and an irritation, because a study of this magnitude justifies a

report in which can be found not only the basic data but also a full interpretation. True, the authors say "analysis continues and further papers in the series will be published"; but if a comprehensive report finally becomes available the data will be many years old.

The size of the sample was enormous—$1\frac{1}{2}$ million exposed to risk—and one wonders whether a sampling frame could not have been devised for reasons of economy and expedition, which might have led to quicker analysis and earlier publication.

Discriminant analysis has been tried out on the same data, and one wonders why it was not incorporated in the report. Such an analysis, leading to a method of scoring for assessment of the relative risks of the different factors affecting infant mortality, would have been invaluable to medical administrators. A. T. GORE.

16.—*Census* 1951. *England and Wales. Fertility Report.* London, H.M. Stationery Office, 1959. cxi, 251 pp. $13\frac{1}{4}''$. 90s.

This, the only full Census report on fertility in England and Wales since that of 1911, has had, perhaps, the misfortune to appear comparatively soon after other important publications on the subject: the preliminary and final reports on the Family Census of 1946 and the Census 1951 One Per Cent Sample Tables. It would, however, be a mistake to discount its value on this score, for it takes the story considerably further than the other reports by integrating statistics collected at the 1951 Census with those collected at birth registration up to 1955 and with such of the Family Census statistics as related to England and Wales. It is, moreover, to some extent complementary to the Family Census Report and where the latter was narrow in scope, as for example in the discussion of differential fertility, this report has considerable breadth. Although it was possible to collect more detailed information at the Family Census enquiry than at a normal Census of Population, where fertility can be only one of many subjects of interest, the Registrar General's collation of the three independent sources of information has resulted in a comprehensive and illuminating new set of statistics.

There are 250 pages of tables preceded by some 110 pages of commentary. The text consists of four chapters which discuss the methods used in analysing the data, describe features of the various sets of tables and include summary statements and charts directing attention to the salient features of the tables.

The first chapter is devoted to the purpose and method of the enquiry. As the tabulation of the results relating to all women would have involved an enormous amount of work the burden was reduced by carrying out the analysis of general fertility on a one-fifth sample of the 7·4 million married women under the age of 50 enumerated in England and Wales, and the analysis of completed fertility was effected by means of a four-fifths sample of those aged 45–50. The statistics relating to women married more than once were, however, obtained from a full tabulation. Wherever possible and appropriate, the fertility of marriage has been analysed in terms of completed family size, total incomplete family to a given Census age or marriage duration, and current fertility. These measures, besides being given in the raw state, have frequently been standardized in regard to the main influencing characteristics.

Chapter 2 is a general treatment of the fertility of married women, mainly in terms of mean family size and current fertility rates at various durations of marriage, both in relation to age at Census and to age at marriage. Women married once only, those married more than once and all married women are separately tabulated and, in addition, tables giving the ratios of the fertility and infertility rates and average family sizes of the last two classes of married women are included.

Chapter 3, probably the most novel and interesting in the volume, deals with differential fertility. The data are subdivided according to a number of alternative characteristics of the population, mainly social and geographical. Differential analyses have been carried out in terms of the disparity between the ages of husband and wife, the age at which the husband's education ceased, and the husband's industry group, social class and socio-economic group. Fertility details are given also for socio-economic group cross-classified with the degree of urbanization of the areas of residence. All the tables, except those relating to gainfully

occupied women, relate to once-married women only. Much of this is quite new. The now well-known gradient with social class, as found in 1911 and 1946, still persists. In 1951, among women of completed fertility (age 45–49) there was a regular decline in completed family size through the classes from Class V (the unskilled occupations) to Class I (the professional occupations). When women of child-bearing age were included, however, the class differences became less marked. In Class I, for example, where the average marriage age is rather high, current fertility in relation to age and duration of marriage is now up to the average of all classes. It is of interest to note that the Census report shows their standardized current fertility to be 101 per cent. of the national average while the One Per Cent Sample Tables had given 90 per cent. only—a difference that probably arises from the smallness of the One Per Cent Sample for this group. There are indications that the fertility of this class is rising and indeed, when the finer classification by socio-economic group (first used in this Census) is adopted, the managerial and professional group (a near equivalent to Class I) is found to have a higher current fertility (again allowing for marriage age and duration) than shopkeepers, clerical workers, foremen and shop assistants. A fairly regular variation of average completed family size with terminal education age was also evident: the wives of men who left school before the age of 15 tended to have the largest families (2·11 on average, unstandardized) followed by those with a terminal age of 20 and over, such as university graduates and others of equivalent education (1·58), while wives of husbands with a limited amount of further education tended to have the smallest families (1·48 for terminal education ages 15–16 and 1·44 for 17–19). It is especially interesting to find that within socio-economic group 3 (higher administrative, professional and managerial) fertility is highest for those whose education went on the longest.

Chapter 4 deals with fertility classified by year of marriage, mainly for the cohorts of 1920–1954. Since from the standpoint of population replacement, ultimate family size is the fundamental factor, the report considers in detail how families have been built up at successive durations of marriage, giving also the corresponding fertility rates. The results are illustrated by a series of interesting graphs. The mean completed family size of selected cohorts from 1861 onwards, with projections for the cohorts of uncompleted fertility from 1935–1950, are given in summary form in the text. The 1861 cohort was almost the first to be affected by family limitation (a practice dating from about 1870) in any material degree; for about two generations there was a fall in family size at the rate of some 2 per cent. between each successive pair of annual cohorts, or a fall in average size of a completed family over sixty years from about six children to a little more than two. For marriages after those of 1922 or 1923 the downward trend slackened somewhat, and since 1927 mean family size has been almost constant. In striking contrast, fertility rates, which are made available from 1910 onwards, have fluctuated widely and it is clear that it is the greater relative stability of completed family size which renders the cohort form of analysis advantageous. The impact of the war on family building appears as a postponement of births early in the war, with a counterbalancing acceleration later. The effect on family size of earlier marriage is also discussed: instead of adopting the fertility of the younger marriage age groups of previous generations the women who now marry young complete their families more quickly. In normal times couples have, on average, a little over half their children in the first five years of marriage and about four-fifths within the first ten years.

To analyse replacement fully it is not enough to consider only the average sizes of completed families born to marriage cohorts: the mothers have to produce sufficient offspring to replace not only themselves but also those women who do not have children. For such calculations it is preferable to consider the extent to which the women born in any particular year produce female children. These women are assumed to have been subject to the mortality, marriage and fertility rates recorded during their lifetimes or, for the younger generations, to forecasts of such rates. Shortened methods of calculation have mainly been used, but for investigating the extent of current replacement the appropriate nuptiality table has had to be calculated, and the procedure for doing so is fully described in the appendix to the chapter. The replacement rate based on the nuptiality and

fertility of 1951–1955 is given as 1·01, but there are alternative results according to which of several reasonable assumptions is made; whether, for example, female or male nuptiality is used. The range of values is from 3 per cent. under full replacement to 8 per cent. over, values both more favourably and less dispersed than those obtained by Hajnal for the Royal Commission on Population some years previously. Generation replacement rates, on one method of calculation, are given for women born in successive quinquennia from 1903 onwards. The first generations showed only 70 per cent. replacement but for successive groups of births over the next 40 years there was a steady increase until full replacement was indicated for the 1943–48 generation. More recent generations show promise of doing a little more than replacing themselves. It must be borne in mind, however, that since 1918 there are progressively greater elements of projection in the calculations; the fertility basis used appears to be stable but such favourable rates have not so far been experienced throughout the lifetime of any generation in recent times and it remains to be seen how realistic they are.

A slight weakness of the Census data lies in the fact that about 5 per cent. of women in the samples had to be excluded because the information they provided was incomplete. There was an appreciable amount of understatement of numbers married more than once and it is believed that this resulted from the absence of a corroboratory question in regard to remarriage: the women were asked to state date of first marriage "if married more than once." The inclusion of a further question as to whether they had been married more than once, requiring a "yes" or "no" answer would probably have eliminated the error. It is to be expected, therefore, that the next fertility Census will benefit from this experience. These omissions undoubtedly introduce bias into the results but the Registrar General's statisticians have, by various ingenious means, endeavoured to minimize it. The authors of the report have also been alive to the errors resulting from the inclusion of premarital births among the offspring of a marriage and from the concealment of premarital conceptions. They are able to conclude, however, that the mis-statements which occurred were a good deal less important than in the past—certainly less serious than in the Family Census enquiry, which was voluntary and in which there was a tendency for women who had conceived out of marriage to refuse to complete the form.

There are also errors of sampling, and while the inclusion of confidence limits would have been highly desirable for the statistics given, particularly the ratios, they would have involved a considerable amount of further calculation and their omission is a small price to pay for the wealth of information that the report provides. The use of appropriate signs—asterisks, hyphens, noughts and blanks—conveys some idea of the degree of significance attaching to the figures. Furthermore, as the tables in this volume generally confirm the One Per Cent Sample results, it should be possible to have greater confidence in fertility statistics obtained from samples of this order at future Censuses.

The report is compact, and to have text and tables in one volume is convenient. Compactness has been achieved partly by desirable summarizations of the data and partly by commendable restraint in the choice of tabulations. Further material, however, is promised for a later occasion and its appearance will be awaited with interest. Among the promised tabulations are the distributions of families by their sizes, and fertility rates by birth order. Students would do well to include this volume in their reading, for it is both up-to-date and readily digestible. The text gives a good insight into the technical demographic problems involved in the linking of data collected at different times on dissimilar bases; the first appendix to Chapter 4 describes the great care and attention to detail that was devoted to the solution of these problems and cannot but arouse admiration. The second appendix describing the methodology of the construction of the nuptiality table will be of especial interest, as the details are not usually included in demographic text books.

C. J. THOMAS.

STATISTICAL NOTES

(1) British Offical Statistics

The index of retail prices compiled by the Ministry of Labour, which was 110 in March 1959 (prices at 17th January, 1956 = 100) was at the same level in April and fell to 109 in May to August. Calculated to one place of decimals the figures were 110·3, 109·5, 109·1, 109·3, 109·0 and 109·3. The detailed figures of the weights used in calculating the index and the indices for different commodity groups were as follows:

	Food	Alco-holic Drinks	Tobacco	Housing	Fuel and Light	Durable House-hold Goods	Clothing and Foot-wear	Trans-port and Vehicles	Miscel-laneous	Services	All Items
Weights:	350	71	80	87	55	66	106	68	59	58	1,000
Mar. 17th, 1959	108·9	105·9	107·8	126·2	117·0	100·1	102·1	113·9	113·7	115·6	110·3
April 14th, 1959	108·6	97·8	107·8	127·1	117·0	98·1	102·3	114·3	113·0	115·6	109·5
May 12th, 1959	108·1	97·8	107·8	127·3	111·1	98·1	102·3	114·4	113·1	115·8	109·1
June 16th, 1959	108·4	97·8	107·8	128·1	111·1	98·1	102·5	114·5	113·0	116·3	109·3
July 14th, 1959	107·4	98·1	107·8	128·5	111·1	97·9	102·4	114·7	113·4	116·4	109·0
Aug. 18th, 1959	108·1	98·1	107·8	128·5	111·4	97·9	102·5	114·8	113·5	116·5	109·3

The Ministry of Labour index of weekly wage rates, calculated on the basis of January 31st, 1956 = 100, showed that in March 1959 the level was 116·7. It remained at this level in April; rose to 116·8 in May and June, 117·0 in July and 117·1 in August. In manufacturing industries alone the figures were 116·2 in March, 116·3 in April and May, 116·4 in June and July and 116·6 in August.

The six-monthly enquiry into average earnings showed that in April, 1959, the average earnings per week in the industries covered by the enquiry, i.e. the manufacturing and some of the principal non-manufacturing industries and services, were 262*s.* 11*d.* for men, 137*s.* for women, 114*s.* for youths and boys and 87*s.* 4*d.* for girls. These figures represented increases since April, 1947, of 113, 103, 141, and 117 per cent. For all workers combined the average was 222*s.* 6*d.*, an increase of 115 per cent. This compares with an estimated rise of about 83 per cent. in weekly full-time *rates* of wages in the same industries and services. In manufacturing industries alone the average weekly earnings in April, 1959, were 271*s.* 9*d.* for men and 137*s.* 7*d.* for women. The corresponding hourly earnings were 68·5*d.* and 39·8*d.* The average hours worked per week in all the industries and services were 46·3, a slight increase since the date of the enquiry in October, 1958. The review of the results of the enquiry in the *Ministry of Labour Gazette* also contains figures for the dock labour, agricultural and coal mining industries which are not covered by the main enquiry.

The total working population and the numbers in civil employment in the six months ended July, 1959, were as follows:

(*Thousands*)

Date	Total Working Population			Numbers in Civil Employment		
	Males	Females	Total	Males	Females	Total
February, 1959	16,089	7,816	23,903	15,135	7,662	22,797
March, 1959	16,069	7,812	23,881	15,144	7,660	22,804
April, 1959	16,062	7,837	23,899	15,168	7,692	22,860
May, 1959	16,064	7,854	23,918	15,211	7,724	22,935
June, 1959	16,063	7,905	23,968	15,234	7,786	23,020
July, 1959	16,078	7,932	24,010	15,245	7,809	23,054

The numbers of persons on the unemployment registers of the Employment Exchanges fell by 19,800 in April, by 50,200 in May, by 67,200 in June, and by 18,500 in July. In August the figure rose by 32,200. The total for August represented 2·0 per cent. of the number of employees in Great Britain. The percentages in the different Regions ranged from 1·1 in London and the South-East to 3·4 in Wales and 3·9 in Scotland.

The following is the sex analysis of the figures:

Numbers of Unemployed Persons on the Registers of Employment Exchanges

	Men and Boys	*Women and Girls*	*Total*
March 9th, 1959	395,566	154,979	550,545
April 13th, 1959	379,943	150,810	530,753
May 11th, 1959	344,152	136,391	480,543
June 15th, 1959	299,205	114,106	413,311
July 13th, 1959	288,016	106,777	394,793
August 17th, 1959	307,366	119,596	426,962

Of the total of 426,962 in August 93,483 had been unemployed for not more than two weeks, 108,209 for 2 to 8 weeks and 212,154 for over 8 weeks, while 13,116 were temporarily stopped. In the five weeks ended August 12th, 199,086 vacancies were filled by the Employment Exchanges and the number unfilled at that date was 263,393.

In the week ended July 25th it is estimated that in manufacturing industries 53,600 manual workers were on short-time.

The number of insured workers absent from work owing to illness, including self-employed as well as employed, was 1,171,900 on one day in March, 926,600 in April, 856,500 in May, 824,400 in June, 807,800 in July and 806,500 in August. The numbers of employed persons absent owing to industrial injuries were 61,900, 59,200, 61,000, 60,400, 63,300 and 58,600.

The *Ministry of Labour Gazette* for June, 1959, contains an age analysis of the employee population at May, 1958. Of the total of 14,220,000 males, 185,000 were 70 and over and 564,000 were 65 and over. Amongst 7,600,000 females 46,000 were 70 and over and 168,000 were 65 and over. Figures relating to married women showed a total of 3,830,000, about one-half of the total number of female employees. The review also includes regional and industrial analyses of the figures and some estimates of inter-regional migration.

An analysis of the size of manufacturing establishments appears in the September *Gazette*. It shows that in April, 1959, out of a total number of establishments with 11 or more employees, of 55,739 14,874 had under 25 employees, 14,625 had 25 and under 50, and 11,520 had 50 and under 100. At the other end of the scale 328 had 2,000 and under 5,000 employees and 75 had 5,000 or more. The figures are analysed by regions and industries.

The Air Ministry have issued as No. 100 in the series of Geophysical Memoirs of the Meteorological Office a paper by G. A. Tunnell on the *World Distribution of Atmospheric Water Vapour Pressure*. Part I of the publication is an atlas of the distribution over the whole world of the daily mean of atmospheric water vapour pressure for the months of January, April, July and October. All available data from about 3,500 stations have been used to produce the maps. Part II of the publication gives a brief survey of the world distribution of diurnal variation of vapour pressure and its variation with climate. The price of the Memoir is 10*s.* 0*d.* and is available from H.M. Stationery Office.

(2) Other Statistics

The Institute of Actuaries and Faculty of Actuaries in Scotland have jointly issued a booklet entitled *National Pensions* in which they make a plea that there should be a greater realization that the full cost of changes in pension schemes does not emerge for many years. They suggest that there should be better provision for making the public acquainted with the measurement of the ultimate effect on the economy of promises which are made now but which will not come to full maturity for some time. The two bodies propose the appointment of an independent authority to guide the country through the financial, economic and technical aspects of this peculiarly difficult problem.

The first four numbers have appeared of a new quarterly journal entitled *Technometrics*. This is a joint enterprise of the American Society for Quality Control and the American Statistical Association under a management committee consisting of representatives of the two societies. The editor is J. Stuart Hunter of Princeton University.

The purpose of the journal is to publish papers which contribute to the development and use of statistical methods in the physical, chemical and engineering sciences. The style of the new journal is not unlike that of *Biometrics* except for the different choice of subject matter. Thus both societies should find it a useful addition, complementary to their existing journals rather than competing with them.

Student's Collected Papers.—The Society is glad to welcome, especially under the Presidency of Sir Hugh Beaver, the appearance of a third impression of Gosset's statistical papers. There is no new material in this edition apart from a very short preface by Professor Egon Pearson. Professor Pearson explains why, despite suggestions to the contrary, he has decided to maintain the original form of the volume and confirms his belief that the *Collected Papers* should remain in print to find their place on library shelves with Karl Pearson's *Early Statistical Papers* and R. A. Fisher's *Contributions to Mathematical Statistics*. Apart from *Collected Papers* the volume contains a very well written biographical note by Launce McMullen.

The Society has received the first issue of *Estadistica Española*—a quarterly review published by the National Institute of Statistics in Madrid. This issue contains four papers of a theoretical character and three papers under the heading of Organisation and Methods. The journal also contains book reviews and statistical notes. Most of the papers have English and French summaries and the editorial standard is commendably high.

CURRENT NOTES

The Trustees of the Houblon-Norman Fund, on the recommendation of their Advisory Committee, have made the following awards for 1959–60.

Research Grants.—Dr. L. R. Amey, Lecturer in Industrial Economics, University of Nottingham, "The Movement of Firms between Industries"; Mr. K. F. Dixon, Westminster Bank Ltd. (Renewal), "The Early Development of the London Money Market"; Miss A. G. Donnithorne, Lecturer in Political Economy, University College, London, "The Chinese System of Economic Planning, with Special Reference to International Trade"; Mr. Bernard Edwards, Lecturer in Economics and Statistics, Municipal College of Commerce, Newcastle, "The Structure of Banking in Venezuela"; Mr. J. K. Ghandhi, Research Student, King's College, Cambridge, "The Extent and Impact of Hire-Purchase in the United Kingdom, 1948–1957"; Mr. C. H. P. Gifford, "The Problems of Economic Forecasting—a Case-Study in Timber"; Mr. C. D. B. Harbury, Lecturer in Economics, Uni-

versity of Birmingham, "The Distribution of Capital and the Laws of Succession"; Dr. G. D. Ramsay, Fellow of St. Edmund Hall, Oxford, "The Account Book of John Isham"; Mr. Peter Robson, Lecturer in Economics, Queen's University, Belfast, "The Economic Effects of Index Linking in the Money, Bond, and Insurance Markets"; Mr. Neil Runcie, Lecturer in Economics, New South Wales University of Technology (Renewal), "Hire-Purchase in the United Kingdom and the British Dominions"; Mr. R. M. Titmuss, Professor of Social Administration, University of London, "The History of Occupational Pension Schemes in Britain".

The Trustees of the Houblon-Norman Fund invite applications for Fellowships or Grants in aid of Research into the working of industry and finance in Great Britain and elsewhere and the economic conditions affecting them.

Fellowhips, which are intended for experienced research workers, are awarded for full-time research for one year, which may be renewed for a second year. Grants are awarded for either full-time or part-time work. The amount of the awards, which will take effect from the 1st October 1960, will depend upon the circumstances of the candidate and the probable expenses of his work.

Applicants must be British subjects normally resident in the United Kingdom. The Trustees are prepared to relax this rule only in exceptional circumstances.

Forms of application, which may be obtained from the Secretary of the Fund, c/o the Bank of England, London, E.C.2, should be returned not later than the 1st March, 1960.

OBITUARY

Wilfred Leslie Stevens

The sudden death of "Tony" Stevens, leaving a wife and three young children, in June 1958 at the early age of 46 came as a great shock to his friends, and is a serious loss to the science of statistics.

Stevens was educated at Reading and Cambridge Universities. At Cambridge, after reading mathematics (he was a Wrangler and achieved Schedule B) he took the Board of Education Diploma in Teaching. His real interest, however, lay in statistics, which he studied under Dr. Wishart and Professor Eddington, and after a short interval of teaching he obtained in 1935 a post at the Galton Laboratory under Professor R. A. Fisher. He came to Rothamsted with Professor Fisher at the outbreak of the war, when the Galton Laboratory was evacuated there, though he still continued to reside in London and was indeed bombed out in the air raids.

His increasing desire to take a more active part in the war effort led, for a short time in 1941, to a transfer to the Rothamsted Statistics Department, which was then engaged on work on air raid damage in addition to its agricultural activities. Soon after the transfer, however, urgent representations were made by the Foreign Office and the British Council that Stevens should take a post at Coimbra University, Portugal, where he was personally known to Professor Tamagnini through his work on human blood groups. He eventually acceded to this request, though not without considerable hesitation. He spent the rest of the war years in Portugal, where he met his wife, who was similarly engaged in upholding British interests, and returned to England in 1944 to take a post with Imperial Chemical Industries at Billingham. Although he found much to interest him in the statistical problems arising at Billingham, both on the experimental and control side, he and his wife found life in the north of England uncongenial, and in 1947 he joined the Admiralty Statistical Department (then located at Bath) under Mr. H. L. Seal. The work there, however, did not greatly interest him, and housing difficulties and a longing for sunny climes and a Latin atmosphere led him in 1948 to accept a Chair of Statistics at São Paulo University, Brazil, where he remained till his death.

His knowledge of Portuguese and his considerable teaching skill were, of course, an immense advantage for this post. At São Paulo he rapidly established contact with the Agricultural Research Institute at Campinas, and was appointed their Statistical Advisor. This was a very fruitful liaison, and he did much to further the application of modern statistical methods to agricultural research problems and field experimentation in the State of São Paulo and in other parts of Brazil.

Stevens was a meticulous and thorough worker, capable of putting an immense amount of effort into problems that interested him, though he had little inclination to devote his time to projects in which he found neither theoretical nor practical interest. He had a keen perception for the essentials of a problem and did not accept a solution just because it had been widely promulgated or had received the stamp of high authority. He was a caustic critic, in private at least, but he wisely refrained from committing many of his more pungent criticisms to print.

Stevens will probably be best remembered for his development of a maximum likelihood method of fitting exponential curves by successive approximation (1951), and his method of handling multiway tables of non-orthogonal data (1948) also by successive approximation. Both these methods have proved their value, not only for desk computation but also for computation on electronic computers, for which successive approximation is particularly suited. His paper on control by gauging, read before the Research Section of the Society in 1948, was also a noteworthy contribution to quality control literature, and

demonstrated what was not then recognized, at least in the publications on the subject, that a pair of suitably chosen go and not-go gauges gives a method of control which per article tested is 80 per cent. as efficient as exact measurement, and is of course in many cases immensely simpler to operate.

Stevens was one of the team of authors who produced the text book *Statistical Methods in Research and Production* sponsored by Imperial Chemical Industries and edited by O. L. Davies, which was first published in 1947 and had reached its third edition by 1957. He also produced a number of neat and ingenious tables, three of which (limits of expectation for binomial and Poisson distributions, densities of organisms estimated by the dilution method, and initial differences of the powers of natural numbers) are included in Fisher and Yates's *Statistical Tables*.

F. YATES.

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STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

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Advancement of Science—

Vol. 15 (1959), No. 60—Prospects and probabilities in electronic data processing; *A. D. Booth*.

Applied Statistics—

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LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part 2, 1959, the Society has received the publications enumerated below.

I.—OFFICIAL PUBLICATIONS

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The Economics of Housing and Urban Development

By P. A. Stone

Building Research Station, Department of Scientific and Industrial Research

[Read before the Royal Statistical Society, April 15th, 1959, the President, Sir Harry Campion, C.B., C.B.E., in the Chair]

Summary

Most planning problems have more than one solution. The difficulty of weighing up a large complex of different kinds of factors makes the choice of solution difficult. Many more of the factors can be quantified than is usually thought. The technique is to extend the concept of "cost" to "cost in use"; that is, to estimate the total real costs to the community. In this way a single cost figure can be obtained for construction and development, and for living and operating costs in various environments. This can be set against the subjective valuation of the factors of appearance and utility, such as the preference for city atmosphere, or for life in flats, or in houses with gardens.

This paper is concerned with the techniques necessary for costing different types of housing developments. Costs are derived for houses and flatted blocks of various heights, for estates at various densities, and for constructing neighbourhoods, town extensions and new towns. Estimates are given of other costs arising in connection with overspill schemes and of the costs of living and operating in towns of different sizes.

The initial costs of providing housing vary from about £600 to £1,100 a person according to the size of the dwelling and the height of the block. Commercial, industrial and communal buildings and works cost about a further £500 per person. The costs of moving people and industry add £60 to £70 a person to the cost of creating a new town. Against the extra costs of overspilling people to new areas can be set the future savings resulting from lower replacement and operating costs in new buildings as compared with those available in the redevelopment area of a large city. Estate running costs work out at around £10 per person for two-storey housing and up to a third more for dwellings in high flatted blocks. Transport may add as much as £10 a year to the living costs of people at the outskirts of large towns as compared with those more centrally situated; living costs in small towns may be less than in large towns by a similar amount.

This paper shows how estimates can be derived of the comparative costs of different types of development—for instance for developing central areas at various densities, the balance of the population being treated as overspill to a new area.

INTRODUCTION

1. Urban planning is concerned with the trinity of appearance, function and cost, that is, with the visual impact of the development, its convenience for the people who will live, work and operate in it, and with the cost of providing it. The adequacy of any programme of development must be judged in relation to all three requirements. In essence judgement consists in comparing the "value for money" offered by the proposed development and a value has to be placed on the various attributes of appearance and function to be set against the cost. Such comparisons are extremely difficult, particularly where several alternatives are available, because of the need to weigh up a large complex of different kinds of factors. The subjective factors of appearance can never be quantified nor can those of utility, but other factors of function can be quantified and their costs added to the costs of development. Clearly, in making judgements the development must be considered over its entire life. No appraisal of immediate consequences alone can be adequate.

2. This paper is concerned with the problem of evaluating the cost implications of a development over its life, that is, with the measurement of the "costs in use". This is equivalent to measuring the impact of the development on the national economy and the total real cost to the community. This can be achieved by extending the field of costs to include not only the initial costs of development, including the costs of movement and disturbance, but also the consequential costs for maintenance and servicing over the life of the development and the user cost associated with the particular form of development, allowance being made for the expected lower costs of operating in new buildings and environments designed for current needs, and for living and operating in towns of various sizes. For instance, the economic cost of a housing estate in this sense can be taken as the total cost of construction, of maintaining and servicing the dwellings themselves and the open spaces and roads within the estate, and of providing local authority and commercial services to the estate and its inhabitants.

3. Such an approach cannot eliminate the other aspects of judgment—for instance, the importance of retaining a "city atmosphere" and all that it implies in the redevelopment of central and inner areas, preferences for life in flats, or in houses with private gardens, or for high as compared with low density development, which can only be expressed in terms of value and not of money. It does, however, permit the cost aspects to be quantified and subjective valuations of these other factors to be considered against their cost in terms of national resources in labour and materials. The demand for national resources for development projects can then be compared with the demand for resources to satisfy the community's other requirements and the implications of the various requirements can be studied.

4. The purpose of this paper then is limited to providing a basis of deriving the economic costs of housing and associated urban facilities. The many other factors, which the planner must consider, lie outside the scope of this paper. The absence of any discussion does not reflect on their importance. This paper falls into two parts: Part 1 provides a description of the methods by which the estimates of the various development costs were obtained, the estimates themselves and a detailed discussion of the levels of requirements being given in the Appendices; and in Part 2 these estimates are applied by way of example to some current planning situations, to housing estates, central development, New Town and town extensions and urban renewal. These examples show how the techniques can be applied to assess total costs and provide some indication of the relative

total real costs of various common planning situations. The planner will of course wish to assess where the balance of advantage lies for the alternatives available to him. The techniques and cost data which have been derived should provide a basis on which assessment can be made. The facilities listed in the schemes costed in this paper are put forward only as a basis of costing and do not attempt to set standards. The planner will set his own standards and will often be able to use unit costs within his own experience.

PART 1. THE COSTS OF DEVELOPMENT

The Costs to be Considered

5. In order to obtain a complete picture of the effect of development decisions on costs, it is necessary to consider both initial and running costs. First, a study must be made of land use and densities of development, since this factor affects costs directly through land costs and indirectly through the costs of roads and public utility services. Secondly, consideration must be given to the costs of the buildings and engineering services necessary in an urban development. The running costs of housing developments vary with the type of development, with, for instance, the nature of the services provided. The cost of the development of overspill schemes is not confined to the constructional costs of the new developments; allowance must be made for the costs of moving people and their effects to the new area and for the costs of disturbing existing installations involving, for example, loss of production. But, as a result of the new developments, new assets are acquired in the place of old and some allowance must be made for the difference in value resulting from buildings and services with a longer life and probably lower maintenance costs. Finally, it will be appreciated that living and production costs depend to some extent on environment and that, for instance, costs in small towns are likely to be different from those in the conurbations. No reliable estimate of the costs of the alternative types of development is possible unless these various points are considered.

The Sources of the Information

6. However carefully the principles of costing are developed, the resulting estimates have little value unless they are based on the practical experience of actual developments. Information has, therefore, been obtained from all the New Towns in the Home Counties, from town extension schemes, from large towns, county councils and public utility authorities. In addition, useful data have been obtained from published research notes and from reports of development and building projects. The information obtained related to work carried out to provide varying facilities at different times. The figures have, therefore, been considered in relation to the needs for which they were provided, the cost figures being first adjusted to bring them to the mid-1957 level of costs—which broadly obtains now—so as to obtain a set of typical requirements and costs which could be used as a basis for cost studies.

It will, of course, be appreciated that it is unlikely that the individual estimates given will be identical with the cost of any actual development. The range of size and specification and hence cost is very wide, particularly outside the field of housing itself. The estimates given are intended to be used collectively and care is necessary when introducing alternative figures that the balance of the estimate as a whole is not upset.

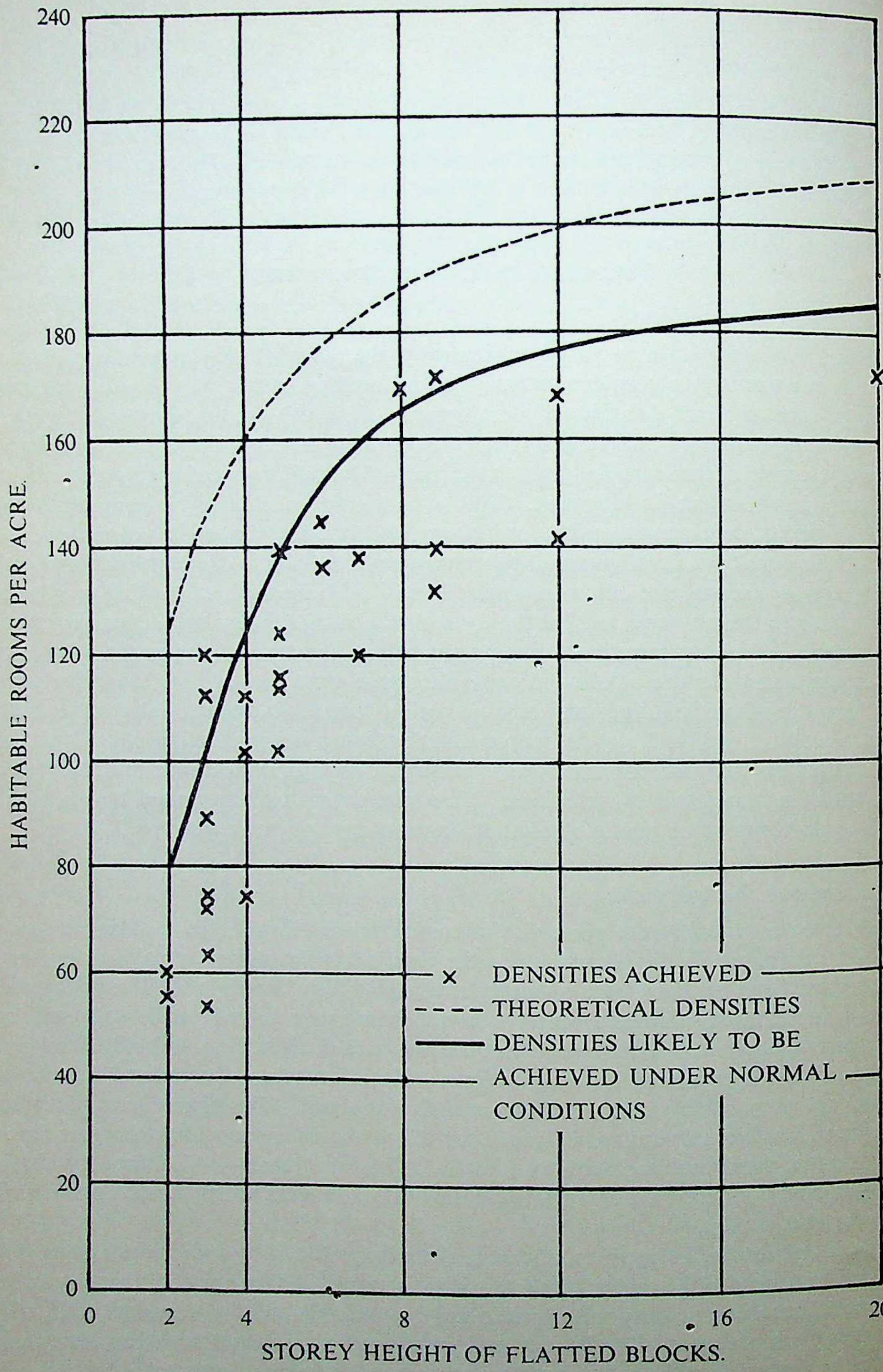

FIG. 1.—Net residential densities.

Land Economy and Densities

7. Somewhere between 300 and 400 acres of land are required for each 10,000 persons in order to provide sufficient space for non-housing needs at current requirements (Appendix A). The area of land required for housing varies with the type of development; in practice, two-storey housing generally requires about 200 acres for 10,000 persons, and while a still larger area may be used, the land requirement can actually be reduced to 100 acres for 10,000 persons with tight planning (Appendix A). The area needed can be further reduced by the use of high flatted dwellings but even in this case 50 acres is about the minimum possible (Appendix A). Land use must be considered as a whole and attention given to the claims of land, for instance, for food production and mineral working, as well as for urban purposes (Appendix A).

The Densities of Housing Schemes

8. The densities at which housing is actually developed are, in general, a good deal lower than is "theoretically" possible. This is, no doubt, partly because of the difficulties of achieving such densities and partly because designers are not always prepared to accept the aesthetic and amenity standards which such densities entail. For the purpose of this study it has been assumed that the densities likely to be achieved will be at about the highest levels consistently achieved hitherto in practice (Fig. 1 and Appendix A). The effect of setting the densities on the high side (Table 1) is, of course, to under-estimate the land, road and public utility service costs per dwelling, but these are small compared with the total building costs and the amount by which the total costs are under-estimated is likely to be of little consequence.

TABLE 1

Anticipated Density of Housing Schemes

Type of Housing	*Two-storey Houses*	*Three-Bedroom Dwellings in Multi-storey Flatted Blocks*									
		Two-storey	*Three-storey*	*Four-storey*	*Five-storey*	*Six-storey*	*Seven-storey*	*Eight-storey*	*Nine-storey*	*Ten-storey*	*Twelve-storey*
Dwellings per acre . .	14	20	26	31	35	38	40	41	42	43	44

The Costs of Land

9. Under the Town and Country Planning Acts, 1947 and 1954, public authorities purchased land at existing use value, plus any established development value and compensation for disturbance, if necessary applying for a compulsory purchasing order for purchase at this figure. In contrast, private developers pay the market price which is roughly the value of the land for the purpose for which planning permission can be obtained. Under the Town and Country Planning Bill, now before Parliament, the public authorities will pay current values.

10. Public authorities are now paying between £80 and £500 an acre for land at the periphery of towns, the average being around £250 an acre, while for New Towns the price is around £80 an acre. Central area land ranges from about £1,000 to perhaps £60,000 an acre, £5,000 an acre being a fairly typical figure. Most central area land is acquired for redevelopment and its cost depends on its present state and use and any

established development value. Condemned property is acquired under the Housing Act 1936 at site value, additional compensation only being payable in certain circumstances for property owner-occupied. It is often necessary to acquire some sound property for redevelopment schemes; sound housing may cost £25,000 or more an acre and industrial property up to four times as much. Exchequer Grants under the Town and Country Planning Acts, 1947 and 1954, can be obtained for land for redevelopment to the extent of 50 per cent. (75 per cent. in the case of land for open spaces) of the difference between expenditure on the acquisition of the land, its clearance and preliminary development, and the value of the land for its new use.* An allowance for fees and handling must be added to land costs, and may amount to about 5 per cent. Further, perhaps another 10 per cent. should be added to cover interest for the interval between acquiring the land and obtaining a return from it.

The Costs of Development

11. The costs of development are likely to vary considerably and will depend not only on the local prices of labour and material and on the natural and historical features of the site, but also on the success with which a layout is devised to make the best use of existing conditions.

12. The cost of clearing, levelling, and constructing roads and sewers is the item most affected by the pattern of layout, and the historical and natural features of the estate. There appears to be a substantial fall in cost per dwelling for these services as density rises, but, as would be expected, the costs level off as the higher densities are reached (Fig. 2). Costs are usually about 50 per cent. higher in central areas than in peripheral areas. This may be largely a difference between the costs of redeveloping and new development. It should be appreciated that the possible range of costs for any particular density is very large; the figures given in Fig. 2 indicate the typical relationship for housing areas. The costs for clearance, levelling, and roads and sewers for neighbourhood centres on virgin land are around £2,700 an acre, the corresponding figure for redevelopment land being £4,200. The corresponding costs on virgin land for industrial areas are about £2,500 an acre, while the figure for town centres is about £8,500 an acre.

13. The cost of services, electricity, gas, water and telephone, can be considered under three headings; supply, distribution to the housing estate, and distribution on the estate to the dwellings. Supply is on a regional basis, and the extent and cost of installation is not greatly affected by the location of the housing and other urban development. The costs of distribution to the housing estate tend to be related to local factors rather than to the type and density of development. Generally, some new mains will be needed to meet additional demands both for central and peripheral areas and while the length of the mains link will usually be greater for peripheral than central developments, the increase in cost incurred is largely balanced by the greater unit cost of installing mains in built-up areas as compared with other areas. An examination of figures provided by the main public utility services suggests that about £80 a dwelling is a reasonable sum to cover the distribution mains for both central and peripheral areas. For these services for all purposes for a town as a whole the figure would be around £30 per person. The cost of distribution on the estate depends largely on its form of development. Greater densities mean shorter

* These Grants will be payable in future only in respect of war-damage redevelopment, as set out in the Local Government Act, 1958.

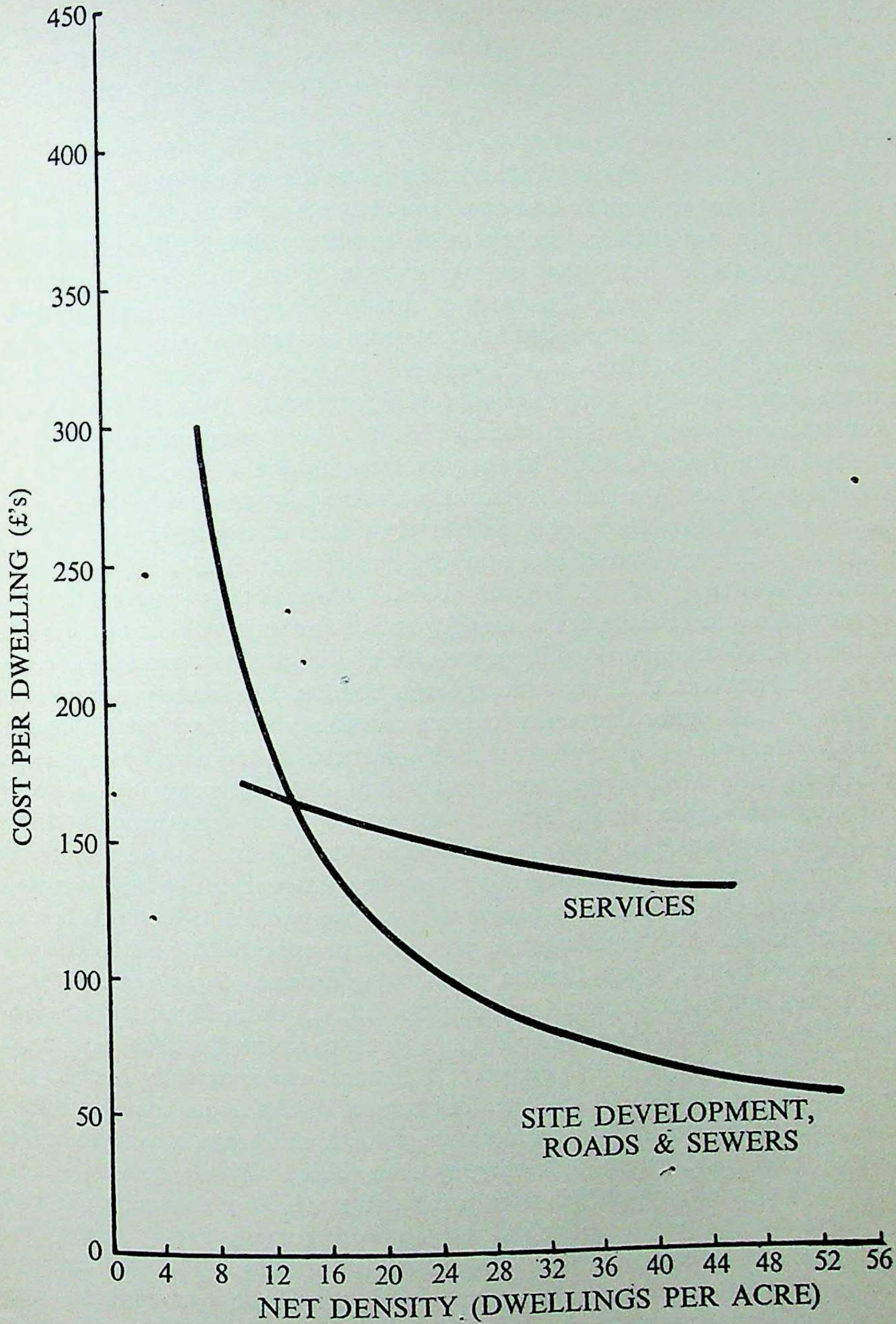

FIG. 2.—Density and costs per dwelling for services (electricity, gas, water and telephone) and for site clearance, etc., in peripheral areas (excluding fees and interest).

runs, large blocks of dwellings allow the elimination of estate road mains and house connections, and sub-stations can be placed within the block itself. The general trend of costs with density is shown in Fig. 2. Where new sewerage and water installations have to be provided for a town, these might cost around £20 and £15 a head respectively.

The Costs of Dwellings

14. The average three-bedroom local authority house has a floor area of just over 900 square feet; the corresponding two-bedroom house has a floor area of around 770 square feet. The average prices of these dwellings in the provinces are £1,470 and £1,323 respectively. Generally local authority flats are smaller than houses, having fewer and smaller rooms; one-bedroom flats have a floor area of around 510 square feet and two-bedroom flats 680 square feet. Flats of three bedrooms or more are unusual, although dwellings of this size are found both as flats and maisonettes. However, dwellings in flatted blocks appear to contain on average about one and a half bedrooms. Flats of this size in two-storey construction average in price around £1,130. The average relationship between floor area and price (Reiners, 1957; Ministry of Housing and Local Government, 1958) for dwellings can be used to adjust the price of a dwelling for changes in floor area. On this basis, the price of two-storey local authority type flats of two bedrooms (680 square feet) is around £1,230 and for one bedroom (510 square feet) £1,031. These figures are consistent with the average of £1,130 given above. It would thus appear that for two-storey construction prices are about the same for houses and flats. Prices tend on average to be a little lower for traditional dwellings than for dwellings of new traditional construction and to be 20 per cent. to 25 per cent. higher in London than in the provinces.

15. There is little doubt that under current conditions dwellings within two-storey blocks are the cheapest. Tender prices rise with increases in the number of storeys but the rise in price is less than proportional. While it is steep from two to five storeys, it tails off amongst the higher blocks, but it is not possible to be very certain of the price relationships for the very high blocks as so few have been built. Because of the small numbers of prices available, conclusions about the price pattern must be based on the data for several years and it is difficult to determine what the position is likely to be even in the immediate future. The usual regional price pattern appears to exist in the case of two- to five-storey blocks, Outer London prices being around 15 per cent. higher than provincial prices, and Inner London (excluding the L.C.C.) being about 30 per cent. up. The regional price pattern for the higher blocks appears to follow the usual lines for most parts of the building, although the percentage differences are apparently not as great as for two- to five-storey blocks. The pattern for prices of blocks as a whole is confused by the incidence of the alternative forms of structure, column and beam, crosswalls and load-bearing brick, and of differences in heating systems. Generally the provinces seem to have favoured designs of a type which were in themselves the more expensive alternatives, showing, for instance, a preference for staircase access as compared with balcony access, and central heating as compared with other forms of heating. The smaller regional differences in the prices of the individual parts were thus masked when combined into the cost for the building as a whole by the choice of cheaper design forms in London, as compared with expensive ones in the provinces. If the effect of these differences in choice of design forms is excluded, the regional differences for the high block appear to be about half the level for other forms of housing (Reiners, 1958).

16. The relationship of the cost of dwellings at different storey heights which will be applicable in the immediate future clearly depends on future trends which cannot be predicted now with any certainty. Such items as the form of construction and plan, standards and the efficiency of design and erection, are all relevant. On the basis of the figures now available, 12-storey blocks are around 60 per cent. more expensive per dwelling than two-storey ones in Inner London, 85 per cent. more expensive in Outer London and around 110 per cent. in the provinces (Table 2). It could be argued that the provincial relative costs will tend to fall to those of Inner London and that the relative costs as a whole will come down but for the present analysis the existing cost levels have to be used.

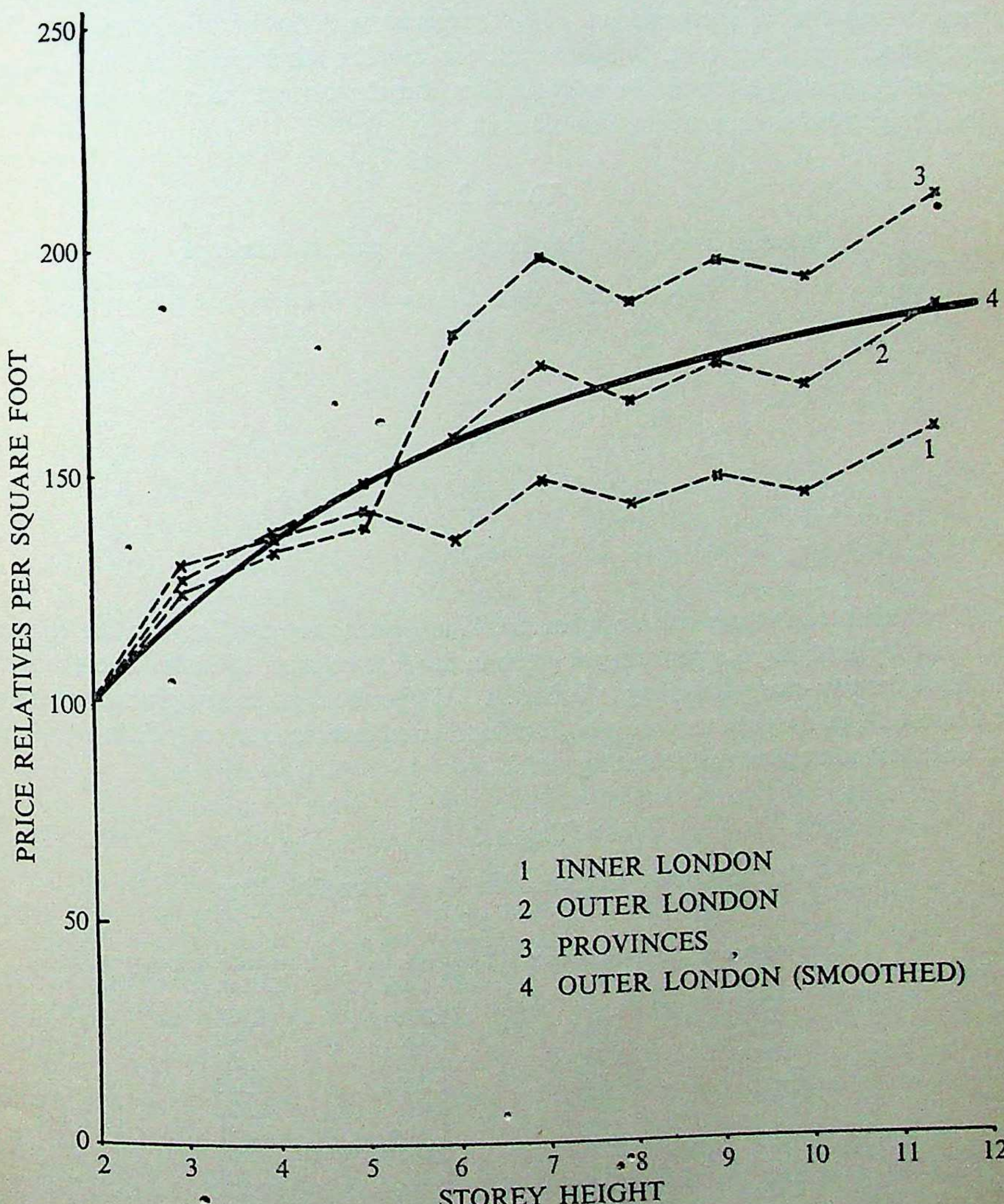

FIG. 3.—Average relative prices for blocks of flats and maisonettes, Jan., 1953 to Sept., 1956

17. The average relative prices given in Table 2 and illustrated in Fig. 3 are based on blocks varying greatly in design. Maisonettes are, of course, normally cheaper than flats and the incidence of maisonettes in the figures for even storey heights is reflected in the consistent pattern of minimum points (Fig. 3). The price relationships for Outer London appear to be the most consistent; the unevenness of the relatives has been removed by drawing a smooth line through the actual averages (Fig. 3). The resulting values are given in the last column of Table 2. These values are used as a basis of costing in this paper. In applying them to provincial developments it is being assumed that the increase in costs of high blocks relative to low blocks is falling, the level being taken as half way between the historical levels of the provinces and Inner London. Prices examined recently lend some support to this conclusion. As compared with lower blocks the figures for five-storey blocks are 2 per cent. to 3 per cent. too low for blocks with lifts and about the same percentage too high for blocks without lifts, but the comparisons between six-storey and block heights other than five are unaffected.

TABLE 2

Relative Prices for Blocks of Flats and Maisonettes

Block Height	*Inner London*	*Outer London*	*Provinces*	*Outer London Smooth Curve*
2	100	100	100	100
3	130	127	124	120
4	136	137	133	136
5	142	—	148	148
6	135	158	180	157
7	148	173	197	164
8	141	165	187	170
9	148	173	196	175
10	144	168	192	179
11 } 12 }	158	185	210	182
				185

18. The estimated average costs of provincial local authority dwellings in flatted blocks are obtained by applying the percentages given in the last column of Table 2 to the figures given above for two-storey dwellings (Table 3). While the figures are given here correct to the nearest £1, as they are used as a basis of further calculations, clearly little significance should be attached at least to the last figure.

TABLE 3

Estimated Net Construction Costs Per Dwelling

Block Height	*Three-bedroom, 910 sq. ft.*	*Two-bedroom, 770 sq. ft.*	*Two-bedroom, 680 sq. ft*	*One-bedroom, 510 sq. ft.*	*£'s Average of One- & Two-bedroom*
2	1,470	1,323	1,230	1,034	1,132
3	1,764	1,587	1,476	1,240	1,358
4	1,999	1,799	1,673	1,405	1,539
5	2,176	1,957	1,820	1,530	1,675
6	2,308	2,076	1,931	1,623	1,777
7	2,411	2,169	2,017	1,695	1,856
8	2,499	2,249	2,091	1,757	1,924
9	2,573	2,315	2,153	1,809	1,981
10	2,631	2,367	2,202	1,850	2,026
12	2,720	2,447	2,275	1,912	2,094

19. Relatively, the dwellings providing one or two bedrooms are more costly per person than those providing three, as shown in Table 4, which is derived from Table 3 on the basis of 0·89 persons per habitable room. The cost per person is 10 per cent. more in two-bedroom dwellings than in three-bedroom dwellings and 40 per cent. more in one- as compared with three-bedroom dwellings. While dwellings of all sizes must be provided, it appears more economic on the basis of these figures to put the smaller dwellings in the lower and cheaper blocks. This, of course, is contrary to the usual preference for housing families with young children at ground level.

TABLE 4

Estimated Net Construction Cost Per Person

£'s

Block Height	*Three-bedroom, 910 sq. ft.*	*Two-bedroom, 680 sq. ft.*	*One-bedroom, 510 sq. ft.*	*Average of One- & Two-bedroom*
2	420	461	581	509
3	504	553	697	610
4	571	627	789	692
5	622	682	859	753
6	659	723	912	799
7	689	755	952	834
8	714	783	987	865
9	735	806	1,016	890
10	752	825	1,039	911
12	777	852	1,074	941

Other Building and Civil Engineering Costs

20. The variability in costs of building types outside the field of low cost housing is likely to be greater than that within the housing sector, since there is a far greater range of specification. But comparison is to be made not amongst individual buildings but as between total developments for communities developed for housing at different densities. It is, therefore, more important that the total development costs should collectively be of the right order than that individual costs shall be correct, and it is sufficient if the individual building costs of general amenities are representative. Information was not available in sufficient quantity for a regional price pattern to emerge but the prices were generally for regions other than London. However, the figures used later should be of the right order; they may be a little higher than normal provincial prices, since prices tend to be a little higher in areas where a high level of building and development is taking place, as usually local supply is inadequate and the combined effect of bringing in supplies at a higher cost and the reduction in the level of competition tends to raise prices. The prices might, however, be on the low side for the London area. A number of sources has been used, estimators' cost books, the prices of buildings as given in articles in journals, and confidential prices and estimates provided by New Town Corporations and other developers, (Appendix B).

The Initial Cost of Developing Urban Areas

21. Having established both the broad physical needs and land requirements of urban areas and their construction costs, it is now possible to build up the estimated costs of various types of development; housing estates, neighbourhoods, industrial areas and the

town as a whole. The housing estates, together with shopping, educational and social amenities, form the neighbourhoods, and the neighbourhoods together with the commercial and industrial areas and certain general public utilities, roads and so on, form the town. By no means all development consists, of course, of complete towns; most will consist of the addition of estates or neighbourhoods to existing towns. Such additions will usually necessitate the expansion of other town facilities, and indeed the development of a new town will itself create the need for some additional facilities outside the town, such as road and bridge widening, water and other public utility expansions. The choice of locations for development should, of course, take account of the amount of under-used resources within the alternative locations.

22. In building up the estimates, an allowance has been made for all the buildings and development which would be provided in a New Town with a population of 80,000 if it were planned at the present day. An allowance has been made for current planning trends which indicate rather higher densities than were used at the time the present New Towns were planned. No allowance has been made for land within the designated area of the town which remains as agricultural land.

23. Throughout, costs have been built up on the basis of provincial prices; broadly these are national prices, prices in London being exceptional. All development work for new towns and extended towns will be on the basis of provincial prices, as will much of the redevelopment and peripheral development work (Appendix C).

24. *The Housing Estate.*—The capital costs involved in housing 10,000 persons in either two- or three-bedroom dwellings have been estimated at provincial prices on the basis of various land prices and using the prices for dwellings, site clearance, roads and sewers and public utility services derived earlier, an addition being made for fees and interest during the period of construction. In practice about half of the low cost houses, although rarely flats, are provided by private developers and these will normally cost a little more than public authority houses; their price complete with all charges might be around £2,100. Moreover, houses will normally be at lower densities than two-storey flats, which raises the costs per dwelling of land and services. Housing in two-bedroom dwellings costs around 20 per cent. more than in three-bedroom dwellings. The costs of housing in flatted blocks rise with storey height and are less than doubled by building at 12 storeys instead of two, the proportional extra cost depending on the price of the land (Appendix C, Table C.1).

25. *The Neighbourhood.*—The conception of the neighbourhood is of a balanced locality providing, in addition to housing, the other main everyday amenities which need to be within easy reach of the home. The neighbourhood unit includes the local shopping centre, primary schools, libraries, churches, open space and other social amenities. A neighbourhood for 10,000 persons would need around 120 acres of land and cost around £760,000 in addition to the housing area (Appendix C, Table C.2).

26. *The Industrial Area.*—The land allocated for the industrial area of a new town comprises not merely land needed for industrial sites but also land for roads, railways and canals, for private recreation (factory sports grounds) and for future expansion. Somewhere around 20 per cent. of the population is likely to be employed in the industrial area of a new town (Central Statistical Office, 1958). On this basis the industrial area for a population of 10,000 persons will be about 60 acres and its development will cost around £1,700,000 (Appendix C, Table C.3). This figure is based on the assumption that

only light industry is involved; for heavy industry, costs would be substantially greater.

27. *The Town Centre.*—The town centre contains the main shopping area of the town, the main administrative and civic buildings, roads and car parks for these, and usually central gardens and possibly a park. The main transport centres, rail and bus stations, are usually located within or on the outskirts of the centre, as are often the higher educational establishments. The shopping area will usually contain, in addition to the basic type of shops needed for everyday shopping, in which field it supplements the neighbourhood centres, the shops selling durable and specialized goods. It will take about 80 acres to develop a town centre for a new town of 80,000 persons and cost around £5,400,000 (Appendix C, Table C.4).

28. *Open Space.*—In addition to the open space provided in the neighbourhood, in the industrial area, and in the town centre, there is usually some further general provision of open space. This often takes the form of a town park, a sports stadium, golf courses and areas of natural woodland and grass. A further 300 acres may be needed for this purpose and the development cost will be around £200,000 for a town with a population of 80,000 persons (Appendix C, Table C.5).

29. *Higher Education.*—So far the only provision made for education is for primary and nursery schools; in addition provision will be necessary for secondary modern, grammar and technical schools and probably for a county college and other facilities for further education. It would appear that for a town of 80,000 persons the following provision might be necessary (Ministry of Housing and Local Government, 1952; Central Statistical Office, 1956; Ministry of Education, 1955).

	Places
Secondary Modern: 9 schools of 600 pupil places each	5,400
Grammar: 3 schools of 900 pupil places each	2,700
Technical: 1 school of 450 pupil places	450
Total Secondary Schools	8,550
Further Education: 1 school of 450 pupil places	450
	9,000

This provision would entail about 220 acres of land and cost around £2,745,000 (Appendix C, Table C.6).

30. *Other Building and Civil Engineering Works Within the Town Curtilage.*—In addition to the main areas already considered there are a number of lesser ones which find accommodation within the designated area of a town. While some of these, agriculture and railways, for instance, are large users of land, they may not be wholly, or to any extent, an essential part of the urban development of the town, and their place in the costs of town development must be considered on their merits. The balance of town facilities might cost about £10,386,000 and require 555 acres of land, where the town population is 80,000 persons (Appendix C, Table C.7).

31. *Works Outside the Town Curtilage.*—For a town of the size under discussion, the only important capital works lying outside the town and not yet costed are the construction and widening of road approaches to the town; this work might cost around £500,000. There remains a certain class of works such as the supply facilities for gas and electricity, terminal facilities for transport, universities, and central administration buildings, which, while a part of urban development and meeting the needs of the town, are yet no part of it. These are best regarded as national assets rather than town assets, since the need for

them is little related to the location of individual towns but almost wholly to total population.

Summary of Land Use and Cost

32. *New Town.*—The figures derived in Appendix C, Tables C.2–C.7, are summarized in Tables C.8 and C.9. The 80,000 person town postulated for the purposes of these estimates would require around 4,200 acres of land (Appendix D, Table D.8) and cost about £85 millions (Appendix C, Table C.9).

33. The largest single element of costs in the town is housing; the housing area including houses, land and development, costs nearly £47 million and requires about 1,600 acres, that is about 55 per cent. of the cost and 39 per cent. of the land. The figures are likely to be higher in practice since the figure for housing is based on local authority standards and on three-bedroom housing. It is probably fair to assume that 10 to 20 per cent. of the housing will be higher cost housing and that perhaps a quarter of the housing will be two bedroom or smaller, rather than three bedroom; this might add 15 to 25 per cent. to the cost and require another 300 acres for housing. Thus on this basis the area of land might be 4,500 acres for the town and the cost:

	£47 millions
+20 per cent. for higher cost and for two-bedroom housing .	£9 millions
Total for housing	£56 millions
Rest of town	£39 millions
Total cost	£95 millions

34. This suggests that with two-storey housing the 80,000 person town will cost between £1,100 and £1,200 a person according to the types of housing provided, the percentage of the cost for housing ranging from 55 to 59 per cent. The increase to be added for higher cost and smaller dwellings (less rooms) is much the same whatever the storey height of development. Developing at an average of four storeys instead of two-storey houses would add about 20 per cent. to the costs of housing, an average of 10 storeys would add 50 per cent. and an average of 12 storeys would add 57 per cent.; there would, of course, be some savings in land. The marginal cost per acre saved by using four-, eight-, and 12-storey instead of two-storey flats is about £30,000, £40,000 and £47,000 respectively. These figures are derived from Appendix C, Table C.1. If a large proportion of the households require allotments no land saving may result at all.

35. It is of importance to consider to what degree the estimates given are reliable. Clearly the total possible cost range of town amenities is very large, since towns vary so much in the type and amount of amenities they provide. While there are few sets of independent estimates against which to set the figures derived above, such comparisons as could be made were satisfactory.

36. The earliest estimate of the cost of a new town was that given by the New Towns Committee (1946). They suggested a figure of £27 million for a town of 50,000 persons, assuming prices in 1946 to be 50 per cent. higher than in 1939. Prices are now probably around 230 to 240 per cent. higher than in 1939. On this basis the cost for a town of 80,000 would be around £97 million, in fact, nearly the figure suggested above when allowance is made for higher cost housing.

37. Lichfield (1956) has estimated the cost of developing an area for 23,000 persons as a part of an existing town. No direct comparison was possible because of different underlying assumptions but there is close agreement for comparable sectors after adjusting for size and price changes.

38. Comparison has also been made against the Master Plan for Stevenage, which detailed the physical requirements; against the costs given for the proposed new town of Allhallows (Adams, 1957; *The House-Builder*, August, 1956; *The Surveyor*, October 6th, 1956); and against those for the proposed L.C.C. new town (*The Times*, September 20th, 1957). The information was often vague and adjustments had to be made for size and existing developments. The resultant estimates were very close to the results given above.

39. *Town Expansion.*—As an alternative to the development of new towns, town expansion offers a potentially cheaper solution. The problem is to find towns which are sufficiently isolated to need to be self sufficient, which have some industrial potential and experience, and which have some unused capacity in terms of general urban amenities. Such development takes place under the Town Development Act and not under the New Towns Act and hence the initiative must come jointly from the exporting town and the town wishing to expand. It is not, therefore, surprising to find that the greatest initiative comes from towns with economic difficulties, a result, perhaps, either of a lack of industrial balance—a one-industry town, or a town whose industries offer employment to only one sex—or where industry has declined.

40. The essential provisions in a town expansion programme are housing, neighbourhood amenities, an industrial estate, water and sewerage. Provision of other amenities depends on the extent to which there is surplus capacity in the existing amenities. Often there is a wish to raise the general level of amenities and some of the development work often undertaken in the course of a town expansion programme is as much for the benefit of the existing inhabitants as for the immigrants. Town centre development, street lighting, baths and stadiums fall into this class and often development work on water and sewerage. It is, of course, often argued that unless the attractions of the town are raised in this way, people will not be prepared to move to it, and such developments must be considered in estimating the costs.

41. It was seen from earlier tables that the cost of a new town per 10,000 persons is of the order of £11–£12 millions where two-storey housing is used, the difference in the estimates depending on the type and standard of housing. The bulk of this cost is for the essential housing, neighbourhood and industrial amenities, leaving around £2·5 millions for the general town amenities. It would appear that around £1 million is being spent on general town amenities for town extension programmes for each 10,000 of new population, mainly on water and sewerage, street improvements, open spaces, secondary and higher education and on local government installations. This figure could easily be doubled if extensive town improvement schemes were undertaken.

The Annual Costs of Urban Development

42. Capital costs do not by themselves provide an adequate basis for decisions on development policy. It is necessary also to consider the running costs. The two types of costs can be combined, either as the sum of the capital cost and the discounted value of running costs, or as the annual cost, which is the sum of the amortized value of the capital cost, and the running costs. The two concepts are, of course, equivalent; the latter

method has been preferred because the meaning of the annual cost is more readily understood as it can be compared against incomes and annual expenditure. Most buildings and works have a life of between 40 and 80 years which, at the range of interest rates likely to prevail, gives an average equivalent annual charge about equal to the rate of interest. Thus, assuming that the long-term interest rate is about 5 per cent., the annual charge can be taken as being about a twentieth of the first cost.

43. *The Maintenance and Servicing of Estates.*—The costs of maintaining and servicing the housing itself, and the other estate amenities, form quite a high proportion of the total estate costs. The costs of servicing the housing and space about buildings have been built up from the results of surveys and returns of costs from various local authorities. They range from £34 to £54 a year depending on the type of housing, being around 50 per cent. higher in tall than in low blocks (Appendix D, Table D.1). The regional cost pattern found for housing is broadly applicable and has been applied to the costs (Appendix D, Table D.2). In addition to the costs for housing and space about buildings which fall to the housing authority it is also necessary to consider the costs of servicing roads, around £800 a mile (Appendix D, Table D.3), which fall to the local authority, and the cost of refuse collection and other delivery and collection services and allotments (Appendix D), which are services directly related to housing and which fall partly to the cost of the local authority and partly directly on the inhabitants, or on the community as a whole.

44. *Housing Subsidies and Exchequer Grants.*—Central government grants can be obtained by public developers of housing both in respect of land and general development, and of the dwellings themselves. The grants, even directly on the dwellings themselves, can be substantial—upwards of £50 per dwelling a year for dwellings in blocks of six storeys or more (Appendix D, Table D.4). In addition, local authorities may receive Exchequer Equalization Grants.* These can be very large and may be affected by decisions on density and overspill (Appendix D). Grants towards development can also be obtained in some cases from the Central Government and the County Councils.

45. *Environment Costs.*—The cost of living varies between large and small towns and is greater for London than for other large towns. It would appear that real costs per person may be £5 a year greater in London than in large provincial towns and perhaps £10 less in small as compared with large provincial towns (Appendix D, Table D.5).

Improvement Values

46. As explained above, the new development has the effect of replacing old buildings and works with new ones which should have a longer expected life and lower operating costs. While it is difficult to find any figures based on actual experience some estimate of the saving can be made on the basis of the likely age distribution of the buildings and works in the old areas. For instance, old assets, forming a normal cross-section of the assets of all ages, will have a worth about 28 per cent. less than newly created assets because of their shorter expectation of life. A similar result is obtained by assuming that the old assets have an unexpired life of about 25 years. Further, the old assets will be less valuable to the user because maintenance, running and operating costs are likely to be greater; for this reason old assets may only be 80 per cent. as valuable as new. The net effect of these two assumptions is to place a value on the old assets of only about 60 per cent. of

* These will be replaced under the Local Government Act 1958, by Rate Deficiency Grants.

those newly created. On these assumptions the effect of replacing the old assets by new ones is to create an improvement value of about 40 per cent. of the value of the assets. In making these estimates no allowance has been made for the residual value of the assets being replaced. Often these are retained and used for some other purpose so that the real improvement value may be a great deal higher than that suggested above. Industrial assets must be considered separately. In general, factories only move to new areas because they are unable to re-build in the area in which they are situated. Thus their move is made with the expectation that the cost of the new building will be paid for out of the increases in efficiency thus made possible. The extra costs of movement and disturbance resulting from moving out of their locality must, of course, be reckoned as a cost against the overspill programme. It therefore appears reasonable to assume 100 per cent. improvement value for factory buildings, although not, of course, for the general development of industrial areas (Appendix D).

Movement Costs

47. All development and redevelopment involve some movement of personnel and effects from one place to another with the associated disturbance to production. The greater the distances over which the moves take place the greater both the costs of movement and disturbance. For instance, a limited move of 80,000 persons within the confines of a large city may cost about £2 million, whereas a move to another area 100 miles away is likely to cost around £5 million (Appendix D, Table D.7).

PART 2. THE ANNUAL COSTS OF ALTERNATIVE TYPES OF DEVELOPMENT

Introduction

48. Having considered the various types of costs which arise in urban development and ways of measuring the impact of development on the costs of the urban complex of amenities and services, it is possible to estimate the costs of any particular type. This is the object of this section of the paper; all the estimates used for building up the total costs are derived from the figures given in the earlier part of the paper, and its Appendices.

49. In considering development costs it is necessary to take into account both the type of development and the point of view from which its economic situation is assessed. For example, housing estates can be considered from the point of view of the privately owned estate, the publicly owned estate, the tenants, local authorities as entities, and the community as a whole. The balance of advantage as between different types of estates from these various points of view will differ according to whether the estate is in London or the provinces, is on redevelopment or virgin land, according to the price of the land itself and according to the position of the estate relative to the other urban amenities. All these aspects are capable of being costed and so their total effect can be reduced to one figure whose significance can readily be grasped. While this paper is mainly concerned with real costs, that is community costs, costs to public and private developers are considered, where these are relevant to the argument.

50. Clearly it is not feasible in one paper to consider all the possible planning alternatives and their costs; both to a large extent depend on local circumstances prevailing in the town for which a solution is required. Consideration will be given here only to a

selection of the main types of problem. The costs for other situations and other assumptions can readily be derived from the basic cost data already given. All costs have been calculated as at the middle of 1957; since then costs have remained fairly steady and the cost data can be taken as broadly applicable today.

51. While the most usual form of dwelling has three bedrooms, flatted dwellings more often have one or two bedrooms. This situation largely arises because of the preference for housing the families with young children at ground level. If the scale for which the alternative is houses or flatted dwellings is large, the flatted blocks will inevitably have to consist of three-bedroom dwellings and the comparison will be in terms of three-bedroom houses as against three-bedroom flatted blocks. If the scale is small the alternative can be considered only for the small families and the comparison can be in terms of one- or two-bedroom dwellings. Usually room sizes and circulation space in flatted dwellings are less than in houses and it might be thought legitimate to compare the costs of the smaller flatted dwelling against the larger house. If this is done, it is, of course, necessary to consider the value of the larger floor area in the house as compared with the flatted dwelling in comparing the costs. For the sake of simplicity the analysis which follows is worked in terms of three-bedroom dwellings of constant floor area; the effect of other alternatives is considered later.

Housing Estates

52. The capital costs of a housing estate can be divided into two parts, the greater part consisting of the cost of the dwellings themselves and of the works within the curtilage, and the lesser the costs of land, estate development, and the estate public utility services. These latter costs depend on the density of development. In general the densities given in Table 1 are unlikely to be exceeded but in special cases higher densities may be possible, although densities much above the maxima indicated cannot be universally achieved. The erection of a very tall building on one site pre-empts the sky space of the locality and necessarily limits the opportunities to build similar high blocks on the adjacent plots. Often the densities achieved will be lower than those given in Table 1; some idea of the effect of changes in density can be seen in Fig. 4. It will be noticed that small changes in density have little effect on the initial cost of high buildings but that the effect increases as the storey height is reduced and at two storeys it is quite marked. Even for two-storey housing initial costs of developing an estate are dominated by the cost of the dwelling itself, which on low cost land accounts for four-fifths of the costs (Table 5). This proportion rises with the height of the block; even on expensive land, say £5,000 an acre, and taking site clearance and public utility services at the higher level of costs applicable to redevelopment areas, the proportions are but little changed (Fig. 5). The most variable element of these costs is, of course, the land cost, but this must rise to very high levels to make very much difference to the proportions.

53. For the estate developer who proposes to sell the finished estate, or who proposes to let with the tenant fully responsible for repairs, initial costs can be taken as a measure of supply costs. Of course, neither the private nor the public estate developer has to pay for the public utility services but the differential effect of these is so small that their inclusion makes little difference to the conclusions. Both the sale price and the rent income will be determined independently of the development cost of any particular estate and the

developer's interest lies, therefore, in reducing development costs to a minimum for dwellings offering the particular range of amenities. For this condition the estate must be developed

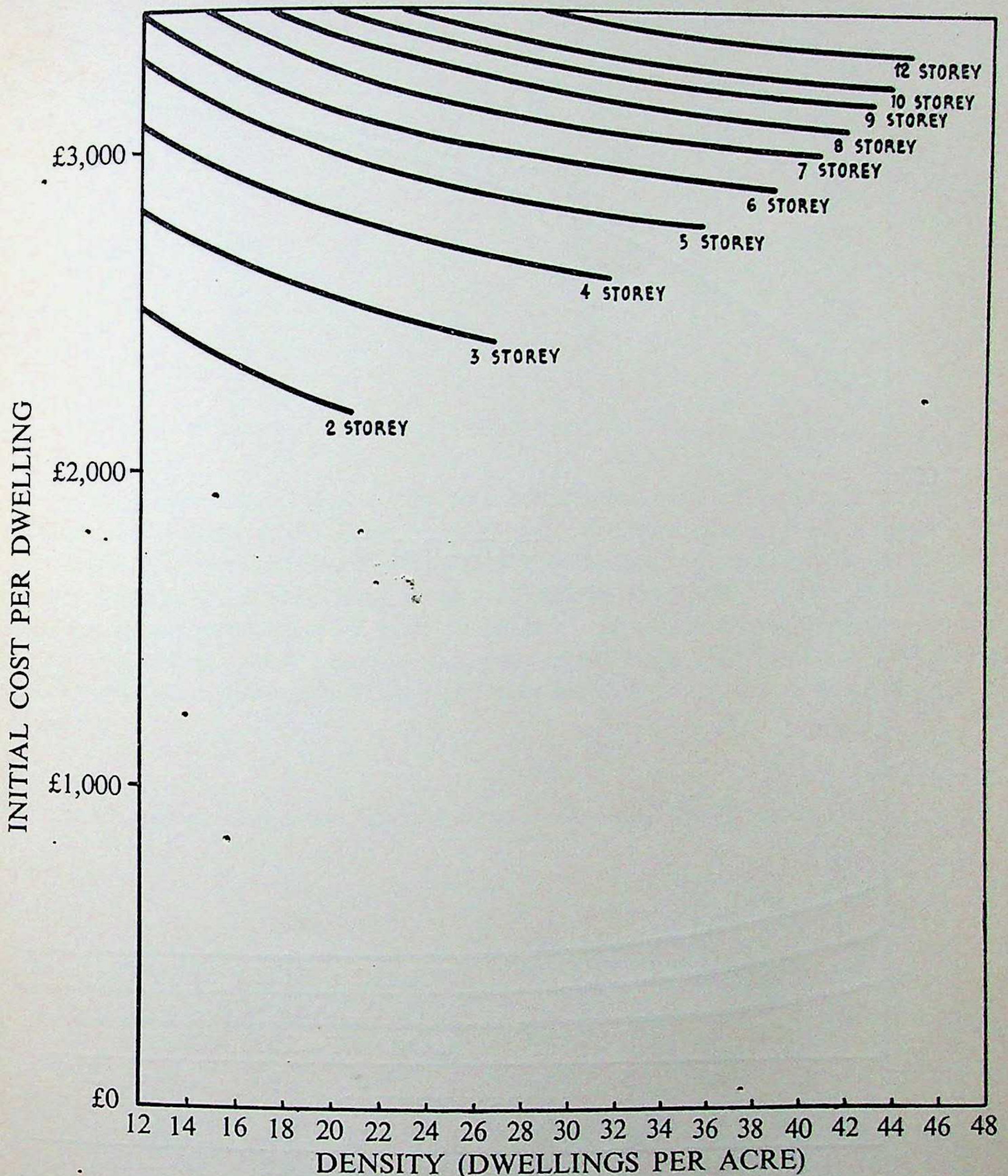

FIG. 4.—Capital costs of three-bedroom dwellings in flatted blocks in central areas in the provinces.

at the maximum density compatible with the block heights used. In particular cases it may be possible to raise densities a little by using a mixed development of blocks at various

heights instead of a development based on blocks all of one height, but this problem has been ignored here for the sake of simplicity. In practice, of course, the costs for the

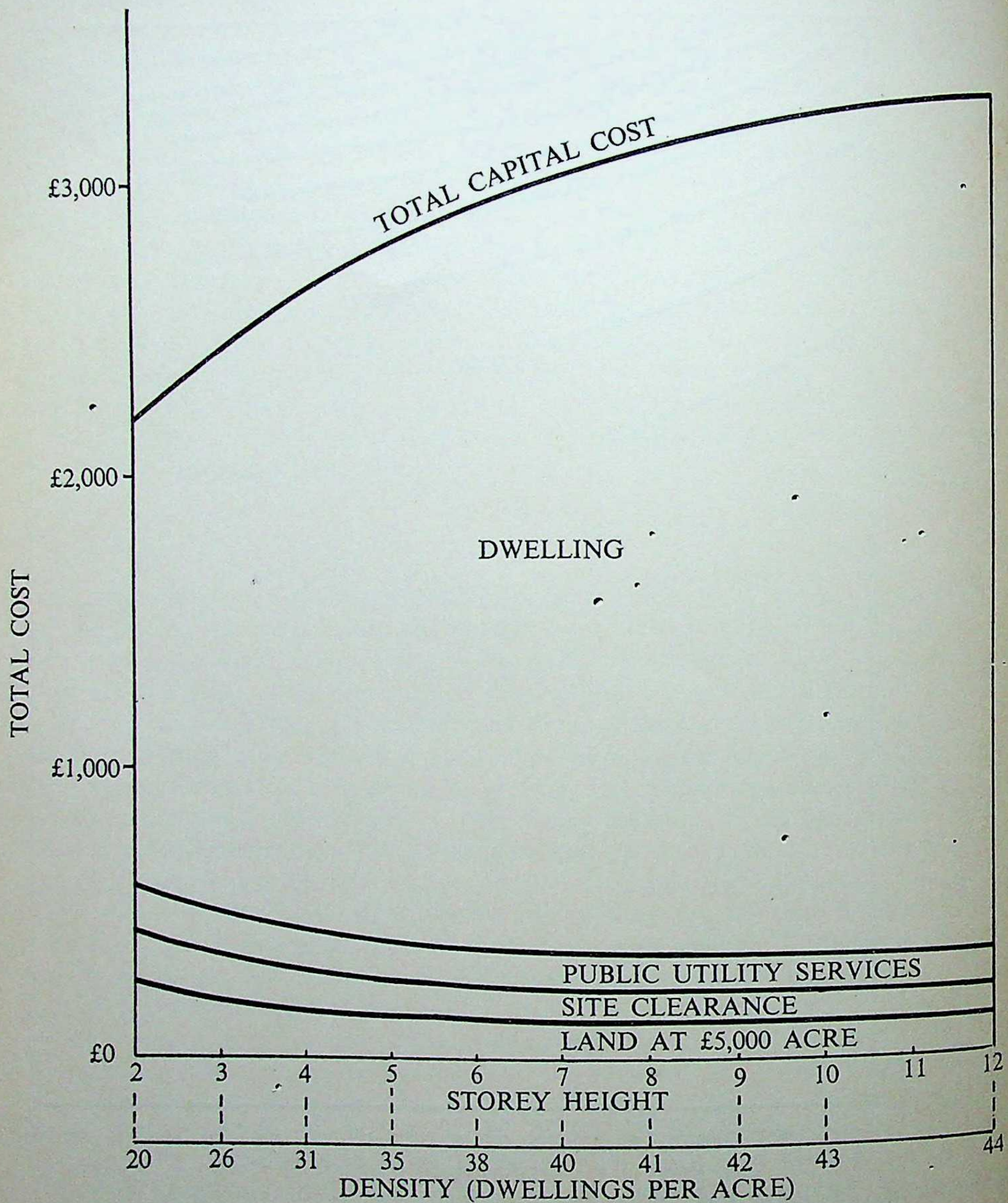

FIG. 5.—Capital costs of three-bedroom dwellings in flatted blocks in central areas in the provinces.

dwelling will not rise regularly with block height but will jump at the block height at which lifts are installed; this will normally occur between four and six storeys.

TABLE 5

Capital Cost of Three-Bedroom Housing in Flatted Blocks—Provinces

Measures of Density			*Construction Costs per Dwelling*				
Block Height	*Habitable Rooms per Acre*	*Dwellings per Acre*	*Land at £100 an Acre* (£)	*Site Preparation,* Levelling Roads and Sewers* (£)	*Public Utility Services* (£)	*Dwellings* including Works Within the Curtilage* (£)	*Total Capital Cost for 3·5 Persons* (£)
2 . .	80	20	5	127	153	1,617	1,902
3 . .	104	26	4	101	145	1,940	2,190
4 . .	124	31	3	88	140	2,199	2,430
5 . .	140	35	3	80	136	2,394	2,613
6 . .	152	38	3	76	133	2,539	2,751
7 . .	160	40	3	72	132	2,652	2,859
8 . .	164	41	2	70	132	2,749	2,953
9 . .	168	42	2	69	131	2,830	3,032
10 . .	172	43	2	68	131	2,894	3,095
12 . .	176	44	2	67	130	2,992	3,191

* Including an addition of 10 per cent. for fees, interest, etc.

54. The main determinants of the total initial costs will be the price of land and the cost of the dwellings at the given block height. For any given set of dwelling and land prices there is a corresponding block height for which total costs are at a minimum. These can be ascertained by reworking Table 5 at different land costs. Some results of doing this on redevelopment land are given in Table 6. It will be seen that in terms of initial costs, for instance, it might pay to build eight-storey rather than two-storey flats on land costing £40,000 and perhaps 12 storeys rather than two storeys on land costing £46,000 an acre.

TABLE 6

Block Height and Land Prices for Minimum Initial Estate Costs

(1) *Storey Height*	(2) *Density (Dwellings per Acre)*	(3) *Price of Land per Acre (Estate Cost per dwelling equal to that for two-storey flatted blocks)* (£)
2	20	—
3	26	24,000
4	31	29,000
5	35	32,000
6	38	35,000
7	40	37,000
8	41	40,000
9	42	42,000
10	43	44,000
12	44	46,000

55. Many private developers will let the dwellings at an inclusive charge and provide all the maintenance and repair services, porters, lifts and attendance of the public circulation space and the space about the buildings. The effect of adding in the costs of these

additional services is to increase further the comparative disadvantages of high blocks. In order to compare the costs it is first necessary to convert the initial costs into their annual equivalent cost, which is to divide them by 20; to these figures the annual maintenance and management costs can be added. Since the maintenance and management costs increase sharply at the point at which lifts and porter and caretaker staff are introduced (Table 7), the costs of land necessary to balance the rise in block costs will be substantially higher above this point than shown in Table 6.

TABLE 7

Comparable Costs in the Provinces of Three-Bedroom Dwellings in a Central Area for Private and Public Authority Housing

	Three-Bedroom Dwellings in Multi-Storey Flatted Blocks									
	Two-storey	*Three-storey*	*Four-storey*	*Five-storey*	*Six-storey*	*Seven-storey*	*Eight-storey*	*Nine-storey*	*Ten-storey*	*Twelve-storey*
Maximum Achieved Densities (number of dwellings per acre) .	20	26	31	35	38	40	41	42	43	44
	(£)	(£)	(£)	(£)	(£)	(£)	(£)	(£)	(£)	(£)
Total Capital Costs per dwelling for Private or Public Authority Housing of Land (£5,000), Development, Services and Construction .	2,211	2,429	2,632	2,793	2,918	3,017	3,108	3,184	3,243	3,336
Equivalent Annual Costs per dwelling . .	111	122	132	140	146	151	155	159	162	167
Annual Maintenance and Management per dwelling	30	30	30	43	43	43	43	43	43	43
Total Expenses for Private Housing per dwelling .	141	152	162	183	189	194	198	202	205	210
Subsidies per dwelling .	25	25	34	40	52	54	55	56	58	62
Net Cost for Public Authority Housing per dwelling	116	127	128	143	137	140	143	146	147	148

56. Public housing authorities building dwellings to replace slum clearance property, or building under overspill arrangements, receive subsidies in the form of an annual payment from the central government. These subsidies are scaled to the storey height of the building and also to the price of the land. The subsidies on the building rise rapidly to six storeys and then more slowly. Even in the case of three-bedroom dwellings, the subsidies go a long way towards offsetting the rise in annual costs with storey height (Fig. 6). Since the subsidies are the same for dwellings of all sizes, their effect in reducing costs is greater in the case of one- and two-bedroom dwellings. The subsidies payable on land are obtainable for land costing £4,000 per acre and upwards and increase with the price, amounting to about two-thirds of the annual costs of amounts in excess of £5,000 an acre. As a result the use of expensive land does not mitigate the costs of building high as rapidly for public as private developers. For instance, for public authority housing the total annual costs of 12-storey blocks, including for land and subsidies, do not fall to those for two-storey blocks until land costs around £75,000, whereas the corresponding figure for private housing is around £55,000 an acre, £9,000 an acre higher than if initial

costs only are considered. A further difference between private and public housing agencies lies in their different economic problems. Private agencies expect a return for each investment and so consider the price of land to the individual investment. Public

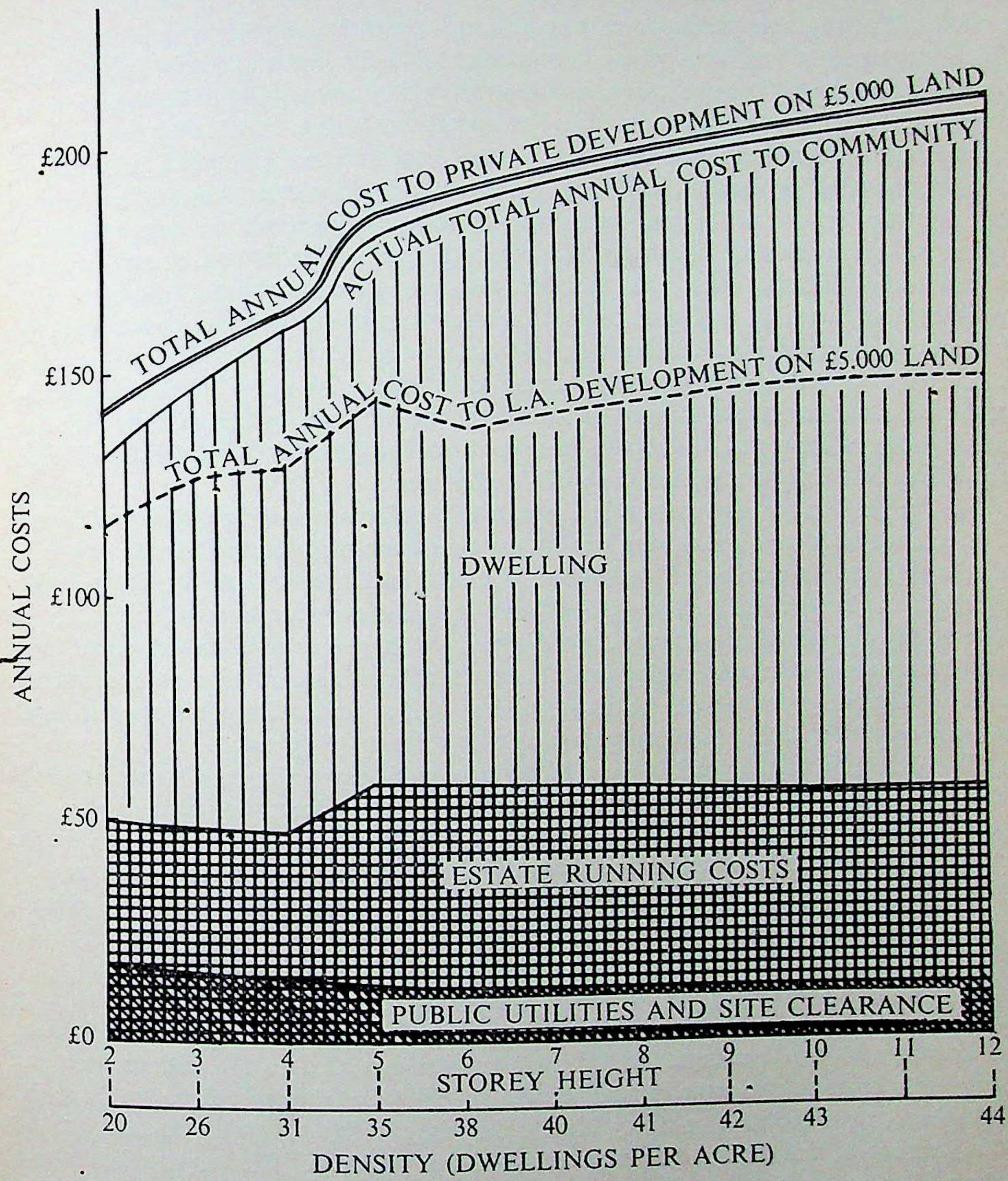

FIG. 6.—Total annual costs of three-bedroom dwellings in flatted blocks in central areas in the provinces.

agencies need only aim to balance revenue and expenditure over the whole of their estates and, therefore, consider the average price of the land, when it would be immaterial to them whether they site high blocks on expensive land in the centre or low cost land at the peri-

phery. For both, however, costs rise more slowly as height increases and, therefore, once committed to high building, say 10 storeys and above, it costs but little more per dwelling to build really high and it may be possible to double the height without incurring much extra cost.

57. Both private and public developers as housing agencies expect to meet their costs. The private agencies will try to obtain a rent to cover their costs together with something over for profits and will not build at costs so high that they are unlikely to be able to obtain adequate rent. Public agencies can afford to spend more since in addition to rents they receive central government subsidies. Both sorts of agencies can, of course, subsidize one set of rents at the expense of another set. Local authorities can additionally subsidize housing at the expense of the rate fund. They tend to charge about the same rent for all dwellings of about the same size irrespective of whether they are in houses, or comparatively expensive high blocks of flats. For instance, they might charge about £2 5*s.* a week for three-bedroom dwellings, sufficient to cover dwellings in two-storey blocks but leaving a deficit on all other forms of housing ranging from around £10 to £30 a year. This would be covered by a subsidy from the rates. Local authorities as an entity are also responsible for the maintenance and servicing of the streets and public open spaces, and for collecting and disposing of the waste. However, these costs are low and the density of development makes little difference to the estate costs of these services.

58. The position for the tenant is the converse of that for the housing agencies. Tenants of private agencies tend in the long run to pay the economic rent and so will pay more rent for high cost development than for low; the tenants of a local authority will normally pay about the same whatever the type of block in which they are housed, subject to the effect of any differential rent scheme. Both types of tenant pay rates as well as rent and since it appears that rateable value bears a closer relation to rent than to construction costs, it is likely that for the private tenant rates as well as rent will rise steeply with block height, whereas for local authority tenants neither will vary very much.

59. Clearly from the point of view of the community as a whole, all taxes and subsidies are merely a transfer of money from one pocket to another; no resources are used and there is no effect on the national income. Subsidies can, from this point of view, be ignored. The costs of land are in a somewhat similar position; the price paid is a transfer of money involving no change in national resources. National capital is, of course, reduced by demolitions and increased by the value of subsequent redevelopment, but this point has been considered in connection with improvement values.

60. However, from the point of view of the community, certain other costs also arise such as the service costs of estate roads and landscaped areas and the costs of collections and deliveries to the dwellings. The former have been added to the maintenance and management costs to give estate running costs; the latter are shown separately (Table 8).

61. As compared with two-storey blocks, dwellings in five-storey blocks cost around £50 a year more, and around £70 more in 10–12-storey blocks. The savings in site and public utility service costs with increases in density are less than the rise in maintenance and servicing costs resulting from the use of high blocks. As a result the capital costs of flatted dwellings would need to fall as the block height rose, in order that annual costs should be constant for all heights. The gross capital costs of 10–12-storey blocks would need to be about £1,450 per dwelling, about £150 less than for two-storey blocks, if their annual costs were to be no greater—clearly an improbable achievement.

62. Inevitably, costs differ from one place to another and some substantial savings in block costs may be achieved. While changes in unit costs might change the detail of the figures it is difficult to visualize changes large enough seriously to alter the conclusions drawn from them. Changes in block prices and levels in subsidies might well change the prices of land at which certain block heights become worthwhile to building owners, but the overall pattern is likely to remain unchanged. The national resources needed to construct, maintain and service flatted dwellings would continue to rise with block height.

TABLE 8

Comparable Annual Community Costs per Dwelling of Three-Bedroom Dwellings in a Central Area in the Provinces

	Three-Bedroom Dwellings in Multi-Storey Flatted Blocks									
	Two-storey (£)	*Three-storey* (£)	*Four-storey* (£)	*Five-storey* (£)	*Six-storey* (£)	*Seven-storey* (£)	*Eight-storey* (£)	*Nine-storey* (£)	*Ten-storey* (£)	*Twelve-storey* (£)
Dwellings including work within Curtilage . .	81	97	110	120	127	133	137	142	145	150
Site clearance, Levelling, Roads, Sewers and Public Utilities . . .	17	15	14	13	12	12	12	12	12	12
Estate Running Costs .	33	33	33	45	45	45	45	44	44	44
Other Community Costs—Expressed as the difference from the costs of two-storey blocks .	—	1	1	2	2	2	2	2	2	2
Annual Cost to the Community	131	146	158	180	186	192	196	200	203	208

Housing in Town Perimeters

63. It is now necessary to consider the effect on costs of the complex of urban amenities which must be associated with the housing estate. This will differ according to where the estate is located and to the local conditions. Many comparisons could be attempted but only the most general cases can be considered here, and those turn on the density at which central areas should be re-developed. The practical range of net residential densities lies between around 14 to 44 dwellings to the acre, that is perhaps from 50 to 150 persons to the acre, although of course extensions beyond this range are possible. Densities much above 20 dwellings to the acre can only be obtained by using multi-storey flatted blocks but under these conditions the costs and densities rise together. Central areas of towns are usually adjacent to all the urban amenities, shops, schools, industry and communal facilities. It is often sufficient to provide housing alone in central development areas, all other urban amenities being drawn from the surrounding district. At most the only addition to housing will be neighbourhood amenities. There are several alternative locations for housing population not housed in the Central Redevelopment Area. Broadly, these are:

(1) on undeveloped land within the town itself,
(2) on land within the town itself at present in urban use, e.g. housing,
(3) on undeveloped land at the periphery of the home town,
(4) on undeveloped land at the periphery of another and usually distant town,

and

(5) on undeveloped land forming part of a new town.

64. There is little undeveloped land still available for housing in large towns and this case need not be considered further. The second case is of more interest. This suggestion is that land lightly developed with sound housing should be cleared and re-developed at a higher density. In many respects this is not dissimilar to housing in central re-development areas and similarly neighbourhood amenities might or might not be necessary. However, the amount of additional housing provided by this type of development is lower, density for density, than either for housing on undeveloped land or on land without sound buildings, for a proportion of the dwellings to be built are needed to re-house the occupants of the sound dwellings cleared to allow the re-development to take place. Again, in Case 3—undeveloped land at the periphery of the town—neighbourhood amenities might or might not be necessary, although usually full neighbourhood amenities will be needed in this case. London and other conurbations and many of the large towns have inadequate or no peripheral area with free land on which to develop. Case 4 refers to the overspill of surplus population to a large planned extension of a small town and in this case it is usually necessary to provide at least full neighbourhood amenities and an industrial estate as well as housing. In Case 5, overspill to a new town, all urban amenities must be provided.

65. Town development is carried out by many different authorities, the various public authorities each having its own statutory duties. Housing estates may be developed by public or private agencies, as may commercial and industrial estates. Such developers are not responsible for the public utility services. Developers are, in fact, often responsible for only a few of the amenities outside the estate. The responsibility for roads, education and many other services is divided among the local, county and central authorities and varies with the status of the local authority. Other amenities are provided by private and institutional interests. The pattern of central government grants and subsidies from other local authorities varies with circumstances. In individual cases the problem of determining the balance of advantage is not difficult but in this field generalizations are of little value. The balance of advantage to this or that authority is only of importance in so far as it affects their decision to proceed with the development most desirable to the community as a whole. Much of the complex of costs affecting decisions in this way is made up of transfer payments, which can readily be adjusted to bring the interests of the developing authorities in harmony with the interests of the community which is concerned only with real costs.

66. The costing given below is in terms of real costs, and is based on the estimates given in the first part of this paper and on its Appendices. The estimates have been applied to a block of 10,000 persons. Each case has been treated on the same basis so that all the costs are comparable.

67. First, the annual costs of re-housing 10,000 persons have been built up as the annual equivalent of the initial costs of constructing the buildings and developing the engineering services. To these figures must be added the difference in the estate running costs as between the standard two-storey housing solution and other forms of housing. The costs on this basis are given for various types of location and various densities (Table 9).

68. The annual costs of the non-housing amenities can be built up in a similar manner. Table 10 shows the figures for the various groups of costs of neighbourhoods,

extended towns and new towns, allowance having been made for movement costs (Table 10).

TABLE 9

Annual Costs of Re-housing 10,000 Persons
(Including housing, roads and sewers, services and movement costs)

£'000s

Type of Dwelling	Density of Development (Dwellings per Acre)	Redevelopment Land: 1 Mile from Existing Housing	Redevelopment Land: 5 Miles from Existing Housing	Virgin Land: 5 Miles from Existing Housing	Virgin Land: 100 Miles from Existing Housing	Differential Estate Running Costs
Two-storey Houses	14	304	304	291	296	—
Two-storey Flats	20	281	281	273	277	9
Three-storey Flats	26	321	321	314	318	10
Four-storey Flats	31	356	356	348	353	11
Five-storey Flats	35	381	381	374	379	47
Six-storey Flats	38	398	399	394	399	48
Seven-storey Flats	40	416	416	409	414	48
Eight-storey Flats	41	427	427	423	428	48
Nine-storey Flats	42	441	441	434	439	46
Ten-storey Flats	43	450	450	443	448	46
Twelve-storey Flats	44	464	464	457	462	46

TABLE 10

Annual Costs for 10,000 Persons (excluding housing)

£'000s

Item	Neighbourhood in Central Redevelopment Area (Provincial)	Peripheral Neighbourhood (Provincial)	Extended Town (Neighbourhood and Industrial Area only)	New Town (All amenities)
Neighbourhood	41	38	38	38
Industrial Area	—	—	105	105
Other Town Costs	—	—	—	122
Total Annual Costs	41	38	143	265

69. An allowance must also be made for the improvement value which arises when existing buildings are replaced by new buildings. The amount of improvement value depends, of course, on the nature of the assets to be replaced, not on the new assets. For instance, whereas the improvement value which arises from a redevelopment of slum clearance houses is 100 per cent. of the annual return that would be obtained from the slum clearance houses if they were replaced by similar new ones, the improvement value in the case of the replacement of sound housing will vary in accordance with the expected life of the existing houses. The improvement value in the case of houses built between the wars might perhaps be about 40 per cent., although if the dwellings had been built immediately prior to the 1939–45 War they might have an expectation of 40 years and an improvement value of 27 per cent. (Appendix D, Table D7). The figures in Tables 9 and 10, adjusted for improvement values, can now be used to build up estimates for the various costs of providing dwellings discussed earlier (Table 11; Fig. 7). The exact assumption as to the number of houses originally on the slum clearance area is of course not of great importance since the number only enters the calculations in calculating the improvement

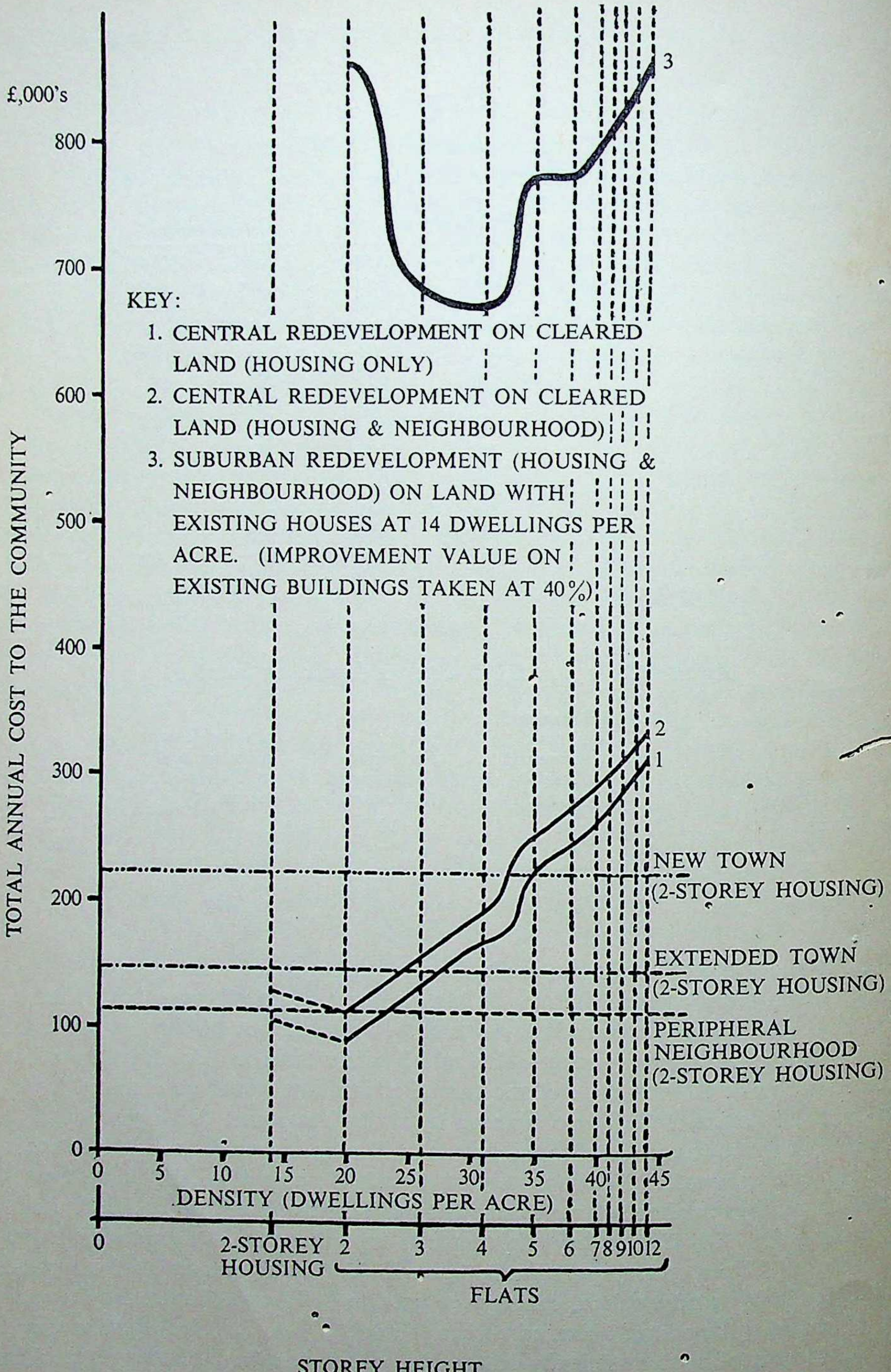

FIG. 7.—Comparison of the total annual costs of rehousing 10,000 persons from slum clearance areas.

value and this is, of course, deducted equally from the costs of all the alternatives. As would be expected, the costs rise both with the height of the flatted blocks and with the scale on which the amenities are provided. The peripheral developments are of course a little cheaper than the central area developments since development is cheaper on virgin land than on land previously developed. The increases in costs with block height are of course similar wherever the development is situated. It will be seen that the costs for the new town alternative in two-storey housing are a little less than housing at five storeys in a central redevelopment area where no neighbourhood facilities are provided. Naturally the costs of other peripheral developments are lower and the break-even points as compared with central development come at lower block levels.

70. The exception to these generalizations occurs in the case of suburban re-development on sound housing land. In this case no general rule can be laid down because the relative costs with block height depend on the unexhausted life of the existing assets and hence on their improvement value, on the existing density of development and on the scale of provision for non-residential amenities. The figures given are based on 14 dwellings to the acre. On this basis if the area is redeveloped at 20 dwellings to the acre, only six dwellings an acre are available for occupation by people from a slum clearance scheme, but generally it will be necessary to provide such people with neighbourhood facilities and this will have the effect of halving the land available for their housing. The net yield of dwellings for slum clearance persons will then be reduced to three dwellings to the acre. Thus in these circumstances, for every 10,000 persons re-housed from slum clearance around 60,000 persons must be re-housed in all. It is not therefore surprising to find that the costs for this sort of redevelopment can be as much as ten times as great as re-housing on slum clearance land or on peripheral land. Clearly in such circumstances the increase in net housing available for re-housing slum clearance persons with increases in block height will be very large and at first the marginal rate at which the amount of net housing increases with block height will exceed the marginal rate at which the costs of such housing rise. However by the time five-storey blocks are reached, the marginal increase in costs is already greater than the marginal increase in housing and the total cost of housing each 10,000 persons from the slums starts to rise again. Whether or not this cost eventually exceeds the cost of two-storey flats depends on the cost ratios, the existing density of development and the improvement value.

71. For constant improvement values, the cost rises with the density of the existing housing, and rises disproportionately when the new densities are low. For moderate existing densities, say 14–18 dwellings per acre, minimum costs are obtained at around four-storey blocks; as the existing density rises, so does the density at which minimum costs are obtained. The costs where existing densities are low must be considered as special cases, since the type of existing housing will be different from that which would normally replace it.

72. In practice it seems clear from the figures given in Table 11 that the redevelopment of areas of sound suburban housing as a means of dealing with the overspill from slum clearance projects will not ordinarily be economically justified. Clearly, the lower the existing density of development and the shorter the expected future life of the existing dwellings, the more likely the redevelopment of the area can be economically justified. Generally the choice for rehousing the surplus from slum clearance schemes will be virgin land at the periphery of the existing town, in an extended town or in a new town. The

break-even points illustrated do not, of course, directly determine the most economic level of block heights in the central area. This depends on the relationship between density and cost per person in the central area and cost per person in the peripheral area.

TABLE 11

*Comparative Annual Cost of Re-housing 10,000 Persons from Clearance Areas**

£'000s

Type of Dwelling	New Density (Dwellings per Acre)	Central Redevelopment on Cleared Land: Housing only Within 1 Mile of Existing Housing	Central Redevelopment on Cleared Land: Housing plus Neighbourhood Within 1 Mile of Existing Housing	Suburban Redevelopment plus Neighbourhood for 10,000 on Land with 14 Dw. per Acre: 40% Improvement Value on Existing Dwellings	Suburban Redevelopment plus Neighbourhood for 10,000 on Land with 14 Dw. per Acre: 27% Improvement Value on Existing Dwellings	Peripheral Neighbourhood on Virgin Site	Extended Town	New Town
Two-storey House	14	104	130	—	—	114	148	223
Two-storey Flat	20	90	116	870	1,029	104	138	213
Three-storey Flat	26	131	158	687	778	146	181	255
Four-storey Flat	31	167	193	672	743	182	217	291
Five-storey Flat	35	228	255	772	833	244	279	353
Six-storey Flat	38	246	272	775	831	265	299	374
Seven-storey Flat	40	264	290	794	848	280	315	389
Eight-storey Flat	41	275	302	811	863	294	329	403
Nine-storey Flat	42	287	313	828	880	303	337	412
Ten-storey Flat	43	296	322	839	889	312	347	421
Twelve-storey Flat	44	310	337	863	912	326	361	435

* An improvement value of 100 per cent. is allowed for the dwellings vacated by the 10,000 slum dwellers re-housed. This is taken as £100 per dwelling and for 2,000 dwellings.

73. In the simplest terms the housing problem of a given city is to house P persons on an area of A acres in the central area and on such land as may be necessary elsewhere, at the periphery, or in an extended or new town.* From the point of view of the community it is one estate and the criterion is the density in the central area at which the average annual estate costs are at a minimum.

74. Comparison for the various alternative developments can easily be built up on this basis; by way of illustration the cost of various ways of housing 154 persons (the number that could be housed on one acre with 12-storey blocks) is given in Table 12. For instance, with two-storey housing, a density of 70 persons per acre can be achieved, so that if two-storey housing is used in the central area, 84 persons must be housed elsewhere. If these 84 persons are housed in a new town, the real cost to the community will be

$$[(70 \times £9 \cdot 0) + (84 \times £22 \cdot 3)]$$

* The estate costs are given by the formula:

$$Y = C_x Ax + C_0 (P - Ax)$$

where

Y is the total annual cost of the estate;
C_x is the annual cost per person in the central area at density x;
A is the area in the centre in acres;
x is the density in persons in the central area;
C_0 is the annual cost per person in the peripheral area;

and

P is the total number of persons to be housed.

or around £2,500. The costs are likely to be higher if the number housed in the centre is increased, since three-storey blocks will be necessary.

75. Clearly estimates of this kind inevitably necessitate a number of assumptions and it is important to examine how far the conclusions can be changed by varying these. Perhaps the most controversial concept introduced is that of improvement values, allowance having been made for reductions in replacement costs, maintenance, servicing costs, and user costs. In estimating these a difference in treatment has been made between industrial and non-industrial amenities, the former being regarded as a voluntary move and the latter as an involuntary move. As long as industry is free to move out to other towns this is reasonable, but if industry were directed to the overspill towns then the move would be involuntary and the economic advantages might be less. Under these conditions the improvement values might be no greater than estimated for non-industrial amenities. The cost per person for an extended town would then rise from £14·8 to £19·0 and for a new town from £22·3 to £26·5. Even in this case the economic advantage of three-storey blocks as an alternative to the new town is doubtful. While it is impossible to deny that some allowance for improvements must be made, it is interesting to consider the results of excluding this allowance altogether. The costs per person for housing would then be increased by £20 in each case; the costs per person for peripheral neighbourhood, extended town, and new town then become £32·9, £43·9, and £56·1 respectively. Even on these unrealistic assumptions, it is still cheaper to accept the alternative of new town development than to build five-storey blocks in the centre.

76. Lifts are expensive to service and operate. Once they are introduced it is usual to employ caretakers and other staff to supervise their use, to clean the public access areas, and to maintain and supervise the space about buildings. The omission of lifts and the accompanying services would save around £12 a dwelling a year. Normally lifts are introduced when the buildings are of five storeys. Their omission would not, however, make the use of five-storey blocks economic (Table 12).

77. It is also interesting to consider to what extent the conclusions would be changed if the construction costs of high blocks relative to two-storey blocks were lower than has been assumed. Some further reductions in the relative costs of blocks of different storey heights can be expected in the next few years. Even if these ratios were about halved it would still not be cheaper to use five-storey blocks instead of the new town alternative unless the lifts were excluded. The addition of 50 per cent., or even 100 per cent., to the non-housing costs of the new town would make but little difference to the conclusions unless the improvement values were heavily discounted at the same time.

78. The smaller the amount of accommodation provided by the dwellings the greater are the costs for each person housed, and the greater therefore the costs of housing relative to the costs of other urban facilities. Thus the level of block height which is just economic would be higher in each of the situations studied if the estimates were based on four-bedroom dwellings and lower if they were based on two-bedroom dwellings, and lower still, of course, on the basis of one-bedroom dwellings. Similarly, of course, the economic block height is lowered if two-bedroom dwellings are used in the centre and three-bedroom dwellings elsewhere. Even if the rooms are made smaller in the two-bedroom dwellings so as to give a floor area of 680 square feet, as against 910 square feet for the three-bedroom dwellings, the economic block height never exceeds three storeys and then only if the comparison is against the New Town solution and on the assumption that the ratios by

which costs rise with density are halved. More favourable situations for the economic use of high blocks occur either if the comparison is made entirely in terms of reduced area (680 square feet) two-bedroom dwellings, or if a 770-square-feet two-bedroom house is compared with 680-square-feet flatted two-bedroom dwellings.

TABLE 12

*Comparative Annual Costs of Alternative Ways of Housing in Central and Peripheral Estates**

			Flats				
Storey Height			2	3	4	5	5 ex lifts
Density in Centre (Persons per Acre)			70	91	108·5	122·5	122·5
Persons remaining to be housed at Periphery . .			84	63	45·5	31·5	31·5
A. *Based on the Estimated Costs of Constructing Dwellings Given in Text* (*Italic Figures denote Minimum Costs*)							
Relative Costs of Constructing Dwellings			100	120	136	148	148
Costs in Centre Per Person for Housing Only . .			£9·0	£13·1	£16·7	£22·8	£19·4
Type of Overspill		*Costs Per Person for Housing Overspill (Two-storey Houses)*	£	£	£	£	£
(*a*) Peripheral Neighbourhood . . .		£11·4	*1,600*	1,900	2,350	3,150	2,750
(*b*) Town Extension	Industrial Move .	£14·8	*1,850*	2,100	2,500	3,250	2,850
(*c*) New Town	Voluntary .	£22·3	*2,500*	2,600	2,800	3,500	3,100
(*d*) Town Extension	Industrial Move .	£19·0	*2,200*	2,400	2,700	3,400	3,000
(*e*) New Town	Involuntary .	£26·5	*2,850*	*2,850*	3,000	3,600	3,200
B. *Based on Reduced Costs of Constructing Dwellings*							
Relative Costs of Constructing Dwellings			100	110	119	125	125
Costs in Centre Per Person for Housing Only on this Basis			£9·0	£10·8	£12·8	£17·5	£14·1
Type of Overspill		*Costs Per Person for Housing Overspill (Two-storey Houses)*	£	£	£	£	£
(*a*) Peripheral Neighbourhood . . .		£11·4	*1,600*	1,700	1,900	2,500	2,100
(*b*) Town Extension	Industrial Move .	£14·8	*1,850*	1,900	2,050	2,600	2,200
(*c*) New Town	Voluntary .	£22·3	2,500	*2,400*	*2,400*	2,850	*2,400*
(*d*) Town Extension	Industrial Move .	£19·0	2,250	*2,200*	2,250	2,750	2,300
(*e*) New Town	Involuntary .	£26·5	2,850	2,650	2,600	3,000	*2,550*

* These costs are based on housing 154 persons using 1 acre of land in the centre and unlimited land at the periphery—figures based on provincial prices.

79. In theory densities can be higher than those assumed for purposes of estimating the level of costs, although in fact the densities taken are a good deal higher than is generally being achieved. While some saving in costs is possible in special cases by the use of higher densities, this is not likely to be very considerable, especially where high blocks are used to obtain high densities. In fact, the annual cost per person is not likely to be altered by more than £1 or £2. For purposes of simplicity the estimates have been based on the

assumption that all blocks in a development will be of equal height. In practice mixed heights are more likely. The use of mixed block heights appears unlikely in general to reduce costs in terms of the achieved densities. Small savings might be achieved by obtaining the densities normal for five- and six-storey blocks, by use of a mixture of higher and lower blocks, so as to balance the increase in service costs normal at these heights against the comparatively lower rates of increase in block costs as against the increases in density. But such savings might be lost in practice if the greater services normal for high blocks are extended to the whole estate. Costs might, of course, be reduced if a development with equal block heights was replaced with a mixed development of more economically designed blocks, making a better use of the potentialities of the site.

80. The cost advantages to developers to a large extent run contrary to the cost advantages to the community. As shown earlier, building high blocks on expensive land can result in a reduction of the costs of the dwellings both to private and to public developers. On really high cost land private developers are likely to build higher than public developers working with subsidies. Costs are, of course, always lower for public than for private developers. But local authorities as an entity have interests distinct from their interests as housing developers. The cost criterion for the local authority is, in the final analysis, the level of the rates. The consumer of local authority services is interested in the services provided against the rate level. This often means that local authorities have an interest in keeping populations within their boundaries. While in general the average rate income from low cost housing is probably less than the average cost of the services provided to its occupants, the value of rate income from a block of housing may exceed the marginal savings resulting from exporting the occupants, especially when the associated Government grants are considered. Furthermore, the loss of a block of housing may result in a loss of a part or all of any Rate Deficiency Grants. Thus the net effect of exporting a block of low-cost housing may result in a loss of revenue far exceeding the saving in expenses. This loss may be increased if the exporting authority has to pay the receiving authority an annual contribution on dwellings built for population exported under an agreed overspill scheme. The value of these net losses may offset the costs of retaining the population and providing heavy rate subsidies to house them in high blocks of flatted dwellings. One authority calculated that on this basis it would cost around £40 a dwelling a year less to house in 10–12-storey blocks within its boundaries as compared with exporting a block of housing and developing with two-storey housing outside its boundaries. Four-fifths of this saving was a result of an Exchequer Equalization Grant. Furthermore, the interest of the receiving local authority may not benefit from town expansion. While it would obtain Exchequer grants towards the development of certain amenities for its town, and might receive contributions from the exporting authority, the rents and rate income from the expansion might not offset its costs. Administration difficulties may also arise when a small town authority embarks on problems of expansion. On the other hand, the development of commercial and industrial areas can be very profitable, especially if sufficient land is purchased at the outset by the developer to cover all his likely requirements for future development.

81. Directly and indirectly the cost of moving all urban services, including local authority services and transport, has been considered. An allowance has been made for the cost of new fixed capital, buildings and works, and of moving plant and people. But there is the possibility that the reduction in the cost of town services in the old area may be less than

the expenditure in the new, in respect of the exported population. Local authority services cost around £30 a head, and about three-quarters of this is wages and salaries. A good deal of the rest will be supplies, transport, and other movable machinery, or buildings and works in small units which can be closed. It would therefore appear that, at most, any excess expenditure of the type considered cannot exceed £1 or £2 a head a year. The effect on transport will be even smaller. Moreover, no allowance has been made for the residual value of the assets made redundant in the exporting towns, the value of which may well more than offset any excess expenditure of the kind described above.

82. So far, in comparing the costs of different types of development, no allowance has been made for the differential costs in different locations arising from travelling, food costs and transport. The acceptance of the figures derived earlier in effect reduces the comparative costs per person of the alternatives of moves to extended or new towns as compared with development in provincial central areas by £10, and by £15 as compared with London central areas. Similar savings would not, of course, occur in the comparisons between peripheral and central development in the same town. In such cases longer journeys to work might be necessary and these would increase costs per person for the peripheral development although it is doubtful if the increase would be as much as £10 a year, since it is unlikely that even initially the additional travelling would be as great as the distance moved for each of the workers concerned. In the long run jobs would be changed, further reducing travelling. In given circumstances the costs of travelling can be estimated but this is not possible in the general case and no allowance has therefore been made in Table 12.

83. The comparisons for London can be worked out in the same way as has been used for the provinces, but redevelopment in London is likely to compare even less favourably with peripheral development than is redevelopment in the central areas of provincial towns. Generally it appears that prices for all housing up to and including five storeys are around 30 per cent. higher in Central London, and 15 per cent. higher in Outer London, than in the provinces, so that all central development costs will be higher by these percentages than shown for the provinces, whereas the peripheral costs will be as shown for the provinces since peripheral development from London must take place in the provinces. In addition, of course, the differential living costs discussed above further widen the central peripheral cost comparisons for London as compared with the provinces.

Conclusions

84. It is clear from the cost analyses given above that in the general situations considered the scope for using high block residential buildings as a means of reducing the costs of providing housing and its accompanying urban amenities is very limited. The only block heights which, at current relative costs, appear in general to fulfil this requirement are three- and, rather more doubtfully, four-storey blocks. Higher blocks would only be economic if their relative costs can be lowered not merely in relation to two-storey blocks but also successively to each other. This involves design and construction economies relatively greater for the higher than the lower blocks. Lower service charges for the higher blocks would also contribute.

85. Further it would appear that small provincial towns have lower real costs as a location for urban development than the larger and older urban centres. This advantage arises partly from lower environment costs, partly from lower prices, and partly from the

advantage of virgin sites over previously developed sites. These conclusions should not, however, be extended too far, for the estimates, particularly for environment costs, were not intended to carry conclusions on the location of population and industry. Moreover, the limit to a large-scale migration of population and industry to small towns is set by the number of firms who can hope to cover the costs of moving out by improvements in their comparative efficiency, which in itself is limited by the cost of attracting workers to move with their firms to small towns. In turn the limit to the economic use of small town expansion as distinct from the more expensive new town solution is set by the prevalence of existing towns with unexpended general facilities.

86. The individual town planner will, of course, wish to consider the balance of advantages for the alternative developments with which he is faced. The cost situation can be assessed in a similar way to that described above, allowance being made for the local situation. The planner can then weigh the costs of the various alternatives against the other more imponderable desiderata that he has to consider.

Acknowledgments

87. This paper forms part of the programme of research of the Building Research Board and appears by permission of the Director of Building Research. Thanks are due to the many authorities who provided the detailed information on which this paper is based.

APPENDIX A

Urban Land Requirements

*Net Residential Areas**

1. The densities which can be achieved in what is loosely termed the "housing area" depend on the layout and the types of dwellings involved. A clear division must be made between dwellings with private gardens and those without, since the amenities they provide are very different. Clearly, really high densities are not possible if private gardens are provided with each dwelling. Moreover, the rise in densities with an increase in storey height is markedly greater where no private gardens are provided.

2. Some indication of the maximum net residential density† possible with two-storey

* Net residential area is the land within the curtilage of the residential buildings, plus small public or private open spaces within the area of residences, plus half the width of streets abutting on the area of residences, except where the street is a principal traffic road, when only up to 20 feet of its width is included.

† *Net Residential Density:*

This may be expressed either as

$$\text{Net accommodation density} = \frac{\text{Number of habitable rooms}}{\text{Net residential area}}$$

or as

$$\text{Net population density} = \frac{\text{Number of persons}}{\text{Net residential area}}.$$

A habitable room is a room intended for eating, sleeping or living as distinct from washing and cooking.

houses can be obtained by considering a simple rectilinear grid layout (Table A.1). Most peripheral and new town housing developments provide a good deal of housing with frontages of 24 to 30 feet, sufficient to allow some private garages and back gardens about the size of small allotments. Such development would provide 14 to 9 dwellings an acre (Table A.1), say 70 to 45 habitable rooms. Roads and footpaths occupy about a seventh of the area. While the introduction of cul-de-sacs does provide some scope for reducing this proportion (Keeble, 1956), it is difficult to achieve much saving if awkwardly shaped and undersized gardens are to be avoided. Small savings in land can also be achieved by the use of access paths in place of roads, by reducing the set-back of the building line from the highway to the minimum necessary for daylighting, about 7 feet, and by the introduction of flats and shops without private gardens on corners and side roads, instead of isolating them from the two-storey housing. On the other hand, small areas of planting are usually thought desirable and these will reduce densities. In practice, grid layouts are not very acceptable and the land is seldom ideal for achieving maximum densities, so that actual densities will be lower than the figures given in Table A.1.

TABLE A.1

Plot Sizes and Density for Houses with Private Gardens

Size of Plot				*Area or Density*				
Width (Frontage) (feet)	*Length of Back Garden (feet)*	*Depth of House (feet)*	*Depth of Building Line (Length of Front Garden) (feet)*	*Area of Back Garden (sq. ft.)*	*Area of Plot (sq. ft.)*	*Area for Roads and Footpaths (sq. ft.)*	*Total Area per Dwelling (sq. ft.)*	*Dwellings per Acre*
16	35	25	20	560	1,280	275	1,555	28·0
18	40	25	20	720	1,530	311	1,841	23·7
20	50	25	20	1,000	1,900	350	2,250	19·4
24	66	25	20	1,584	2,664	432	3,096	14·1
30	90	25	25	2,700	4,200	556	4,756	9·2
40	120	25	25	4,800	6,800	768	7,568	5·8
50	150	25	30	7,500	10,250	1,000	11,250	3·9
60	180	25	30	10,800	14,100	1,241	15,341	2·8

Assumptions.—In calculating the figures for this table it has been assumed that the paths and carriageway will require 30 feet of width and that the houses will be laid out in parallel roads with side roads every 440 yards.

3. Clearly there are serious practical difficulties in providing private gardens for multi-storey dwellings but it is practicable to do so for four-storey maisonettes, although not perhaps popular (Willis, 1954). Theoretically, with private back gardens of 1,800 square feet to each maisonette, the density rises from about 60 rooms per acre with two-storey buildings to 80 rooms with four-storey buildings and this is probably the practicable limit for dwellings with private gardens.

4. The full advantages of building high can be achieved once private gardens are omitted, but it is difficult to determine any useful averages for net residential densities, because the several expedients for increasing density tend to interact to each others disadvantage. In general, densities of dwellings without gardens do not increase in proportion to the number of storeys; possible densities are less than doubled by raising storey height

from two to 20, other things being equal (Table A.2). In practice, other things rarely are equal and densities can be raised by reducing the size of rooms, by making blocks wide enough to enable dwellings to be placed either side of a central corridor, by using special block shapes and arranging them, for instance, in a broken cruciform pattern and by using special features of the site gradients and proximity to open space, or to lower buildings for which daylight and sunlight are of little importance. Normally densities are unlikely to rise much above 200 habitable rooms per acre, although in really favourable circumstances it is possible to achieve higher densities and still to provide adequate daylighting. However, other factors must be considered as well as daylighting and while, in general, if daylighting is adequate other things will be satisfactory, this is not necessarily true at the higher densities (Ministry of Housing and Local Government, 1952).

TABLE A.2

Habitable Rooms per Acre for Dwellings without Private Gardens

Number of Storeys	*Habitable Rooms per Acre of Net Residential Area*	*Distance Between Parallel Blocks (Feet)*	*Space about Buildings Per Habitable Room (Square Feet)*
1	85	25	261
2	124	44	227
3	146	63	215
4	160	81	209
5	170	100	207
6	178	119	204
7	183	138	202
8	188	156	201
9	192	175	200
10	194	194	199
11	197	213	198
12	199	231	198
13	201	250	197
14	203	269	197
15	205	288	197
16	205	307	196
17	206	325	196
18	207	344	196
19	208	363	196
20	209	382	195

Densities based on the assumption of the equivalent of 250 square feet of floor area per habitable room in parallel blocks (440 yards long, 25 feet deep, 30 feet end to end and with a rise of 8 feet 9 inches for each storey) with 25° Daylight Angle.

5. The densities being achieved in practice are however far less than those possible in theory (Fig. 1). In new towns the densities range from 11 to 16 dwellings per acre, say 41 to 69 habitable rooms per acre, mainly in two-storey housing; such densities are achieved in some cases with back gardens reduced to 25 feet in length (Turner, 1956). Densities in other peripheral areas are perhaps a little lower. Higher densities are secured in central areas where building is higher; outside London densities range from 70 to, exceptionally, 200 habitable rooms per acre.

6. Average existing net residential densities over towns as a whole are much smaller, 50 to 60 persons per acre in large towns and even less in small towns. Future plans envisage densities of 30 to 50 persons per acre or about 35 to 55 habitable rooms per acre.

7. It appears a reasonable conclusion that high building, by itself, is not necessarily the most effective method of saving land (Table A.3). Even on the basis of the theoretical densities derived above, less than 40 acres per 10,000 persons can be saved by raising the height of flatted dwellings from two to 20 storeys, and over half this amount could be saved by going no higher than four storeys, at which height no lifts are required and the problems of high building are largely avoided. On a similar theoretical basis 100 acres can be saved by reducing a moderate frontage by eight feet, say from 24 to 16 feet in the case of dwellings with gardens (Table A.3).

TABLE A.3

Housing Land Per 10,000 Persons

Type of Dwellings	*Frontage (in feet)*	*Persons Per Acre**	*Land Per 10,000 Persons (acres)*
Two-storey with Private Gardens	30	32	313
	24	49	204
	20	68	147
	16	98	102
	Storeys		
Flatted Dwellings without Gardens	2	110	91
	3	130	77
	4	142	70
	5	151	66
	10	173	58
	15	182	55
	20	186	54

* Calculated at 3·5 persons per dwelling or 0·89 persons per habitable room.

Gross Residential Area†

8. It is usually considered desirable to provide the every day amenities within easy reach of the housing, and the concept of the "neighbourhood" has grown up—this term covers the housing itself, the local shopping area, primary schools, libraries and open space. Something around 120 acres per 10,000 persons of non-housing land is required in order to provide reasonably self-contained neighbourhoods (Table A.4). At first sight the allowance for open spaces may appear very large. In fact, this standard for playing fields—60 acres per 10,000 persons—allows for 17 per cent. of the inhabitants to play outdoor games (Ministry of Housing and Local Government, 1956*b*). The limited wear which pitches will stand, 6 to 7 hours a week (Chinn, 1956), precludes an intensive use of the space for games. The 10 acres allowed for parks and gardens works out at only 5 square yards a head. No allowance is made in Table A.4 for allotments, though 24 acres would be required per 10,000 persons for this purpose to provide allotments for 12 per cent. of the families (Ministry of Housing and Local Government, 1956*b*), the national average holding of allotments (Ministry of Agriculture, Fisheries and Food, 1956). Clearly the demand for allotments is likely to increase as the provision of private gardens of adequate size declines.

† Gross Residential Area is the net residential area plus curtilages of primary schools, local business premises, cinemas, churches, other public buildings and local open space and service industry serving the locality, plus half the width of abutting streets with a limitation of 20 feet in the case of Trunk and Class I and II roads.

TABLE A.4

Amenity Land Requirements of Neighbourhood Areas
(Acres Per 10,000 of Population)

Use	*Open Development*	*Outer Ring*	*Inner Ring*	*Central*	*Central*
Primary Schools* (3–11 years of age) including playing fields	17	17	17	17	17
Open Space	70	70	60	50	40
Shops, Offices, etc.	9	8	7	6	5
Community Centre, Churches, etc.	7	5	4	3	3
Public Buildings	4	3	2	2	2
Service Industry and Workshops	7	6	5	4	4
Main Roads including half Boundary Roads up to maximum of 20 feet, and Parking	35	28	20	17	14
Total†	149	137	115	99	85
With net densities of	30	50	75	100	120
The gross densities are	21	30	40	50	60

* Nursery, Infant and Junior schools.
† In order to obtain the total Amenity Land it is necessary to add in land for Allotments.

Source: *Design of Dwellings*—Ministry of Health—H.M. Stationery Office, 1944.

TABLE A.5

Land Use: Neighbourhood Areas and "Areas Primarily Residential"
(Acres per 10,000 of Population)

Type of Land Use	*Major Ports*		*Major Industrial Towns*		*Smaller Towns*		*New Towns*	*Suggested**
	Survey	*Plan*	*Survey*	*Plan*	*Survey*	*Plan*		
Primary Schools	5	14	4	11	6	17	31†	13
Other Building Uses‡	29	18	18	21	17	19	31	40
Public Playing Fields and Open Spaces	8	22	14	28	34	39	47	70
Private Playing Fields	2	3	5	8	22	14		
Allotments	7	5	12	10	59	24	18	24
Total	51	62	53	78	138	113	127	147
Net Residential	179	219	192	234	334	334	255	—
Net Density (Persons per Acre)	56	46	52	43	30	30	39	—
Gross Density (Persons per Acre)	43	36	41	32	21	22	26	—
Number of Town Plans Analysed	5	5	9	9	3	3	2	—

* The figures given under this heading are taken from Table 4 and the text of this section.
† This figure includes for secondary as well as primary schools.
‡ Shops, Service Industries, Churches, Social Buildings, etc.

9. In practice, the provision of space in existing towns is far lower than generally accepted as adequate, especially in the large industrial towns and ports. It is planned to increase the provision of space for "neighbourhood" amenities up to about a half the

above standard for large towns and three-quarters for small towns (Table A.5). These figures are more a reflection of what is considered feasible than what is considered adequate. In practice, further amenity land is often provided elsewhere in the town, although in less convenient locations. The importance of the concept of "neighbourhood" amenity land lies in the need it expresses for urban land in close proximity to the housing area. The amount of land required for this purpose is, of course, the same wherever it is located.

TABLE A.6

Analysis of Planned Land Use in Towns
(Acres per 10,000 *population)*

Acres

Classification	*Major Industrial Towns*	*Major Ports*	*Smaller Towns*	*New Towns*	*Averages (Un-weighted)*
Housing (Net Residential Area)	219	213	272	319	256
Industrial Areas	56	42	96	72	67
Business, Shopping, Civic and Cultural	35	40	33	36	36
Education	32	32	43	57	41
Open Spaces and Playing Fields	104	122	167	131	131
Railways and Waterways	25	46	50	18	34
Other (including large establishments)	43	100	134	44	80
Total	514	595	795	677	645
Number of Town Plans analysed	9	6	5	4	—

Town Areas

10. Outside the residential area, land is needed for town centres, higher education, industry, large establishments and open space. As explained above, the concept of land for "neighbourhood" amenities is only a transitional one, establishing a need for land at certain locations, rather than setting a firm target for land. This concept can now be put to one side in favour of an overall assessment of town land requirements. It is, however, difficult to determine any firm standards of land requirements as they vary considerably with the nature and business of the town. The best approach is to compare the standards planned for existing towns and new towns, although it will be recognized that standards in existing towns depend to a large extent on the present provisions. Probably the averages in Table A.6, obtained by an analysis of development plans, would provide a fair measure of town land use. These correspond to an overall town density of 15·5* persons per acre and a net density of 39 persons per acre. The New Towns Committee (1946) recommended densities of 12 and 22 respectively. About 40 per cent. of the land is used for housing, 20 per cent. for playing fields and open space and 10 per cent. for industry. The plans analysed represent a reasonable cross-section of their type, except perhaps in the case of the smaller towns.† An analysis of land for industry based on 250 plans puts the figure at 52 acres per 10,000 persons (Beaufoy, 1956), which lends some support to the

* Overall Town Density is equivalent to the total number of residents in the town divided by the total developed area of the town.

† The figures for Major Industrial Towns and Ports are based on Plans for towns with populations over 100,000; figures were analysed for all towns for which plans were obtainable; the original list covered all towns with a population over 200,000 and a selection of the smaller ones. The figures for the smaller towns were obtained from County Plans.

figures given. It would appear that 300 to 400 acres of land per 10,000 population are required for town purposes outside the housing area.

Housing Density and Town Land

11. While it is difficult to compare the theoretical figures given in Table A.4 with the Planned Land use given in Tables A.5 and A.6, these tables do provide a fair estimate of town land use. For purposes of comparing the effect of net housing density on total land it seems reasonable to take total town land per 10,000 persons as the net housing land, plus 300 or 400 acres for other purposes (Table A.7). At the range of normal densities the savings of land possible per 10,000 persons, by reducing private garden sizes, are impressively large, around 60 acres for five feet of frontage, say from 21 to 16 feet. As compared with the minimum size of private gardens, the use of flatted housing without gardens saves only a further 11 acres, while a further 22 acres can be saved by building four- instead of two-storey flatted buildings, but only a further 16 acres by the use of 20- instead of four-storey flatted buildings.

TABLE A.7

Town Land Acres per 10,000 Persons

Type of Dwelling				With Non-housing Land at 300 Acres per 10,000		With Non-housing Land at 400 Acres per 10,000	
	Frontage (in feet)	Net Residential Density (Persons per Acre)	Net Housing Land (Acres)	Total Town Land (Acres)	Overall Town Density (Persons per Acre)	Total Town Land (Acres)	Overall Town Density (Persons per Acre)
Two-storey dwellings with private gardens . . .	30	32	313	613	16·3	713	14·0
	24	49	202	502	19·9	602	16·6
	20	68	147	447	22·4	547	18·3
	16	99	102	402	24·9	502	19·9
	Storeys						
Flatted dwellings . . .	2	110	91	391	25·6	491	20·4
	3	130	77	377	26·5	477	21·0
	4	142	69	369	27·1	469	21·3
	5	151	66	366	27·3	466	21·5
	10	173	57	357	28·0	457	21·9
	15	182	54	354	28·2	454	22·0
	20	186	53	353	28·3	453	22·1

Land Economy and Densities

12. While there may be some disagreement about the actual densities possible, especially with high buildings, the effect on town land use is likely to be insignificant and will certainly not change the broad picture. Savings in land for broadly residential purposes can in the main be achieved most easily by reducing private gardens or open land. The savings possible by high building, particularly above four storeys, are very limited and tail off rapidly with height (Fig. 8). The size of the contribution which high building can make to land saving can be seen by considering its effect in terms of slum clearance. Of course, only a small proportion of those to be rehoused are likely to be rehoused in dwellings in high buildings. Flats are often considered unsuitable for families with children and over

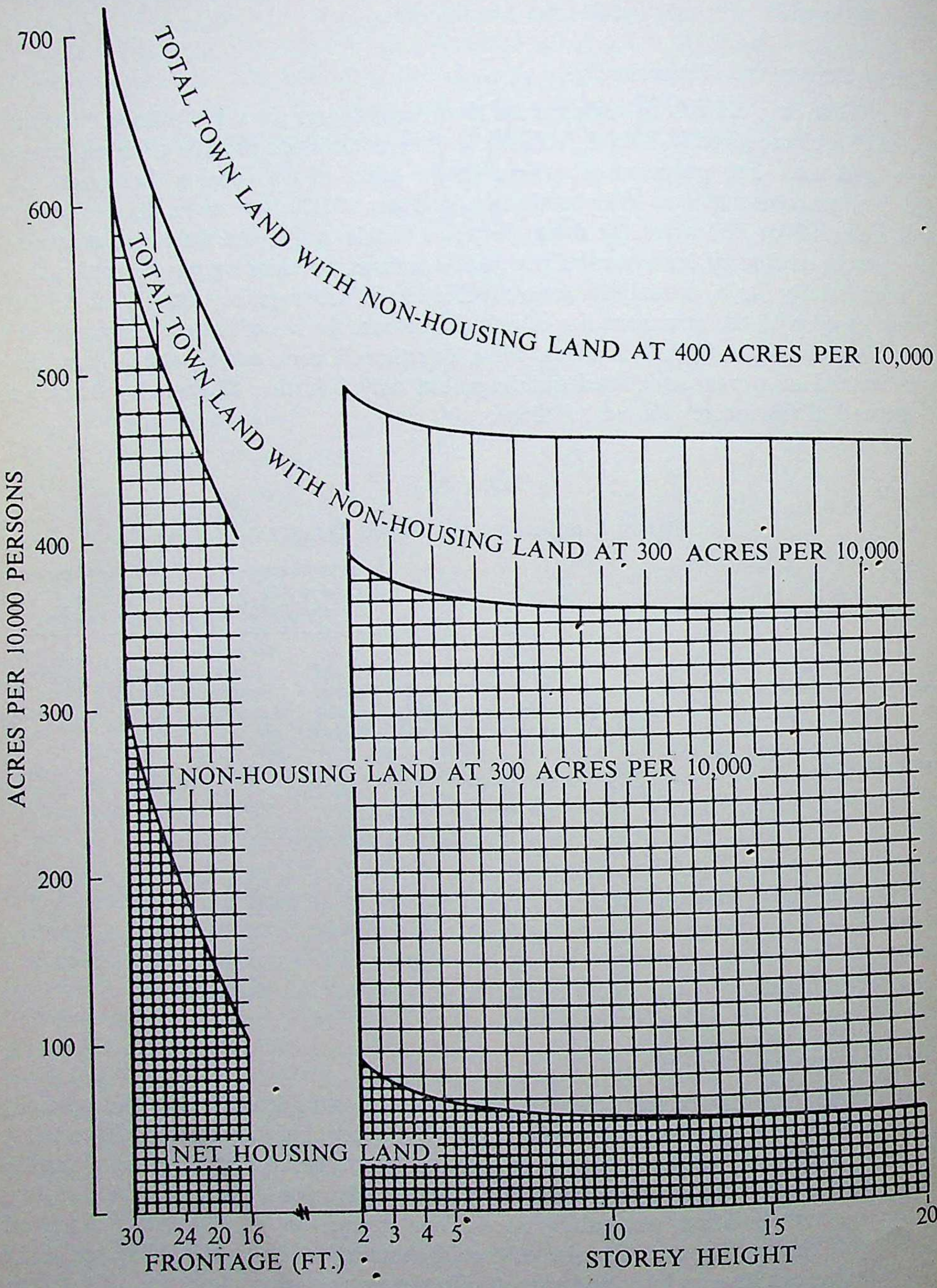

FIG. 8.—Urban land requirements.

large areas the danger of subsidence as a result of mineral working precludes the use of buildings higher than two or three storeys. Perhaps not more than a fifth of the 3,000,000 persons to be rehoused (Ministry of Housing and Local Government, 1955*b*) could be housed in flats, when the saving resulting from using 20 instead of four-storey buildings for these would be less than two square miles (Table A.7). On the same assumption the use of four- instead of two-storey flats would achieve a slightly larger saving. The use of two-storey housing instead of two-storey flats would involve the use of an extra square mile (Table A.7). On the other hand, increases in the size of gardens have a rather larger influence on land requirements. An extra eight feet of frontage, say from 16 to 24 feet, for two-storey houses with private gardens at the usual densities would entail about an additional 43 square miles for the 3,000,000 slum clearance persons (Appendix A, Table A.7). On the other hand, in England and Wales there are 58,000 square miles of land (Central Statistical Office, 1957) of which about a tenth is used for urban purposes (Best, 1957).

Gardens and Agricultural Produce

13. No discussion of urban land use would be complete without some assessment of the agricultural value of gardens. On average about 14 per cent. of each house plot is used for growing fruit and vegetables, a figure which applies broadly over the range of five to 14 houses per acre; the proportion is about 20 per cent. for local authority estates as against 10 per cent. for private estates (Mackintosh and Wibberley, 1952). The proportion cultivated for food crops declines as density rises, it is a third less for 12 houses to the acre than for eight (Agricultural Land Service Research Group and Ministry of Housing and Local Government Research Division, unpublished paper). Estimates of the value of food at shop prices produced from gardens vary from about £67 based on the National Food Survey (Ministry of Housing and Local Government, unpublished paper) to £42 per house plot acre based on studies of garden and allotment output (Best and Ward, 1956). The comparable value from an acre of better than average farmland is about £90 but of this about half is represented by transport and distribution costs which are avoided with garden produce. This suggests that the value to the community of an acre of house plots is about equal to that from the land had it remained as farmland. It would appear that, at worst, the loss of home grown food by value is only marginal when farmland is taken for housing with private gardens and developed at normal densities, and that the use of farmland for allotments would result in an increased output. Farmland taken for other urban development does, of course, involve a loss of produce.

Urban Land Requirements

14. Land is taken from agriculture not only for town development, but also for mineral working, transport, water storage, afforestation and defence purposes. While it is desirable to have land located in convenient reach of centres of population for recreational purposes, this use of land, in so far as it is not for organized games, does not for the most part itself create an additional demand for land.

15. An analysis of the available plans suggests that over the next two decades about half a million acres of land will be required in England and Wales for all town purposes and for mineral working (Ministry of Housing and Local Government, 1955*a*). Allowing

another 100,000 acres for roads and other urban purposes this would give a total of say 600,000 acres, less than 2·5 per cent. of the area at present used for arable and permanent grass (Ministry of Agriculture, Fisheries and Food, 1957). Various suggestions for replacing this potential loss have been made. There are 10,000 to 20,000 acres of tidal lands (Stamp, 1956) and 126,000 acres of derelict land which might be reclaimed (Ministry of Housing and Local Government, 1956*c*). But land reclaiming is often very expensive. Dr. Wibberley (1954) has suggested that £200 an acre—the cost of upgrading hill-land so as to make good the loss of an acre of good farmland—is the upper limit of worthwhile expenditure in reclaiming land. It is doubtful, therefore, whether much of this potential land is worth saving. The Ministry of Housing and Local Government (1955*a*) has suggested that perhaps 36,000 acres of derelict land could be brought back into use.

16. In England and Wales there are about 4 million acres of poor quality mountain and moorland and 2 million acres of other poor land (Ordnance Survey Office, 1950). There is, therefore, potentially sufficient land to replace the land expected to be taken from agriculture for urban development over the next two decades. Even if all the land had to be replaced at £200 per acre it should not cost more than £120 millions, or £6 millions a year. In fact, the necessary replacement is probably quite small. Over the decade 1940–50 farm output per acre in terms of calories increased on average nearly 2·5 per cent. a year (Wyllie, 1954). An increase in output of about this size would be sufficient in one year to off-set the loss of land anticipated over the 20 years without the need for up-grading. It would not appear, therefore, that the loss of agricultural land to urban development is likely to reduce the amount of home-grown food.

APPENDIX B

Building and Civil Engineering Costs

TABLE B.1

Total Unit Cost of Buildings for Urban Development

Items	*Cost*
Educational	
Nursery School	£100 per pupil place
Primary School	£175 ,, ,, ,,
Secondary School	£300 ,, ,, ,,
College of Further Education	£350 ,, ,, ,,
Cultural	
Branch Library (Neighbourhood)	£3,000 for 10,000 persons
Main Library (Town Centre)	£40,000 ,, 80,000 ,,
Museum and Art Gallery (Town Centre)	£40,000 ,, 80,000 ,,
Commercial	
Shop	£3 per square foot
Public House (Neighbourhood)	£10,000 for 10,000 persons
(Town Centre)	£13,000 ,, 80,000 ,,
Commercial Offices	£3 per square foot
Branch Administrative Office (Neighbourhood)	£5,000 for 10,000 persons
Hotel (Town Centre)	£45,000 ,, 80,000 ,,
Petrol Station and Garage (Town Centre)	£30,000 ,, 80,000 ,,
Restaurant and Cafes (Town Centre)	£40,000 ,, 80,000 ,,
Industrial	
Service Industry	£2 10*s.* per square foot
Factory	£2 16*s.* ,, ,, ,,
Entertainment	
Cinema (Town Centre)	£75,000 for 80,000 persons
Civic Theatre (Town Centre)	£40,000 ,, 80,000 ,,
Community Centre (Town Centre)	£10,000 ,, 80,000 ,,
Swimming Pool (Town Centre)	£80,000 ,, 80,000 ,,
Other Buildings	
Hospital (Town Centre)	£7,000 per bed
Health Centre (Neighbourhood)	£5,000 for 10,000 persons
Bus Station and Garage (Town Centre)	£40,000 ,, 80,000 ,,
Fire Station (Town Centre)	£40,000 ,, 80,000 ,,
Church (Neighbourhood)	£15,000 ,, 10,000 ,,
(Town Centre)	£40,000 ,, 80,000 ,,
Cemetery and Crematorium (Town Centre)	£130,000 ,, 80,000 ,,

TABLE B.2

Total Costs of Civil Engineering Works and Other Development

Item	*Cost*
PUBLIC UTILITY SERVICES	
Gas, Electricity, Water, Telephone	
From supply to meter—	
Neighbourhood ex-housing	£1,000 per acre
Industry	£2,500 ,, ,,
Town Centre	£3,500 ,, ,,
Water Supply	
Water works and reservoir (Town) (for 4,000,000 g.p.d.)	£1,200,000 for 80,000 persons
Sewerage	
Sewage works and mains (Town)	£1,600,000 ,, 80,000 ,,
Refuse Disposal	
Disposal works (Town)	£100,000 ,, 80,000 ,,
ROADS AND TRANSPORT	
Roads	
Town roads, footpaths, sewers, etc.	£6,000 per acre
Trunk, spine and other main roads	£8,000 ,, ,,
Bridges	
70 feet span—60 feet wide	£33,000 each
Railways	
Station, sidings, bridge, track and works (Town)	£500,000 for 80,000 persons
Car Parks	
For 200 cars	£5,000 each
OPEN SPACES	
Ornamental Gardens—Town centre	£1,000 per acre
Neighbourhood Parks, general landscaping	£500 ,, ,,
Playing fields	£500 ,, ,,
Allotments	£300 ,, ,,
Woodlands	£150 ,, ,,
Natural open space	£100 ,, ,,
Golf course (64 acres) (Town)	£60,000 each for 80,000 persons
Town park and Stadium (Town)	£100,000 ,, ,, 80,000 ,,

APPENDIX C

Initial Costs of Urban Development

TABLE C.1

Capital Cost of Housing 10,000 Persons

Type of Dwelling	Density (Persons per Acre)	Provinces: Agricultural Land*: Two-bedroom £000's	Provinces: Agricultural Land*: Three-bedroom £000's	Provinces: Development†: Three-bedroom £000's	Provinces: Redevelopment‡: Three-bedroom £000's
Two-storey Houses:					
Fourteen dwellings to the acre	38	6,900	—	—	—
	49	—	5,814	5,846	7,066
Flats in Blocks of (Storeys):					
2	70	6,289	5,434	5,457	6.317
3	91	7,263	6,257	6,274	6,940
4	109	8,063	6,943	6,957	7,520
5	123	8,681	7,466	7,477	7,980
6	133	9,152	7,860	7,871	8,337
7	140	9,530	8,169	8,177	8,620
8	144	9,841	8,437	8,449	8,880
9	147	10,107	8,663	8,674	9,097
10	151	10,315	8,843	8,854	9,266
12	154	10,637	9,117	9,129	9,531

* Agricultural land (without development rights)—taken at £100 per acre.
† Development land (with development rights)—taken at £250 per acre.
‡ Redevelopment land—taken at £5,000 per acre.

TABLE C.2

Neighbourhood Costs for 10,000 *Persons*

Item and Unit Cost	*Total Cost* £000's
Acquisition of Land	
120 acres at £100 an acre	12
Development (say equivalent of about 50 acres)	
Roads, paths, sewers, landscaping, paved areas at £3,000 an acre	150
Public utility services at £1,000 per acre	50
Schools	
Four Nursery Schools—160 places at £100 a place	16
Four Primary Schools—1,000 places at £175 a place	175
Shopping Area	
40 shops (40,000 sq. ft.)—at £3,000 each	120
3 licensed premises—at £10,000 each	30
3 commercial offices—at £3,000 each	9
1 branch administrative office—£5,000 each	5
1 community centre—£10,000 each	10
1 health centre—£5,000 each	5
1 branch library—£3,000 each	3
3 churches—at £15,000 each	45
40,000 sq. ft. of service industry—at £2·5 per sq. ft.	100
Open Space	
40 acres of playing fields, viz.: 7 senior and 7 junior football pitches crossed with 7 cricket tables; 20 rinks for bowling and 13 tennis courts including 3 acres of children's play space	25
10 acres of parks and gardens	5
17 acres of allotments	
Total Cost	760

TABLE C.3

Costs of the Industrial Area for 2,000 Persons

Item and Unit Cost	*Total Cost* £000's
Acquisition of land	
60 acres at £100 an acre	6
Development (say equivalent of 40 acres)	
Roads, paths, sewers, etc., at £3,000 an acre	120
Public Utility Services at £2,500 per acre	100
Factories	
500,000 sq. ft. at £2·8 a sq. ft.	1,400
Petrol Station, shops, etc.	50
Open Space—24 acres at £500 an acre	12
Total Cost	1,688

TABLE C.4

Town Centre Costs for 80,000 Persons

Item and Unit Cost	*Total Cost* £000's
Acquisition of Land	
80 acres at £100 an acre	8
Development (say 70 acres)	
Roads, paths, sewers, etc., at £6,000 per acre	420
Public Utility Services at £3,500 per acre	250
Buildings	
Shops—900,000 sq. ft. at £3 per sq. ft.	2,700
Offices—450,000 sq. ft. at £3 per sq. ft.	1,350
Two Cinemas	150
Three Public Houses	40
Church	40
Library	40
Museum and Art Gallery	40
Civic Theatre	40
Hotel	45
Restaurant and Cafes	40
Petrol Service Station and Garage	30
Omnibus Garage and Station	40
Swimming Pool	80
Fire Station	40
10 acres of Gardens at £1,000 per acre	10
Total Cost	5,363

TABLE C.5

Costs of Open Space (Outside Neighbourhoods) for 80,000 Persons

Item	*Total Cost* £000's
Town Park and Stadium	
50 acres of gardens	
30 acres of playing fields	
34 acres of stadium	
Land, development and buildings	100
Golf Courses—64 acres	
Land, development and buildings	60
Landscaped Areas—24 acres	
Land and development	15
Woodlands and natural areas—100 acres	
Land and development	25
Total Cost	200

TABLE C.6

Costs for Higher Education Provision for 9,000 Places

Item and Unit Cost	*Total Cost £000's*
Acquisition of Land—220 acres at £100	22
Secondary Schools	
8,550 pupil places at £300 a place	2,565
College of Further Education	
450 pupil places at £350 a place	158
Total Cost	2,745

TABLE C.7

Costs of Other Building and Civil Engineering Works for 80,000 Persons

Item and Unit Cost	*Total Cost £000's*
Railways	
Acquisition of Land—50 acres at £100	5
Works, through track, sidings, station, bridge	500
Trunk, spine and other main roads	
Acquisition of land—225 acres at £100	22·5
Works—225 acres at £8,000	1,800
Bridges	
Three bridges, 70 ft. span, 60 ft. wide	100
Gas	
Acquisition of land—5 acres at £100 (Service costed elsewhere)	0·5
Water (Works for 4,000,000 gallons a day)	
Acquisition of land—50 acres at £100	5
Works including reservoir	1,200
Sewage	
Acquisition of land—130 acres at £100	13
Works	1,600
Refuse Disposal	
Acquisition of land—5 acres at £100	0·5
Development and works	100
Cemeteries and Crematoria	
Acquisition of land—25 acres at £100	2·5
Development and works	130
Hospitals	
Acquisition of land—65 acres at £100	6·5
Development and buildings—700 beds at £7,000 per bed	4,900
Total Cost (land 555 acres)	10,385·5

TABLE C.8

The Use of Urban Land for a Town of 80,000

Acres

Use	*Neighbourhood*	*Industrial Area*	*Town Centre*	*Higher Education*	*Open Space*	*Other*	*Total*
Houses and Gardens	1,404	—	—	—	—	—	1,404
Commercial and Public Building	152	12	30	—	—	—	194
Industry	—	160	—	—	—	—	160
Schools with Playing fields	104	—	—	220	—	—	324
Open Space	536	192	10	—	302	—	1,040
Transport	—	36	—	—	—	50	86
Public Utilities	—	—	—	—	—	190	190
Roads, etc.	396	80	30	—	—	225	731
Other	—	—	10	—	—	90	100
Total	2,592	480	80	220	302	555	4,229

TABLE C.9

Capital Cost of Constructing a Town of 80,000

£000's

Item	*Neighbourhood*	*Industrial Area*	*Town Centre*	*Higher Education*	*Open Space*	*Other*	*Total*
Land	256	48	8	22	30	56	420
Houses (22,857)	38,628	—	—	—	—	—	38,628
Commercial and Public Building	2,616	400	4,675	—	—	—	7,691
Industry	—	11,200	—	—	—	—	11,200
Schools with Playing Fields	1,528	—	—	2,723	—	—	4,251
Open Space	240	96	10	—	170	—	516
Transport	—	—	—	—	—	500	500
Public Utilities	4,103	800	250	—	—	2,900	8,053
Roads, Sewers, etc.	5,223	960	420	—	—	2,400	9,003
Other	—	—	—	—	—	5,030	5,030
Total Cost	52,594	13,504	5,363	2,745	200	10,886	85,292

APPENDIX D

The Annual Costs of Urban Development

The Maintenance and Servicing of Estates

1. *Dwellings.*—Maintenance in its wide sense, that is, including external painting and internal decoration, depends more on policy than on the incidence of failures and breakages, since these account for less than half of the cost. Experience in the case of high flats is limited but it would appear that for most items costs will be no higher in flats, whatever the height, than in houses. The important exceptions are for external painting and repairs, where contract figures suggest that costs increase with height (Table D.1) and, of course, the extra incidence of lifts and external services, outbuildings and public gardens, in the case of blocks of flats. The maintenance and service charge for lifts appears

to be about £5 net per dwelling and the charge for external services is perhaps about £4 net. Rent collection and administration* generally cost in the region of £3 with an addition of about £6 a year in the case of flatted estates for porters and caretakers. These costs are based on costs in Central London (Table D.1). The figures for internal decoration include the cost of work falling to the tenant as well as to the building owner.

TABLE D.1

Annual Maintenance and Service and Management Costs of Dwellings in Central London*

Item	Houses £	Two-, Three- and Four-Storey Blocks £	Five-, Six-, Seven- and Eight-Storey Blocks £	Nine- and Over Storey Blocks £
Internal Decoration	10	10	10	10
Internal Repairs, Replacement of Fittings and Repairs and Replacement of Services	8	8	8	8
Lifts, maintenance, repairs, insurance and power	—	—	5	5
External† Painting and repairs	6	7	8	9
External Services, including buildings and publicly maintained gardens	2	4	4	4
Total	26	29	35	36
Overheads (20 per cent.)	5	6	7	7
Communal lighting	—	1	2	2
Porters, caretakers, rent collection and general administration	3	3	9	9
Grand Total	34	39	53	54

* Mid-1957 costs.
† External to the dwelling.

2. The regional price pattern obtained earlier for housing construction seems to apply broadly to maintenance costs and has been applied to the maintenance costs given above. There were no grounds for making a similar adjustment to the lighting and administrative costs (Table D.2).

TABLE D.2

Regional Annual Maintenance and Management Costs† of Dwellings

	Houses	Two-, Three- and Four-Storey Blocks	Five-, Six-, Seven- and Eight-Storey Blocks	Nine- and Over Storey Blocks
Central London	31	35	42	43
Outer London	26	30	37	38
Provinces	23	26	32	32
Communal Lighting, Porters, Caretakers, etc.	3	4	11	11

† Mid-1957 costs.

* In addition to information collected from Local Authorities, data were taken from Alford and Edwards (1957), "The maintenance and management of high density housing estates". Extensive use was also made of Reiners (1955), "Maintenance costs of Local Authority housing", which gives information on the effect on costs of age and region.

3. Additional expenditure is incurred for special facilities, such as about £3 for the maintenance of central heating, an extra £1 to £2 for Garchey refuse disposal over and above the normal chute system, and sums for the maintenance of laundries, common rooms, garages and so on, but these provide services additional to those normal in public authority housing, and their costs have therefore been excluded.

4. *Roads and Landscaped Areas.*—Estate roads have to be maintained, lit and cleaned, and their verges have to be kept trimmed. A number of authorities provided cost data for this work and some information was obtained from published sources (Mercer, 1955; Axford, 1956; McHarg, 1956). It would appear that at provincial prices the total maintenance and servicing costs work out at around £800 a year a mile, that is about 4*s.* 6*d.* a yard of frontage. The corresponding figures for Outer and Inner London would probably be about 5*s.* 2*d.* and 6*s.* a yard respectively (Table D.3). The costs of maintaining and cleaning large paved areas probably range from £50 to £100 an acre, planted areas would probably cost about £150 an acre.

TABLE D.3

Annual Provincial Costs of Maintaining, Resurfacing and Dressing Roads and Maintaining and Re-laying Footpaths

Item	*Maintenance Resurfacing and Dressing Roads, and Maintaining and Re-laying Footpaths* £	*Lighting (One Lamp every 120 feet of Road), Fuel, Lamp Replacement, Cleaning and Maintenance* £	*Verges (1½ yd. each Side of Road), Grass Cutting, Attendance to Trees and Shrubs* £	*Sweeping, Cleaning and Gully Emptying* £	*Total per Mile* £
Cost per Mile . .	300	250	120	130	800

5. *Refuse Collection and Deliveries to Dwellings and Allotments.*—The cost of the actual collection of refuse from the individual dwelling and taking it to a central point is about 30*s.* a dwelling a year. Clearly site layout can make but little difference to the order of these costs. However, chutes are usual in high blocks and might save up to £1 a dwelling a year for collection. Removal expenses to or from dwellings in high blocks are higher than in the case of two-storey dwellings. For removals to or from dwellings on fifth and sixth floors costs are about £6 higher per dwelling than for two-storey houses where lifts are available, and about £10 above where no lifts are available. It might be assumed that on average each tenancy will last perhaps 20 years. Generalizing, these figures suggest that for a cycle—moving in and out again—the extra equivalent annual costs over those arising in two-storey blocks are 2*s.* (three-storey blocks), 4*s.* (four-storey blocks), 6*s.* (five-storey blocks), and 8*s.* (six- to twelve-storey blocks). In the light of these results it would appear that other deliveries, tradesmen and postal, must also be higher for dwellings in higher blocks; the costs of occasional services such as ambulance and fire are also likely to follow the same pattern. However, the cost differentials cannot be large; even if the above figures are multiplied by five the cost differences are not appreciable. Allotments as a substitute for private gardens should not cost more than around 10*s.* a year in peripheral areas but might be prohibitively expensive in central redevelopment areas.

Housing Subsidies and Exchequer Grants

6. Exchequer subsidies on dwellings erected by public authorities are now governed for England and Wales by the Housing Subsidies Act, 1956. This Act, as recently modified, now provides no subsidy for houses built for general needs, except in certain exceptional cases, but concentrates subsidies on dwellings built for slum clearance and overspill. The subsidies are paid in two ways, on the dwelling and on the land, and are payable over 60 years (Table D.4).

TABLE D.4

Housing Subsidies—England and Wales

Subsidy per Dwelling per Annum

Purpose	*Up to Three Storeys*	*Four Storeys*	*Five Storeys*	*Six Storeys*	*Seven or More Storeys*
For housing people from slum clearance, redevelopment and unsatisfactory temporary housing	£22 1*s.*	£32	£38	£50	£50 plus £1 15*s.* for each storey in excess of six storeys
For town development, for urgent industrial needs, or new town development . .	£24	£32	£38	£50	£50 plus £1 15*s.* for each storey in excess of six storeys

Subsidy on Area of Housing Site per Acre per Annum

	Cost of Site as Developed per Acre		
	£4,000 *or Less*	*More than* £4,000 *to* £5,000	*In Excess of* £5,000
Subsidy	Nil	£60	£60 plus £34 for each £1,000 or part of £1,000 in excess of £5,000 per acre

Source.—Housing Subsidies Act 1956 and *Circular* 59/56, Ministry of Housing and Local Government.

7. Grants can also be obtained under the Town and Country Planning Acts 1947 and 1954 towards the cost of acquiring, and the preliminary development of, land with extensive war damage, bad layout, or obsolete development, and for land for open spaces, for re-housing displaced population and industry, and for the restoration of derelict land. The grants are payable for a period of 60 years at a rate of 50 per cent. (75 per cent. in the case of land for open spaces) of the difference between the expenditure and the value of the land for its new use during the year for which the grant is payable. Under the Local Government Act, 1958, these grants will be limited to the redevelopment of areas of war-damage, but County Councils have the powers to make grants to assist development and redevelopment.

8. Under the Town Development Act 1952 the primary responsibility rests with the local authority for the area to be developed. Ministry grants can be obtained of 50 per cent. of the estimated extra costs of water and sewerage works attributable to the town expansion scheme. Further, it is quite common for the exporting town to make a contribution towards each dwelling, for a period of ten years, of something like the old statutory rate fund contribution. In addition, the Ministry has said that further financial assistance will be given if the expanding town is in difficulties at the end of the 10-year period. In Staffordshire the County Council have made an arrangement of this kind.

9. Exchequer Equalization Grants* are made under the Local Government Act 1948 to county boroughs and administrative counties for distribution to the other local government units, in order to offset the higher rate incomes enjoyed by the wealthier authorities. The basis of assessment is a comparison of the individual rateable values per head of weighted population, with the average for England and Wales. The weighting makes allowance for the proportion of children and the mileage of roads as compared with the population. Some authorities receive a large proportion of their revenue in this way, in some cases the Exchequer Equalization Grant is equivalent to a half to two-thirds of the relevant expenditure.

Environment Costs

10. A number of studies have been made of the costs of travelling within urban areas (City of Birmingham Development Plan; Munby, 1951; Westergaard, 1957; Portsmouth District Survey and Plan; London Transport Executive, 1956), the differences in food prices in different types of town (Ministry of Agriculture, Fisheries and Food, 1954), and the effect of built-up areas on transport costs (Glanville and Smeed, 1957; Ministry of Housing and Local Government, 1956*a*). Taking these as a starting point it is possible to make some estimate of the differences in costs of living and working in different areas (Table D.5).

TABLE D.5

Differential Costs Per Person Per Week in Greater London, Other Conurbations and Large and Small Provincial Towns

Item	*Conurbations* Greater London	*Conurbations* Other Conurbations	Large Provincial Town	Small Provincial Town
Weekly Travelling Cost for a Wage-Earner	10/–	—	5/–	2/–
Multiplier for Family of Four	1·75	—	2·5	2·5
Weekly Travelling Cost for Family of Four	17/6	—	12/6	5/–
Average Travelling Cost Per Person Per Week	4/6	4/–	3/3	1/3
Food Per Person Per Week	30/–	29/–	28/–	26/6
Difference in Cost of Living from Greater London per Week	—	1/6	3/3	6/9
Saving on Commercial and Industrial Transport Costs Differential per Person per Week from Greater London	—	—	3*d*.	1/3

11. Using the above figures some estimate can be made for the difference in costs of living and working as between large provincial towns and Greater London, and as between large and small provincial towns. It is difficult to evaluate such differences but, on the basis of the estimates derived earlier, it would appear that annual costs may be around £8·75 per person lower in large provincial towns than in Greater London and perhaps £11·25 lower in smaller as compared with large provincial towns (Table D.5). These figures must be examined to ensure the absence of double counting and transfer payments. The costs of travelling are real costs because they are based on journey miles at constant

* In future these grants will be replaced by Rate Deficiency Grants payable under the Local Government Act 1958, and both the basis and method of payment will be slightly modified.

charges per mile. Food prices are cheaper partly perhaps because of lower wages and lower ground rents and partly because of the larger element of garden produce; the former should be discounted. The saving in transport costs is a real saving but a small part of it may contribute to lower food prices and, therefore, should be discounted. Even if these figures are discounted generously the differences can hardly be put at less than £5 and £10 a year respectively. For 10,000 persons savings on this basis would be £50,000 a year for large provincial towns as against Greater London and £100,000 for small as against large provincial towns.

12. Some attempt was made to estimate the unit cost of services in the various types of urban areas considered but consistent information was not available. The only Local Government services for which the costs per head were available were fire and police. In each case the pattern was similar to that given in Table D.6

TABLE D.6

Costs per Head of Population for Certain Local Authority Services

	Fire Service	*Police*
London	15/–	—
Other Conurbations	10/–	37/–
Other Large Towns	9/–	35/–
Small Towns	8/–	31/–

Source: *Return of Fire Services Statistics* 1956/7. *Return of Police Force Statistics* 1956/7. Both published by the Institute of Municipal Treasurers and Accountants and the Society of County Treasurers.

13. It would seem reasonable to assume that the costs of most other services will follow a similar pattern and that at least costs will be no more in the small than in the large urban areas. Normally the services for which there are potential economies of scale are provided by a large central authority such as the County Council.

Value of Improvements of Buildings

14. In the course of the developments and redevelopments old buildings and assets are replaced by new. In part therefore the eventual need for replacing the old assets as they stand is met by the new developments. Therefore the value of the new developments which corresponds to the real cost of replacing the old can be discounted. In practice, of course, the old assets would not normally be replaced unless their useful life was exhausted and the new development, in effect, anticipates the gradual replacement of the old assets.

15. The annual worth of the old assets is lower than that of the corresponding new assets because they have to be replaced at an earlier date and are likely to be more expensive to maintain and to service and less convenient for the purposes for which they are intended. The difference in the annual worth between the old and the new assets can be taken as the difference in the average net return that can be derived from the two sorts of assets allowing for the difference in their respective lives and for maintenance, heating and lighting and carrying on a given activity. Clearly, the data available will not allow such precise accounting. Moreover, the cost differences and indeed their very nature will vary from one type of asset to another. The problem is to find some approach which will provide an estimate of the difference in the annual worth between the new and old assets.

16. Perhaps the simplest approach is to consider the average annual return obtainable

from buildings and works of different ages. Suppose, first of all, that the existing assets form a normal cross-section of assets of all ages, each with an initial normal expected replacement life of, say, 60 years, and further suppose that the gross annual return of both new and old assets is unity. On this basis the old assets would give a worth of about 72 per cent. of that of the new. But more will have to be spent in maintenance, servicing and adaptation in the case of the old assets than the new, in order that the two may be made equally useful. The net annual return of the old assets will therefore be less than the new. If it is assumed that the old assets would for these reasons be 20 per cent. less valuable than the new, then the net effect of these two assumptions is to place a value on the old assets of only about 58 per cent. of those newly created. If the initial expected life was longer than 60 years, as it might be for civil engineering works, say 80 years, the corresponding value would be 62 per cent. Thus it does not appear that the ratio of the net annual return of an old to a new asset is very sensitive to the expected life of the asset and it seems reasonable to adopt a figure of 60 per cent. In effect, this means that by creating new assets now, and thus anticipating the replacement of existing assets at some time, an improvement value has been created equivalent to about 40 per cent. of the gross annual return.

17. An alternative approach particularly applicable to housing is to assume that, instead of the assets representing a normal cross-section of all ages, they are distributed in a block of common age. In such cases the improvement values will depend on the expected future life of the existing assets. Values have been worked out for various expected lives, again assuming that the old assets will be 20 per cent. less valuable than the new (Table D.7).

18. For instance, with an unexpired life of 25 years, for an asset with a total expected life of 60 years, the appropriate improvement value would be about 40 per cent. (Table D.7).

TABLE D.7

Improvement Value when an Existing Asset is Replaced by a Comparable New Asset

Estimated Future Life of Existing Assets in Years	*Improvement Value as a Percentage of a New Comparable Asset with an Initial Expected Life of:* 60 *Years* %	80 *Years* %
0	100	100
5	82	82
10	67	68
15	56	58
20	47	49
25	40	42
30	35	37
35	31	33
40	27	30
50	23	26
60	20	23
70	—	21

19. In practice, of course, the assets in the old centre are often retained and used for other purposes so that the whole of their value is not lost and the real improvement value may be a good deal higher than the values given in this paper.

20. For the purpose of improvement values, assets can be divided into three classes, residential buildings, non-residential buildings catering for local needs, and non-residential buildings catering for national needs.

21. Normally, housing will not be replaced until its useful life is over. Most of the housing now being demolished is slum clearance housing which by definition has no useful life and hence no value. The useful life of recent assets of housing, even that built immediately pre-war, might be so long that their value is not dissimilar to that of new housing. Even housing built at the beginning of the century might be judged to have a life of 25 years and hence give rise to the fact that for an asset with a total initial expected life of 60 years the appropriate improvement value would be about 40 per cent.

22. The second class of assets such as schools, shops and public buildings becomes redundant in one area as a result of the movement of population out of the area and is replaced by corresponding assets in other areas. It can be assumed that such assets form a normal cross-section of buildings of all ages and hence their replacement gives rise to an improvement value of 40 per cent. of their gross annual return.

23. The third class of assets, mainly industrial buildings, moves quite independently of the overspill migration of population. The factories which move are most likely to be those whose premises are inadequate and who wish to build new premises. Left to themselves they would probably rebuild next door to their existing site. Unable to build locally they move to another town. Thus the rebuilding is done with the expectation that the costs will be covered by lower running costs and gains in efficiency; that is, that the net annual return of the old buildings as compared with the new is zero. But it can be argued that in moving to a new location, a solution probably forced on the management by the negative control of planning regulations, extra cost for movement and disturbance is incurred, and that this should be reckoned as a cost against the overspill programme. There may of course also be compensating gains in moving to a new location but no account will be taken of these.

24. It will be appreciated, of course, that the improvement values must be applied to the gross returns of new buildings similar to the buildings being replaced as a result of the development. The results of applying figures in this way for individual assets and for groups of assets are given in Table D.8. It will be appreciated that in applying the figures to groups of assets such as neighbourhoods and industrial areas, it has been assumed that the assets correspond in principle and type both in the old and new developments.

Movement Costs

25. All development and redevelopment necessitates movement of personnel and goods from one place to another and such costs are as much a part of development costs in the broad sense as are the costs of construction. The problem in the context of this study is to estimate the comparable costs of movement within the confines of a central area, for a central to peripheral move and for a central to new town move. Estimates on these lines have been built up for each type of entity, allowance being made for disturbance costs as well as transport costs. Disturbance costs are, of course, heaviest in the case of industry where allowance must be made, not only for the dismantling and refixing of machinery, but for the grants made to persuade staff to move, the cost of training staff replacements and the loss of output during the period of shut-down and subsequent reorganization.

Some of these costs are, of course, transfer payments and must be discounted. In practice not all the costs will arise in the form assumed since buildings will be equipped with new fittings. The re-equipment is, however, irrelevant to the problem here. The estimated movement costs are given in Table D.9.

TABLE D.8

Improvement Values for Various Urban Developments

Item	*Expected Future Life*	*Current Annual Worth of Asset to be Replaced* £	*Improvement Value* %	*Value of Improvement when Old Asset is Replaced* £
Examples of Buildings				
Two-storey house (14 dwellings/acre)	40 years	100	27	27
	25 ,,	100	40	40
	0 ,,	100	100	100
Two-storey house (1 dwelling/acre)	40 ,,	500	27	135
	25 ,,	500	40	200
Primary School (250 pupil places)		2,188	40	875
Secondary School (855 pupil places)		12,825	40	5,130
Neighbourhood Library (for 10,000 persons)		150	40	60
Shop (1,000 square feet)		150	40	60
Public house		500	40	200
Commercial Office (1,000 sq. ft.)		150	40	60
Factory (20,000 sq. ft.) (voluntary move)		2,800	100	2,800
Cinema		3,750	40	1,500
Health Centre (10,000 persons)		250	40	100
Church		750	40	300
Neighbourhood (10,000 persons) built up as in Table C.2				
General Development and Buildings		37,400	40	14,960
Industrial Area (10,000 persons) built up as in Table C.3				
Industrial Buildings		70,000	100	70,000
Other Buildings and Development		14,100	40	5,640
Total Industrial Area		84,100	—	75,640
New Town (80,000 persons) built up as in Table C.9 excluding dwellings				
Building and General Development		1,365,900	40	546,360
Industrial Buildings		560,000	100	560,000
Total New Town		1,925,900	—	1,106,360

TABLE D.9

Movement Costs for a Neighbourhood and for a Town

£000's

Type of Cost	*Central Area* (1 *Mile*)	*Central to Peripheral* (5 *Miles*)	*Central to New Town* (100 *Miles*)
Neighbourhood (10,000 *Persons*)			
Persons (10,000)	Negligible	Negligible	7
Furniture and Personal Effects (10,000 persons)	24	27	109
Offices (25 persons)	Negligible	Negligible	2
Shops (Equivalent 48 units)	2	2	5
Schools (1,160 pupils)	2	2	6
Social and Other Buildings (20,000 square feet)	Negligible	Negligible	2
Total Neighbourhood	28	32	131
Industrial Area (10,000 *Persons*)			
Factories (2,000 persons)	200	200	420
New Town (80,000 *Persons*)			
Persons (80,000)	1	3	58
Furniture and Personal Effects (80,000 persons)	190	217	869
Factories (16,000 persons)	1,600	1,600	3,360
Offices (5,000 persons)	60	61	330
Shops (Equivalent 1,400 units)	56	60	147
Schools (18,280 pupils)	27	31	101
Social and other Communal Buildings (320,000 sq. ft.)	5	6	29
Total New Town	1,939	1,978	4,894

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DISCUSSION ON MR. STONE'S PAPER

Sir FREDERIC OSBORN: I move a vote of thanks to Mr. Stone with special pleasure for his remarkably comprehensive paper. For ten years I have been putting out shirt-cuff calculations showing that re-housing in cities at high density is far more costly to the public than thinning out and partial dispersal to new towns or country-town extensions, and vainly nattering at economists and experts in public finance to tackle the subject more fully than I was able to do. Perhaps it serves me right that when at last I get the sort of analysis I asked for it overwhelms me a bit, but I am not a trained statistician.

I think this paper is heroic in its daring. The complexities of public, commercial and personal costs and satisfactions in urban developments are so great that I confess I have thought an assessment embracing all of them to be impracticable. My own studies of the comparative costs of central housing at high density and of partial dispersal, including the one I sent to the Ministry of Housing and Local Government in 1958,* have been confined to such estimates as I could make of public costs only. This 1958 study took account of Exchequer and local grants and subsidies, initial losses on new towns, public costs of closing vacated factories, and cost of raising the standard of open space in London to a miserable $2\frac{1}{2}$ acres per 1,000 persons. Open space is one of the things Mr. Stone has omitted, but I hesitate to criticize his paper until I understand it better.

My result showed that for a 50 per cent. dispersal (reducing central housing from 40 to 20 dwellings an acre) a public saving of £2·7 million per 10,000 persons re-housed could be made. The Ministry went carefully into my figures and cut my £2·7 million saving to £1·9 million on grounds which I did not accept. (They wanted to equate a 750 sq. ft. with an 850 sq. ft. house, which I do not think is a valid comparison, and I am glad Mr. Stone sticks to a basic floor area.) In any case, this seemed to be a good enough saving to justify abandoning the madness of multi-storey flat building, even if this kind of housing were not disliked.

For the purposes of public policy I still think my case was unanswerable. I cannot see why the taxpayer should pay out the bulk of this enormous unremunerative cost to enable a city to intensify its central development and to force an increasing percentage of its citizens into flats. Why should a nation pay out cash to counteract city costs which are the nemesis of over-centralization and which would otherwise operate as economic checks on it?

* "Higher density or more dispersal—a note on some relative costs," *Town and Country Planning*, (1958), **76**, 259–266.

Mr. Stone's concern for costs is all-inclusive. He pursues the analysis into the financial effects on firms and individuals. He also considers costs in terms of annual outgoings, for maintenance, etc. This is valuable although I cannot accept his view that transfer costs do not matter, especially when I, as a taxpayer, am the transferor and dwellers in cities are the transferees. I congratulate him on his useful pioneer assessments, including the "improvement value" assessment, which seems however to be scarcely separable from the unquantifiable value judgments he mentions in paragraph 1.

I have enjoyed his adventure very much, but I wonder if Mr. Stone might not agree that it is still something of a shirt-cuff calculation, although he wears much longer cuffs than I can afford. Can the productive value of a new factory in a new town with workers housed within walking distance be weighed against the transfer costs, the different rent, and the cost of town amenities, taking into account also the saving of time and money in daily journeys and so on? Is there not something to be said for an analysis limited to public costs and for accepting as a fact that firms or persons who move, essentially because an intolerable situation has evolved in old cities, do survive and seem prosperous and content in surroundings fairly obviously superior?

If the production of new and better surroundings were more costly to the public and demonstrably imposed a heavy burden on firms and persons, we should have to consider whether the advance we want is worth the cost. My calculations, and I think Mr. Stone's, show, however, that here we have a case of the most unusual kind, where there are two alternative methods of urban development, and the socially better is the cheaper. Mr. Stone does not put the total savings as high as I put the public savings, but if I have understood his final result, he assesses that to disperse 84 of 154 persons saves £1,000 a year as compared with dispersing only 31·5 of them and putting the rest into five-storey flats with lifts. That means, if I have got it right, that dispersing 10,000 persons would save over £60,000 a year—very roughly equivalent to a capital saving of £1,200,000. He does not give the figures of comparison with ten-storey flats, but I have done another calculation, using his data, and it seems to be over £2 million, which is near enough to the Ministry's revision of my figures for public costs alone, so it looks as if Mr. Stone's brilliant calculations of moving costs and "improvement values" just about balance each other. I have to admit to being a little out of my depth in his paddling pool and I am open to correction.

It should be pointed out that in his estimates of new town costs he included some amenities that the new towns still lack; and it would appear that there are more than ample savings out of which they could be provided. He finds the expansion schemes under the Town Development Act more economical because existing small towns already possess amenities. But most new towns would be small town extensions and the difference is really one of machinery; extensions might be built by corporations of the New Town Act type, which has proved in many ways to be better.

I do not like Mr. Stone's idea of reducing frontages to 16 ft.; it would spoil houses and gardens and be a retrograde economy. An average of 20 ft. should be the minimum for any housing scheme. I appreciate his demonstration of the dis-economy of the very small house, and of its absurdly high capital and subsidy cost per person housed. The fear that some families may have a spare room seems to me a disgrace to contemporary housing reformers, and quite out of time in an age of rising standards of production and income.

I like Mr. Stone's debunking of the argument that decent standards of space in urban development would be a menace to the nation's food supply, and his recognition of the fact that food from gardens is comparable in value to that of the farms taken for housing.

I think this is a fine paper and a great contribution to the study of a much neglected subject.

Dr. Rutland: It gives me great pleasure to second this vote of thanks, and like Sir Frederic I should like to praise Mr. Stone for tackling a most complex subject with courage and facility. He has been most assiduous in collecting a vast array of figures and presenting them in a comprehensive and comprehensible way. I think he will agree that the final word on this subject has not been said and probably never will be. That is not a

criticism, nor is what I am going to suggest later, so much as a suggestion for extending the analysis a little further. I want to stress how vitally important this problem is. The results of Mr. Stone's labours have been awaited by and will no doubt be used by a wide circle of persons and organisations.

In spite of the great complexity of the figures presented the problem is in many respects still grossly over-simplified. In order to make the figures applicable and useful to individual developers they need to be adapted to local and individual circumstances. In the first part of the paper we are told of typical costs. I doubt if there are any such things or if they can be represented by a single figure applicable generally and uncritically. No doubt these averages will be seized upon with eagerness by people (none of them in this audience and Society) who will use them as a yardstick for measuring their own type of development. Mr. Stone has put in numerous caveats against misapplying his conclusions and these I should like to reinforce. The ranges of values, densities and costs, quite apart from the controversial "improvement values", are so wide that different assessments will be required in each individual case. No single scheme will conform to the norms here laid down.

Moreover, some of the costings can, at present, be based only on inadequate information. For instance, only very few schemes containing 12-storey blocks have been completed by provincial local authorities and none of these consists of three-bedroom dwellings. In these circumstances it appears somewhat adventurous as well as unnecessary to quote derived figures for four different types of 12-storey development in Table 3 and to claim that they are significant to three figures. At best, such estimates must be somewhat academic.

Then, again, it does not seem admissible to use the early tables as an adequate basis for everything that follows and to equate, e.g. three-bedroom two-storey development invariably to 20 dwellings, to 80 rooms per acre and to 71 persons per acre at 0·89 persons per room. Variations round these ratios are wide and constantly changing. The 0·89 ratio, for instance, was an estimate made before the publication of the 1951 census and now appears to be much too high as a long-term general average whilst it may well be too low, at least for the next decade, in estates containing many young families. Nationally such densities will continue to fall—a phenomenon common to all countries where the standard of living is rising. On the other hand, in many new estates it is bound to rise temporarily.

As a basis for comparison, the cost per person has many drawbacks. It will be one thing on the completion of an estate, a different thing when the families have reached their maximum size and different again when the estate or new town has achieved some stability. The real units of demand and supply are not persons and rooms but households and dwellings. Comparisons in these terms are far more useful and less subject to change. For instance, the increase in population due to the excess of births over deaths often represents no housing demand until the additional children grow up and want to set up their own households twenty or more years later. Conversely a static population may experience an increasing housing shortage because the proportion of actual or potential heads of households contained in it is increasing.

This leads me to a further point, namely to what extent trends should be taken into account when assessing costs. After all, lasting assets are being created and one usually looks forward to periods well beyond the completion of the development. This would imply a further complication of Mr. Stone's analysis in that time series would need to be substituted for some of the static factors. Like all practising statisticians I dislike forward-looking time series, but where populations refuse to remain at a pre-ordained level it is inevitable that such practices should be adopted. To ignore trends is tantamount to forecasting no change, which is usually most unrealistic. I should therefore urge Mr. Stone to exercise his talents in this direction. It is desirable that someone with his background should undertake this study. Many of the data are quantifiable within limits. As an example, I would quote maintenance and running costs which are normally lower in the first few years and higher later.

There are two small points I am not clear about: Mr. Stone points out that tall blocks

usually contain smaller dwellings than do low blocks. He attributes this to the desire to house families with children near the ground. Is there any evidence for this? Is it not more likely that dwellings are kept small in tall blocks simply in order to get more of them into this costly form of development and to increase revenue? Secondly, whilst large dwellings may at first contain families with many children, so giving a reduced cost per person, the census shows that the person-per-room ratio tends to vary inversely with the number of rooms per dwelling. In due course, it may well be that this national picture will be reproduced even in new towns.

These are minor points which I hope will not obscure the very great value which we all place on Mr. Stone's paper.

The vote of thanks was put to the meeting and carried unanimously.

Dr. Marian Bowley: As an economist it gives me very great pleasure to find a paper on this exceedingly important subject coming from the Building Research Station. It seems to me most important that the Building Research Station should take a serious interest in the economic aspects of building, as distinct from the technical and design problems with which it has been associated for so long. The economists there have the special advantage over people outside that whenever they are puzzled by some technicality of building they can ask somebody who knows about it. I personally have found the experts at the Station exceedingly helpful in answering questions by correspondence, but that is not quite the same thing. The evidence that the Research Station is exploiting its own potentialities is very encouraging.

I have not found time to explore in any detail the method of compiling statistical data, and I think it is perfectly possible that an improvement might be made. Nevertheless, the fact that an attempt has been made, and the results are so consistent, encourages me to think that essentially the conclusions based upon these results are a very good starting point for discussion of policy.

It has been emphasized in the paper and by other speakers that development in high blocks of flats is exceedingly costly. This is becoming familiar from other discussions but it is difficult to underestimate the importance of its demonstration in this paper. The reason for this, I think, is that trends in architecture are strongly in favour of high blocks. There is a marked tendency for people who are not architects to feel that for some obscure reason the architects must know best, and the aesthetic section of the community is privileged to lead the rest of us by the nose. Land, however, is not a produced commodity; building materials are. Thus, to a considerable extent, if building goes upward we are substituting produced commodities for a commodity which is not produced and we are increasing the proportion of these commodities to land. We are substituting something for the production of which we have to pay for something which is already there.

Not unnaturally, this is apt to lead sooner or later to increased costs. Improvements in technology and in building materials are simply ways of helping to offset the cost of using produced materials in place of land. Moreover, they provide an excellent illustration of the law of diminishing returns. If you go on increasing one or more factors of production in relation to another (in this case the proportion of building resources, materials and labour to land), sooner or later diminishing returns may be expected to step in and costs to go up. From the point of view of economists, Mr. Stone has produced a very nice statistical demonstration which will be very useful in lecturing! He should, I think, consider the point that those who suggest that town planners and architects should take costs seriously into account have this fundamental justification for claiming that the costs of the ordinary development with low densities should be the starting point for comparisons.

The calculations of improvement value in the paper are rather difficult to follow, perhaps because they are hypothetical. Something seems to have been left out in comparing the costs of high density development with other types. Has Mr. Stone taken into

account the advantages of low density peripheral development in terms of saving the costs of car parks, road improvements and traffic congestion in central areas?

One of Mr. Stone's conclusions is extremely alarming. This is that the system of Government housing subsidies and the interest of local authorities in maintaining rateable values tend to obscure part of the costs to the community of high density development. Whether Mr. Stone's statistical calculations are precisely correct or not, it seems to me that that conclusion is of fundamental importance. It suggests that a serious re-assessment of housing policy and of the basis of housing subsidies in connection with different types of development is needed.

Dr. J. C. Weston: The work described by Mr. Stone this evening has arisen naturally from the Station's interest in the construction of high flatted dwellings. These cost considerably more to build than comparable two-storey accommodation and we have been trying to find ways of substantially reducing their costs. We wanted to know how far it was economically worth while to spend more on accommodation in these high blocks than in houses and what part such blocks were likely to play in the future. This information would provide an indication both of the target prices at which we had to aim and of the amount of research effort worth expending. In order to obtain this information it was necessary to examine the cost of high flatted blocks within the whole field of urban development and it was obvious that the results would have a wide range of usage within the field of planning and development.

The planner is faced with two problems with which a study of this nature can be of real assistance. The first is the difficulty of assessing what the plan is likely to involve in terms of physical requirements and costs. In the end a detailed estimate is necessary and a long and costly analysis cannot be avoided, but while the plans are fluid there is a need for easily obtained estimates and these can readily be obtained from a paper such as this. The second and more difficult problem is to predict the total impact of the development plan on the community and to choose between one plan and another. To do this one must weigh up the large complex of facts relating to appearance, function and cost so as to gain some idea of the value for money offered by the various alternatives. This involves looking at the development not just at the present moment but over its entire existence and looking at its total impact on the inhabitants of the area to be developed and on those around it. An adequate evaluation of this type is not really feasible until the economic conclusions have been extracted and reduced to a single figure which can then be set against the more subjective criteria of appearance and utility.

In this paper Mr. Stone has limited himself to the problems of quantifying all those factors susceptible to this treatment, in putting forward techniques of costing, and of providing estimates of physical requirements and costs as a guide to the planner.

The factors of appearance and utility have not been considered in this paper, not because they are unimportant but because they cannot be quantified and manipulated in the same way as economic data, but clearly they must be taken into account by the planner, and we will hope that his views on such matters as the importance of "city atmosphere", life in flats, the possession of a garden and proximity to the town centre or to open spaces will gain perspective from the measurement and isolation of the economic impact. It is important to obtain value for money in planning as in any other field of activity and it would be as foolish to ignore the values which people set upon the various attributes of the development as it would be to ignore their monetary impact.

We have no axe to grind and no planning policy to put forward except perhaps that planning decisions should be taken in the light of all the facts, believing with the mathematician Babbage that "the errors that arise due to an absence of facts are more numerous and more durable than those which arise due to unsound reasoning respecting true facts".

Sound economy means obtaining good value for money and this is by no means inevitably achieved by buying the cheapest article. The fact that in the examples given in the paper, low blocks and particularly two-storey houses provide the cheapest accommodation is no justification for their universal use, for housing and development units of all types

have their part to play. Moreover, since the local situation will differ from the assumptions made in costing the examples given in the paper, the planner may find that for his situation the costs break even at different points from those given in the paper.

We hope that the planner will find the techniques and estimates given in this paper of assistance to him in his work and that they will help him to obtain for us even better value for money than is already being provided.

Mr. P. J. O. SELF: I want to take up a point made by Sir Frederic Osborn. This sort of analysis is very useful but it is also exceedingly complex and easily leads one to wonder whether an analysis initially based on public costs might possibly be more useful. The paper is so comprehensive that anyone reading it may have missed some of the assumptions and calculations, but let us consider the point that public services and other things are more costly in the London area. There are various examples of this, and London appears to be at a disadvantage as a location. However, while costs are higher so are wages, so it does not follow that people in London are worse off.

According to classical economics, the assumption would be that the higher the wages, the higher the productivity; that is to say, there are real economic advantages in London which enable employers to pay these higher wages which, in turn, justify higher levels of costs. The ultimate demonstration of this would be that if it were not so the hard-headed employer would move elsewhere. On the other hand, it may well be that employers pay higher wages out of convention and without doing the kind of hard-headed thinking I have mentioned.

That leads me to a further point. It seems to me that an analysis of public costs would enable us to dive into Government policy more easily. One sort of analysis could be obtained by taking the different public costs for different patterns and densities of development, including housing and all other public services. In common with Sir Frederic Osborn I feel fairly sure that the conclusions which would emerge would support the conclusions in this paper. That analysis would show, first of all, by distinguishing separately all its costs, how far government is subsidizing different locations, and secondly, as a corollary, how far subsidization of private enterprise might in certain cases be a preferable alternative. To take a simple example, suppose it emerges that the costs of putting more people in higher densities in the inner area of a big city are substantially greater than those of the other alternatives mentioned here, including the new town locations, there would be a margin to work on, and it could be considered whether it is worth while using part of this margin to subsidize or induce industrial firms to move into the cheaper locations. In other words, it would enable direct public costs to be brought into equation with the use of subsidies. In saying this I do not discount the importance of the sort of thing which Mr. Stone has done here. I think it is a most interesting and admirable task, and one needs to look a good deal more closely into some of the background assumptions and calculations which are made.

Dr. LICHFIELD: As someone engaged in town and country planning and interested in the economics of what we are trying to do, I should like to add my tribute to Mr. Stone for attempting something not previously attempted on this scale. We knew that it cost more to provide dwellings at high density than at low density in the centre, in order to save overspill of population. We also knew that it cost more in urban facilities to spread people around the countryside rather than re-house them near their old homes. Just what the comparative costs were, we did not know. Mr. Stone has shown for the first time the orders of magnitude, and for that we must be grateful to him.

This particular subject is complex, particularly for someone who does not spend his time in planning. Mr. Stone has dived in at the deep end and made various assumptions and included various facts and figures which are open to challenge from various quarters. But challenge on the individual figures will not, I think, undermine the general tenor of the paper. I would hope to see this as a beginning of further work on the subject. To assist in this I would like to confine my comments to random points of detail.

Firstly, in paragraph 3 there is a suggestion that we can only put costs and not judgement of value in terms of money. I would suggest that some money values can be expressed in money terms, including many of the values implicit in this problem. For example, I have recently been looking at the rents obtainable for the kind of dwellings to which Mr. Stone refers. It is interesting to find that, in certain localities at least, people are prepared to pay more rent for a house than for a flat of equal size and amenity. Furthermore, with certain exceptions, they are prepared to pay the same rent for a flat of given size and amenity whatever floor of a tall block it is on. Furthermore, they are prepared to pay the same rent for a flat of given size and amenity whether it be in, say, a six-storey block or a ten-storey block. This means that while it costs more to build upwards, you get no more value per unit from doing so. This reinforces from the value side the argument against going high.

The second point refers to density of housing schemes in Table 1. Mr. Stone has taken a range of building heights as appropriate to a range of particular densities. But the relationship between building height and density is more subtle. A particular density can be achieved with a considerable range of building heights. Since the average cost per unit varies with height of block the relation of average cost to density is not as simple as is suggested. Furthermore, the average cost per unit of building varies considerably with dwelling size, and if one considers a range of dwelling size instead of a uniform size, as Mr. Stone has, then again the relation is different.

Thirdly, I should like to come to a point of doubt on this difficult concept of improvement value. If you have a new building instead of one with only 30 years life, clearly it is worth more and credit should be taken for the extra value of the improvement. But a debit should also be made, on the cost side, for the cost of scrapping an asset which has 30 years to go. The community is deprived of its services. Has this been included in Mr. Stone's concept?

Mr. STONE subsequently replied in writing as follows:

May I first express my gratitude for all the very interesting observations which have been made in the discussion?

Planning decisions are usually taken in what is thought to be the best interests of the country as a whole, that is of the community, hence the importance attached to the concept of real costs which attempts to measure the total impact of a development on the community. But even the most advantageous policy will not normally be implemented unless it is also in the interests of those responsible for carrying out the developments. The function of the central government is to deploy taxes and subsidies so as to create conditions under which the interests of developers and local authorities coincide with what is best for the country. The costs of doing this are presumably what Sir Frederic Osborn and Mr. Self call public costs. While these are of interest they are responsible for only a part of the impact of a development on the costs to the public and the impact on production costs may be overwhelmingly greater.

As Dr. Lichfield has pointed out the complement of an improvement value is not greatly different from the valuation that would be placed on a property to be replaced as part of the re-development. The two concepts differ, however, in detail and might produce very different results in particular cases. The valuation technique is not applicable to buildings and works which have no market price. I am less sanguine than Dr. Lichfield as to the extent to which money values can be attached to the imponderables of appearance and utility and I doubt whether the use of the valuer's estimate of the money values attached by the market would either much increase the field which could be quantified or much change the conclusions which could be reached. Of course, in dealing with precise development proposals close estimates of revenue and expenditure should be possible.

Dr. Rutland quite rightly senses the dangers of pushing the use of typical figures too far and of ignoring trends. In this field the man with one eye is king; the experience of individual planners is necessarily so limited that such collections of typical figures as are provided in this paper and in the handbooks of his Ministry are essential to basic cost

comparisons. We did not, of course, rely on the indications provided by a few blocks of 12-storey flats, but determined our price trends in relation to all types of dwellings and all block heights. At a secondary stage, when detailed alternatives are being developed, closer approximations based on detailed needs can be substituted. The problem of trends has been given some attention since the estimates relate to average conditions over the life of the assets. For instance, the occupation rate of 0·89 is an estimate of the average occupation over the life of the dwelling allowing for families to increase and then decline; servicing costs have also been estimated as the average level over the life of the building.

Sir Frederic Osborn will find that I have allowed 7 acres of open space per 1,000 persons as part of the neighbourhood area and that I have not suggested reducing frontages to 16 ft. but have only shown the resulting densities if this were done. In fact all the comparisons are based on 14 dwellings to the acre which gives something like 24 ft. frontage.

Dr. Bowley is correct that no direct allowance has been made for savings in dealing with traffic in central areas when their population is reduced. In particular cases such an allowance would be possible but it is rather difficult to generalize.

Dr. Lichfield is correct in pointing out that there is no fixed relationship between building height and density. For purposes of simplification I have taken a fixed relationship between height and density based on the highest densities which have been achieved with particular average block heights. This assumption, and the fact that I have considered three-bedroom dwellings instead of smaller ones, tend to make the cost comparison become more favourable to the use of high blocks. In general other assumptions which have had to be made will have the same result. Since it was impossible to determine the actual level of all the various cost concepts at all precisely, it seemed better to allow any bias introduced to tend in the same direction since this simplifies the interpretation of the results.

Finally, may I express my complete agreement with Dr. Weston in stressing the importance of reaching administrative decisions in the light of the fullest possible knowledge of the facts.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:

Peter Michael Childs
John Newton Darroch
Cyril Domb
Michael Gilbert
Paul Stuart Groves
Michael Graham Hall
Theodore Herbert Harris
Edgar Horne
Allan John Langner
Frank Lee
Valerie Lofthouse
Alison Mary Martin
Edward Farquharson Mellen
Howard George Miles
Richard Neil Picken
Barbara Gillian Pye
Louis James Rothman
Leonard George Tostevin
Dennis Gim Wyn Tsui
Ronald Whitaker
James Clifford Wilkinson
Marvin Zelen

Simulation Studies of Industrial Operations

A Discussion opened by P. V. Youle, K. D. Tocher, W. N. Jessop and F. I. Musk

[Held before the Royal Statistical Society on May 20th, 1959, the President, Sir Harry Campion, C.B., C.B.E., in the Chair.]

Dr. P. V. Youle:

Statistical Aspects of Process Development and Plant Design

Development projects for new products are key problems for the management of industrial research organizations. The following notes are based on considerations of such problems in the chemical industry.

The field of process development is one on which a great many new techniques are impinging at the moment, and it does not seem that all these new techniques are applied sufficiently effectively in current statistical and engineering research programmes. There is a subject nowadays that might be called "plant study" by analogy with "work study". While "work study" concerns the whole operation of the particular man being studied, "plant study" concerns the whole operation of the plant. The subject has not yet achieved the coherence of "work study", and "plant study" is not a good formal title for what one means. "Process development" is not entirely accurate either and a better term is needed. The subject involves taking a plant and looking at the sum total of its performance. It means taking the plant, if possible, before it is built and examining what you want it to do, how you think it should do it, and how you are going to know that it is doing it, and what you are going to do about it if it does not do it. In this subject there is clearly a great part for chemical engineering and statistical teams to play. Obviously, they are involved in the design of the plant, but in addition they have an indirect part to play in putting these new ideas across: and on both those scores they may have things to learn.

"Plant study" means studying a chemical plant as if it were some sort of organism with raw materials going in, processes going on inside, and product coming out. Product is, however, not the only thing that comes out: so does information. Although it is old-fashioned to regard a plant as solely a machine for making a product, this idea persists. To many plant managers it is revolutionary to look at their plant as a machine not only making a product but also giving information about that product and, moreover, giving the information so clearly and so fully that the information can be fed back into the system so as to improve the product and increase the yield. The easier end of this is concerned with improving yield: this is easy because yield is a fairly tangible thing. If what is in question is quality, that is more difficult, even when applied fairly simply to a quality such as percentage purity. A still more difficult problem arises if the absolute level of the response is less important than a very low degree of variability around the mean. All such problems may enforce modifications in the way statistical techniques are employed in process development work.

An expanding chemical industry must plan for an increase in the output of its intermediates. Thus, over a quarter of a century output might rise from a small trickle at the pilot plant stage to a huge manufacture when several full scale plants are in operation, the rise in output being accompanied by a commensurate improvement in quality. This should be thought of as an evolutionary process. The urgent question is: are the statistician and the chemical engineer making a sufficient contribution to this evolutionary process? An attempt will be made to answer this question under three headings, first as regards the plant itself, then in relation to antecedent design work, and finally in connection with improved ways of planning laboratory and semi-technical experiments.

Studies in Relation to an Existing Large-scale Plant

Plant Data

Fig. 1 represents a control loop for the intermediates plant. It is wrong to think that a cycle like this must necessarily represent no more than an instantaneous control. If a continuous monitor were available, information could be fed back instantaneously to

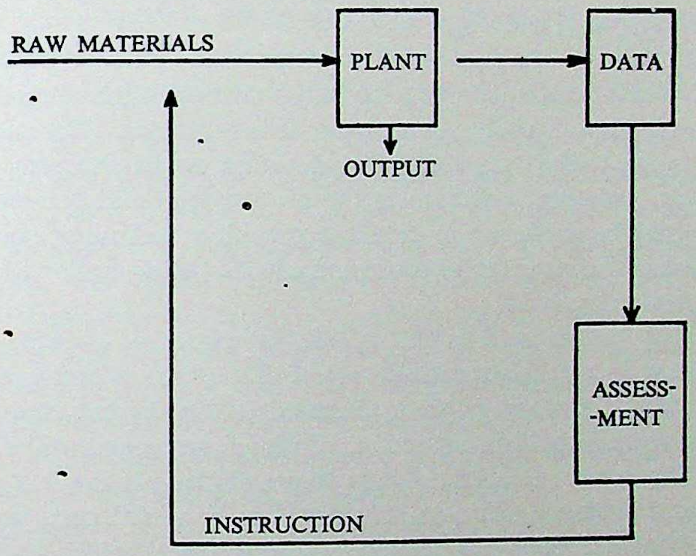

FIG. 1.—A control loop for the intermediates plant.

some point in the plant and used to control temperature so as to maintain quality constant. The time scale can, however, be extended, and delayed analytical results can be assessed by a suitable committee, which can pass appropriate instructions to the plant. This represents a long term control loop: analytical data are taken and passed to a Specifications Committee through some sorting-out mechanism, and the Specifications Committee then gradually starts further discussions that ultimately lead to changes on the plant and to narrower quality specifications.

On such a plant, how is it possible to decide whether all the necessary information is being collected for such a control loop? The first raw materials for such a study are the

plant records, and these may not be good enough to give the information required. There is a big controversy between statisticians and others about the value of plant records as basic data. Many plant records are necessarily ambiguous. If a man is told to go to the plant and adjust the heating so that the temperature stays at 145° C., he will try to do this. If also he is told to record the temperature, he may be at a loss to know whether to record the temperature as it was when he got there, or as what it is after he put it right. From that dilemma perhaps he slides into writing 145° C. anyway. In this sort of discussion there is a further temptation to argue that process labour will never be able to give proper plant records because of this propensity to slide away. Such an argument may well be true now and may have been so in the past, but may be short-sighted when one is looking at the next twenty years.

The old picture of the process man as a pure fetcher-and-carrier is bound to go, especially on complicated polymer plants, so that process records may well be plotted with more and more acumen and co-operation. In designing a new plant and in trying to analyze the performance of old plants, some of the things that ought to be studied are the nature of the plant records that are taken, who they are taken by, and whether in fact what is plotted is what ought to be plotted, and if not why not.

Many plant records, of course, depend on instruments, but it is doubtful if the instrumentation of all plants is adequate to give fully meaningful plant records. Process development implies a very carefully planned use of instruments. Such instruments would obviously help the day-to-day running of the plant and would also provide better records.

Then, of course, granting perfection in the process man and perfection in the instrumentation, there may be confusion in data interpretation because of coincidences in time trends or confusion in the sequence of cause and effect. Allowing for all this, what should be done with such plant records?

They should first be expressed as charts, but the science of charting has been making advances in recent years. It is not enough to plot charts "for information". They should lead to action signals. A recent paper by Barnard (1959) suggests more efficient ways in which such action signals could be arrived at. In brief, he recommended plotting cumulative totals and using cursors of special design to detect changes in plant performance.

In a complicated chemical plant, an instrument, to be useful, need not be coupled to a control point for instantaneous rectification. An instrument which gives data leading to a chart which is used efficiently to detect trends can act to give a warning signal that something has gone off-standard. New methods of charting now make this more of a practical possibility.

The charts can be put to a further use, in trying to sort out cause and effect. This is much more difficult than is usually supposed. It depends absolutely on having accurate and meaningful data, and on the most efficient possible method of detecting changes in the mean level of data. Intermediates quality is often believed to affect final product quality. Proof of this belief is adduced if changes in the one lead to changes in the other and if a coincidence of change-points can be detected.

The statistical technique of multiple regression looks for such relationships in an organized way, and can be made semi-automatic by the use of a "data logger" and electronic computer. If the data are unsuitable for analysis, these devices will only create confusion more rapidly. Pitfalls may thus arise in the use of data loggers where they are applied to unsuitable and inadequate plant measurements and where they are confined to dealing

only in the results of accidental plant happenings. In a carefully selected case they might be useful. Some prior knowledge of likely cause-and-effect mechanisms would be helpful, and physical chemistry can often supply this.

Physical Chemistry

For the above reasons, statistical and chemical engineering research ought to involve programmes of work to sort out efficient ways of charting data. There ought also to be programmes for setting up rules for drawing the morals from such data, and that is where work on process research is often starved of effort. Such starvation means that the work never succeeds in setting up the equations which are needed, such as, for example, an equation giving intermediates product quality in terms of raw material purity. Possibly, in many fields of chemistry, these factors will never be expressible in an exact mathematical form, but to sort out data it is vital to have some rules formulated, and the more mathematically this can be done, the better. Perhaps in a search for panaceas one does injustice to plain physical chemistry. Perhaps a criticism of the pure statistician in this field is that he does not attach sufficient importance to straightforward physical chemistry, just as some physical chemists fail to see the importance of statistics.

Evolutionary Operation (EVOP)

With adequate physical chemistry in support, then at least quasi-mathematical expressions will be available to provide the interpretation of the charts that are based on routine plant records, or perhaps to guide the programming of information from the data logger on to the electronic computer. Information obtained in this way could be valuable—far more valuable than that obtained at present from such sources—but it has one major limitation: it is based on accidental happenings on the plant. The charts may, for example, be dealing with the effect of a particular temperature on intermediates quality. If that temperature can vary for a multitude of accidental reasons, then no clear conclusion is likely. If, on the other hand, there is one principal factor which affects the temperature, then a single cause-and-effect relationship exists and might be picked up. An alternative approach is to make changes in plant variables on purpose.

"Evolutionary operation" is a technique for planning such deliberate experiments. It has been fully discussed in papers by Box (e.g. Box, 1957).

It is sometimes supposed that there is a direct antagonism between plant records and data logging on the one hand and evolutionary operation on the other. This may not always be true. There is this distinct difference though: that in evolutionary operation a variable is deliberately changed, whereas in analyzing plant records reliance is necessarily placed on the accidental variations that occur in it.

If the accidental variations have many causes, evolutionary operation is more use. If the accidental variations have only one cause and this is known, the two techniques may coincide. So far, EVOP has mainly involved experiments on yield. A small Works Committee is set up, the variables are chosen, and the experiment is planned to run in turn at eight different sets of conditions, repeating each set of conditions until a significant result is obtained. There need be no loss of production. If the technique is to be applied to studying quality and not yield, one might wish to reinforce the regulations by which the experiment is confined to the safe limits of the normal product specification.

Quality Control; Work Study; Operational Research; Linear Programming

Many of the process development jobs described so far come within the ambit of "quality control" as defined in its broadest sense. Some come under work study or method study. Operational research workers would claim others. The argument of this paper is that a process development or chemical engineering section should look into all these specialist techniques, consult what experts are needed, attempt pilot problems in relation to some of the full-scale plants, report the results and so sort out those techniques of most value in process development chemistry.

Studies on Design

Up to this point the discussion has been wholly about what to do on and with the large-scale plant. Moving backwards in the argument from plant to laboratory, the next part of the discussion is concerned with the collection and mobilization of design data. These data are based on laboratory and semi-technical experiments, and the design of such experiments will be the last aspect of the subject: one would expect design considerations to influence the planning of such experiments as well as such experiments to influence design.

Simulation: Analogue Computers

Many useful studies of large-scale plants are being made by studying models in which the operations are simplified enough to allow the principles to be disentangled. These simulation techniques are now a fairly conventional sort of tool in operational research studies. For example, for an iron and steel plant one makes a theoretical model of the way it works, and, by looking at that, one tries to find ways of streamlining the operation. If such a survey could be made before the building of the iron and steel works, a great many things in design could be improved. What is said here thus applies with most force to plants where the design is not yet finished.

At the very start of semi-technical and design effort, the target would be to set up a model of the plant, that is to say, a theoretical model and not a physical model. One would aim at describing the exact behaviour of the plant, with all its control loops and instrumentation. To make the problem more precise, one might well set up the task: to design an analogue computer which could simulate the final plant operation. Setting down the circuits and details for this would focus attention on just those mathematical relationships which govern full-scale operation and would mean a greater likelihood of their being discovered at the earliest possible stage. Whether or not the analogue computer was eventually built, the attempt to design it would thus be rewarding. Alternatively, the whole of these calculations could be carried out on a digital computer. In this event, the separate parts of the digital computer programme simulate the separate parts of the plants. Given all the physical chemical data, simulation can predict the full operation of the large-scale plant.

In the most favourable case, the analogue computer design would seem cheap and compact. Then it would be worth while to construct the computer and use it to study plant behaviour. Its especial merit might lie in giving practice in the use of whatever control loops the final plant is to contain.

Studies on the Laboratory and Semi-technical Scale

Process development chemistry on the laboratory and semi-technical scales will fail in its usefulness unless it is backed right at the start by a clear idea about the features of the final plant. This need arises the moment the original invention has been defined. Quite different programmes of work emerge at once, depending on the objective. If an academic paper is required, the work that is done will most likely miss out many features essential to plant design. Plant design is, in fact, jeopardized unless the very earliest laboratory process development experiments have the final plant in mind. Certain useful new techniques (Box and Wilson, 1951; Box and Youle, 1955) are now available to help in final plant design.

The essential feature of these techniques is that they try to impose a mathematical expression of the results from the very earliest moment. Sets of experiments are planned to study the variables along lines planned to indicate the quickest way to the optimum, the so-called method of "steepest ascent". The first set is perhaps a long way from optimum yield, but shows that, for example, higher temperatures, lower pressures, and less catalyst give better yields. The next set of experiments moves to a new base line that conforms with these discoveries. The sequence may have to be repeated several times.

Simultaneously with this, the physical chemist sets up for the system the best model that he can think of, balancing chemical truth, which often goes with complexity, against ease of calculation, which often demands over-simplification. Armed with a suitable model, the physical chemist brings in the mathematician and they quickly try to sort out equations to describe the system. At this stage it is very important to make enough simplifications and approximations. Once a workable model is constructed, calculations can begin and the complexities can be introduced.

The results of the first sets of experiments are then tested against this model. These results serve a dual purpose. They have already guided the chemist toward a better process: they now enable the mathematical model to be improved. As many constants as possible in the equations are now worked out from the experiments. Perhaps more experiments are needed to give reasonable estimates.

At this stage the physical chemist may feel that his model is being distorted. The physical meaning of the constants may be doubtful. Provided that the equations describe the behaviour of the system accurately in the regions required, this does not matter for the moment. The physical chemist may be able to plan a few simple experiments that will later restore meaning to some of the constants.

Assuming that the equations do give a fair description of the system, the next stage is to sketch out various possible types of semi-technical reactor. Their likely performance is calculated and the most promising one is built, but even before an engineer is asked to begin design work the chemist should have begun to think of the full-scale plant, and should have begun to try to simulate the operation of that plant. With more confidence in these techniques the need for large pilot plants might get less and less: the biggest success possible for the process development chemist is for him to show that there is no need to build a semi-technical plant at all.

Suggestion Action in a Typical Statistics and Process Development Group

If there is any merit in the foregoing ideas, where should they most influence the work

of a process development section? The most important place is clearly in those projects that are in their earliest stages.

Suppose that a new chemical intermediate is proposed for manufacture.

Laboratory work should use "steepest ascent" techniques to reach optimum conditions. These same experimental data should be used to derive constants for a physical chemical model. The equations so derived would then be employed to predict the likely behaviour of various types of semi-technical unit. In making predictions for these units, deficiencies will be revealed in the equations, and fresh experimental programmes will be written to fill out these deficiencies.

The most useful semi-technical design will be chosen, partly on the basis of the above predictions. A report will be prepared covering the full range of programmes planned in the semi-technical unit; a number of these will be worked out in complete detail to make sure that all necessary changes in instrumentation and design are made in good time. In particular, estimates will be made of the likely accuracy in the measurement of yields (or in the measurement of whatever other response is involved); if the accuracy is so low that a great deal of replication of experiments will be necessary, the plant procedure should be modified, if at all possible, to improve the accuracy of measurement, using work- and method-study techniques if necessary. The necessary electronic computer programmes should be prepared simultaneously so that results can be processed quickly when the plant is built.

As soon as calculations are complete on the various possible semi-technical plants, one such plant will be chosen. Calculations should then begin at once on the full-scale plant. A theoretical model of the full-scale plant should be drawn up, based on, and in addition to, the usual flowsheets and drawings: the model should enable the actual behaviour of the plant to be predicted, if necessary, of course, using an electronic computer. As far as possible, the model should anticipate the instrumentation and control systems of the full-scale plant, and calculations should be done to check the operation of the controls.

If the project continues successfully, the full-scale plant will be built without the necessity for a large-scale pilot plant. The equations based on laboratory work, the results of semi-technical work, and the simulation of the full-scale plant will give enough confidence in the likely performance of the full-scale plant to avoid an intermediate-scale pilot plant.

Start-up plans for the main plant should have been detailed enough to show what supporting data were needed from semi-technical work. Then when the main plant is running normally, its behaviour should be compared with predictions previously based on the theoretical model. In some respects plant performance may fall short of the target. In other respects, new targets may be set up. Together, these will define the objectives of further process development research. Cause-and-effect relationship can be sought through multiple regression techniques. Before these can be applied, method-study should be used to check that the plant results to be used in the multiple regressions are accurately measured, responsibly recorded, and painstakingly charted: instruments may need to be checked, or replaced. If reliance on accidental plant variability is unlikely to make the necessary advances towards the new objective, evolutionary operation schemes should be instituted. Finally, there should be a progressive feed-back of information from the chart-room to the plant to ensure that the maximum use is made of all the information that accrues from the actual operation of the plant.

Clear responsibilities lie in a chemical engineering or process development section at all stages in this sequence. The mixture of skills and technologies required in such sections can be deduced from the wide range of those responsibilities.

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Dr. K. D. Tocher:

Techniques for the Application of Computers to Industrial Simulation

Without entering into the controversial area of defining operational research, it is agreed that a commonly employed technique used by operational researchers is that of model building. These models are usually mathematical and, in the early days of the subject, simple models sufficed which were amenable to analytic attack. As the research teams have developed and extended their work, so the detail in the description of plants has increased, and now many of the mathematical models used to describe the problems can be studied only by an operational process often called a simulation study or the "Monte Carlo" method.

Applied to plant studies, this method consists of describing the plant in terms of a mathematical model in which the variates representing the uncontrolled factors in the plant behaviour are imitated by random variables. Such a model gives rise to a problem of finding the distribution of some function of the plant condition which represents, in some sense, its behaviour. Thus, in principle, from the point of view of a statistician, the problem is nothing more than to find the sampling distribution of an intricately and irregularly defined statistic. The irregularities of the definition ensure that a theoretical solution is impracticable and recourse is made to a sampling procedure. From the point of view of the operational research worker, the method consists of making an abstract model of the plant, and working the model by supplying the random elements from tables of random numbers or from random number generators.

Two main problems are raised by this approach. The first is the actual execution of the sampling and the second the evaluation of the significance (in a non-technical sense) of the results obtained. Clearly, the latter is one in which the statistical theory of the design of experiments is most relevant and in which the experimenter has perfect control over his material and is guaranteed no spoilt experiments. Although there are many interesting problems still to be solved about this aspect of simulation, this paper will concern itself with the practical problems of performing the simulation itself.

The conventional procedure analyses the structure of the plant to be simulated, describes this in mathematical terms, manipulates the equations to suitable forms and then uses them to generate the sampling values. In earlier days, this last stage was done by hand, using desk machines and tables of random numbers; some attempts have been made to

use punch card installations for this purpose, but before the full possibilities of this equipment had been explored, automatic digital computers became an accepted tool for the research worker. Thus it is natural that simulators have turned to computers to ease the burden of calculation. However, the various simulations made so far have all revealed that the lightening of the burden of calculation has only been done at the expense of a protracted and tedious stage of analysis, programming and coding which, in many cases, has resulted in the overall time to complete the simulation not being very much shortened by the use of the computer.

Thus there is a strong need for a procedure for constructing simulation models which reduces to a minimum (*a*) the time to make the necessary mathematical analysis of the plant structure and (*b*) the programming and coding time for the problem.

Various groups have attempted to meet the latter requirement by providing a library of sub-routines which can perform some of the commonly recurring tasks arising in simulation. This has not been particularly successful as simulation programmes are mainly organizational and the work of writing them is not eased by the sub-routine technique. Moreover, this approach does not have any impact on the first problem of analysis of the works.

An alternative is to obtain a generalized simulation programme in which the organization part of the programme is pre-built and to which special sub-routines are added for each individual simulation. This approach has the advantage that the difficult parts of simulation and programme writing are solved once and for all. Such a technique will require to find a generalized expression for the structure of any plant or, in other words, a mode of description of the operating procedure which is applicable to all plants. This common description will direct the simulator's analytic efforts and thus ensure that his previous experience is utilized to the full. This should lead to significant reductions in the time for analysis. The simulator needs to make a precise statement of the conditions under which activities in the plant can begin, and when this has been done exhaustively for all possible activities the logical implications of this set of statements need not be explored but will be incorporated in the model.

The requirements on a universal programme are as follows:—

(*a*) There shall be no loss of speed in running the programme compared with a "tailor-made" simulation programme. Although it is most likely that for any specific simulation a special programme *could* be written which will be faster than one given by a general programme, the effort to make a fast programme will usually be too great to make it worth while for a single simulation. That effort is worth while for a general programme to be used many times and the general programme is likely to be faster than any special programme that *would* be written. Even if this is false in any particular case, the use of a universal programme is likely to speed up the coding and the development of the programme and so the overall time from starting the simulation to obtaining the results is minimized.

(*b*) A translation from "everyday" language into a code acceptable by programme must be reasonably easy and must be designed so that the process of codification itself reveals errors or omissions in the specification of the problem.

(*c*) The storage used by the programme must be minimized in order to reserve as much storage as possible for data. Experience shows that simulations consume a large amount of storage for this purpose.

(*d*) Finally, and this is the most important requirement, the programme must be written so that any limitations present in it (and of course a truly universal programme is never achievable) can be overcome by additions to the programme and do not need any alteration of the programme. It is well known that once a carefully designed programme is altered, intolerable development delays inevitably result.

An outline of such a general programme will now be given.

This universal programme is limited to dealing with plants in which "events" occur, where the "event" is some discrete change of activity of some component of the plant. Thus, most continuous chemical plants and oil refineries cannot be treated, as except under emergency conditions there are no discrete changes in the plant. The description of the plant used is in terms of machines and their states rather than in terms of the flow of material. The plant is described as consisting of nothing but a collection of machines and each machine is capable of taking one of a set of clearly defined states. It is easily recognized that most of the plant can be described in such terms and it requires only a little imagination to realize that the remaining items can also be so described. Thus, the "tea break" is a machine, a special clock for marking time in one of two ways: "tea break" or "not tea break". A manager or foreman is a machine for taking decisions. A report form or a written record is a machine for recording information. A store is a machine for holding stock, its states being the number of items in the store. The states of the recording machines are the possible values of numbers to be recorded. The states of a decision machine are code names for possible courses of action. From the point of view of making a general programme, the advantage of giving a complete description of the plant in terms of a common element is enormous and too obvious to need further emphasis. If we look at the plant at any instant of time we can give a complete description of it by naming the state of every machine, but a more natural description is in terms of the "activities" going on; activities are just groups of machines in certain specified states. Thus, the activity of tapping a furnace involves the furnace in the state of being tapped, the crane in the state of holding the right sort of ladle, the ladle in the state of being held by the crane in the right place, and so on. The machines involved in the various activities are "active", other machines will be "idle" or "available" to take part in further activities. The activities in progress at any given instant of time will be of limited duration and will cease, and the machines involved in it will be come "idle" or "available". Thus, there is a natural process by which the activity of the plant dies down and, unless some re-activating element is involved, the plant will cease to function. This in fact happens at the week-ends or during the annual shut down. The reactivating process consists of starting activities when sufficient machines have become available in the correct states and the general simulation programme which has been written hinges on this general way of describing the functioning of any plant.

Machines are divided at any instant of time into two classes, those committed and those available. One phase of the programme searches the available machines for suitable groups to start activities and transfers these machines to the committed group, specifying the time they will cease to be committed; the second phase of the programme advances time by the smallest amount to reach the end of some activity and transfers back to the available group those machines which have ceased activity. This two-phase process continues indefinitely, until a sufficient period has been imitated.

This mode of description enables the same programme to be used to describe a plant in fine detail or in broad outline simply by a change in elements of the plants which are regarded as machines. Thus, in a complete plant simulation a whole shop can be thought of as a machine, whereas in a simulation concerning a shop the individual mechanisms in the shop are classed as machines. A mixed description is also possible, and this is particularly useful where we require a detailed study of one part of the works which is heavily dependent on the remainder of the works regarded as an environment. The environment can be described in broad outline while the point of interest is described in detail.

A most important factor in an adequate simulation of any real plant is the incidence of breakdowns and some care must be taken in selecting the breakdowns to be represented. Breakdowns which are infrequent but which when they occur have calamitous results can safely be ignored in a simulation, because the existence of the emergency conditions usually introduces a whole new discipline for organizing the plant. The exact form that this will take in any of the almost infinite variety of breakdown possibilities cannot be catalogued. Similarly, breakdowns which occur very infrequently and have no serious consequences when they occur can be ignored, because this approximation will not seriously affect the results of the simulation. The exclusion of these two classes is of great practical importance since the data for finding the statistical laws covering infrequent occurrences are very hard to get. The only breakdowns which need to be considered in simulations are those which are endemic. This implies adequate data about the real plant.

Just as we have a generalized description of the behaviour of the plant, so we need a generalized description of the breakdown of plant. It is suggested that an adequate general model is to associate with each piece of plant a "life" measured in arbitrary units and to allow each activity in which the machine takes part to wear this life away at appropriate rates, differing from activity to activity. When the life is worn away the machine breaks down. It then undergoes repairs, the duration of which is fixed by drawing a sample from a frequency distribution or by any other appropriate method, and then enters on a new life. Successive lives are independent samples from a life distribution which may be time dependent or historically dependent on the number of lives the machine has already had. All these possibilities are covered.

In practice, such a model raises the difficulty that a life may expire during an activity. This complicates the description of the plant in that elaborate hierarchies of alternative procedures in breakdown conditions have to be specified. This can be avoided by an approximation which is usually adequate, that machines only break down at the end of an activity. This implies that the activity "limps home" if a real breakdown occurs. This simplification is justified in the steel industry where the hot nature of the material makes the conclusion of hot activity essential. The life time can be replaced by a number—being the number of activities in which the machine can take part before breaking down. The various activities can be weighted to give differential viability under different conditions. The simple version in which each activity either reduces the life by one or nought has been found adequate in many cases.

The list of activities consists of a set of tests on the availability and state of a group of machines followed by a series of changes on the states of those machines and a resetting of the time when they are next available. This is done by setting down a series of statements, executed serially, of the kind $x\theta y$ where x is the name of a machine, y is the value of a state and θ is a relation between the state of the named machine x and the value y.

For the tests θ may take the form $=, \neq, \geqslant, >, \leqslant, <$, and for the actions θ may take the form $\rightarrow, \rightarrow +, \rightarrow -$. These latter symbols mean that the state of machine x is replaced by the value y, is increased by the value y and is decreased by the value y respectively.

In acting on the statement serially within an activity, if any test fails the activity is abandoned and a new one attempted.

A wide variety of methods of naming machines is available, including direct naming, naming as the state of a directly named machine (indirect mention), or as a sum of these two terms. All names can be modified by the contents of a modifier register. Special tests and actions are provided for setting this register. A similar range of ways of naming the value y is available and, in addition, it is possible to obtain the value of y as the result of any sub-routine whatsoever, subject only to the condition that the sub-routine contains a certain amount of organization associated with the general simulation programme itself. It is this provision which enables any conceivable expansion of the programme to be made, since such a sub-routine can perform *any* required calculation in addition to finding the value y.

In a complex plant there are very often parallel streams of production or multiple streams within a given area and this often leads to a very extensive list of activities, many of which only differ from one another by the machine's name. The wide range of methods of naming machines has been introduced in order that such similar activities can be coalesced into one. In particular the modifier is very useful in this respect. Methods of searching groups of machines to specify certain tests are also incorporated as special type statements.

Many machines do not have natural process times associated with them. Examples of such passive machines are stores, decision machines, queue counters, recording machines and so on. Thus, only some of the machines have times associated with them and this fact is utilized to speed the programme. In fact, the name of the machine is arranged so that it is possible to talk about the time of a time-dependent machine as the state of another machine and thus all the mechanisms available for changing states of machines are also available for changing the time associated with these machines. These special machines involving times are treated specially by the programme in advancing clock time as previously explained.

It is often necessary to keep lists of properties of items in stock or in queues and these lists are very easily fabricated as aggregates of non-time dependent machines and the naming system is designed to make referring to lists easy.

The various sample times required are obtained from pseudo-random numbers by the usual inverse cumulative transformation. A method has been devised by my colleague A. H. Russell (and will be published elsewhere) for fitting an approximation to this transformation in a wide variety of cases.

In order that the programme shall operate as quickly as possible very intricate coding and packing arrangements have been used and these form a most unnatural mode of expression for a simulator (but not, of course, for a programmer). Consequently, a translation programme has been written which converts a description of the machines and their activities in a simple language into the form required by the programme itself and this same translation programme will automatically provide extra activities to correspond to the recording and printing information called for.

This computer programme forms part of a much more general programme of work concerned with predicting the behaviour of plant under changed conditions. The first stage in this programme is the analysis of the plant structure and the rules of behaviour. This requires considerable probing on the part of the research worker and it takes anything up to a year for a man to gain sufficient experience of the right type of questions to ask and the kind of answers to accept before the working of a complex plant is properly understood. Using the language of the translation programme to set down the facts discovered helps to provide further questions that must be asked and at the same time is a significant step towards simulating the plant. This same activity discloses the type of data which is required to complete the description of the works and this must then be obtained. It is very often not possible to obtain the information from works records and it has to be collected specially. Since the data will have to be analysed into a form of frequency distributions, it is advantageous to collect these data in a form suitable for machine analysis and a separate phase of the work has been the design and construction of special automatic data collecting equipment which provides the data on punched paper tape in a form suitable for input to the computer. It is then possible to undertake the simulation itself and to proceed to an assessment of the results. It is in this pair of activities that the statistician has a particularly important part to play in the designing of a series of simulated conditions which will be efficient in providing the required answers. The final stage of implementing the results of this work hinges upon convincing the appropriate management of their validity.

This can only be done by demonstrating to the management that the simulation is an accurate model of their own plant. This is always the most difficult part of the whole problem. In proposed plant operating procedures any decisions that have to be taken between alternative courses of action are made on objective rational criteria, but in many plants the present day decision procedures are made on the basis of subjective judgment and, in general, it is not possible to describe the results of those subjective judgments as if they were the results of rational objective criteria. The solution proposed to this dilemma is that the simulation of the works under present conditions should not be free running but should stop at each point when a subjective decision is required, providing at that time all the relevant data on which the person concerned claims to take the decision. He is then invited to take the decision for the model which then proceeds until the next subjective decision is required. In effect, the manager or managers who run the real plant are invited to run the simulated plant. When they can see no difference between the results of running their own plant and the simulated version, the simulator is convinced he has an adequate model and the management are convinced that the model can be trusted to predict the effects of changes to the plant.

The development of objective rational criteria for decision making in plant control raises many interesting and intractable problems. These centre on the existence of a variety of conflicting and interacting local objectives. Industrial decisions can be made to control these objectives and the method of effecting these local decisions into a strategy of decisions for optimizing some overall or long-term objective has to be found. The interactions between these various levels of objectives are too intricate for theoretical analysis (the need to simulate is a proof of this) and a process of learning the best strategy by experiment is required.

The development of this programme has been the result of team co-operation, and I

should like to mention in particular the part played by my colleagues D. G. Owen and R. A. Cuninghame-Green, without whose help the project could not have reached its present advanced stage.

Mr. W. N. Jessop and Mr. F. I. Musk:

Simulation Methods and Policy Decisions in Industry

Scope

This paper is expository, its scope being an examination of the applicability and value of simulation methods as applied to the decisions of an industrial organization.

The Meaning of "Simulation"

Representation of reality by a conceptual model is the essence of applied mathematical thinking. Such models inevitably involve a degree of selectivity, that is, features of the real situation considered irrelevant or of secondary importance, are ignored. In terms of such models analytic solutions can sometimes be found to the problems that are posed. The method of simulation is usually tried when this customary mathematical approach leads to equations that cannot be solved. Indeed, the problems in industry for which simulation is used exhibit a more or less complex "structure"; by the latter word we mean the description in mathematical and logical terms of the relationships between the elements of the problem. A simulation model in general is described by three elements:

(*a*) The system whose performance is being studied;
(*b*) An input, upon which the system operates;
(*c*) An output—the measure of performance or effectiveness.

In an industrial context the system whose performance is being studied is of a dual character; there is firstly the actual flow of materials and the carrying out of manufacturing activities (the physical system), and secondly the executive system which controls those activities. As one would expect, complexity in the physical system (e.g. products, processes and factories) has its reflection in complexity in the executive system. Thus, with increasing complexity in the manufacturing processes goes the need for specialized decisions, often of a technical nature. The input (*b*) in the broadest sense includes all the factors which might influence the system. "Manufacturing requirements" is the most tangible factor, but underlying that are general economic conditions, presence or absence of competition, trade trends, etc. The output (*c*) is a measure of the effectiveness of the system being studied. It may be a criterion such as productivity or profitability.

Now the *system* is a controllable factor; its organisation is capable of modification or adaptation and so are the policies operated at various levels by its executives. The *input* is either not controllable or only partially so. In the cases with which we are concerned it will be statistical in character. What would be the effect for a given set of input conditions on the output or measure of effectiveness of different policies within the system? Problems of this kind can sometimes be solved mathematically by some technique such as linear

programming. That is, we can set up a mathematical model which leads to an answer by calculation. This paper is concerned with cases where this is not readily accomplished. In these cases, however, what can be done is to formulate a model of how the system functions, and, most important, how it would function under a new set of policies. This simulation *model* is thus a descriptive model; it will generally be in terms of only those factors considered important. Having obtained this description of how the system works, the next step is to establish an analogy of its actual working, the simulation itself. The simulation is a discrete input of data, with a series of logical and arithmetical steps representing the functioning of the industrial set-up, yielding an output in the form of the required performance criterion. The continued operation of this method generates data; for the same input, the process may be repeated with a system modified to simulate changed policy conditions and so on. It will be seen that what we have got is, in effect, a method of investigation which is very like experimentation. It is not experimentation in the real world, however, but experimentation "on paper". In fact, it is performing the same function as mathematics would do if the latter were really applicable, i.e. from given assumptions certain results follow. What gives the appearance of experimentation is that data are obtained in a manner analogous to what happens in the real world. We are now in a position to summarize the foregoing in the form of a definition, which we hope that later sections will clarify:

"Simulation in an industrial context is a method of evaluation of the performance of systems under postulated numerical inputs by subjecting the latter to arithmetical and discriminatory operations in a manner which is a reflection of the structure of similar operations in the real world. The nature of the input and of the arithmetical and discriminatory operations can be altered at will, corresponding to policy changes, and thus data relating to performance criteria under different policies obtained in a way which is analogous to actual experimentation".

It will be shown later that simulations may relate to decisions of an impersonal kind, or to decisions in which human judgement is an essential element. In the former case, the discriminatory operations are of an entirely logical kind such as could in principle be performed by a computer, and in the latter case the discrimination is of a complex psychological nature using data of an often highly personal kind. What is required now to put the applicability of simulation in a natural setting is an examination of the decision processes within an industrial organization. This is the work of the succeeding section.

Decisions in Industry

We shall, for convenience, refer to the industrial organization as "the firm", it being understood that this may refer to the public as well as to the private sector of industry.

The firm is acted upon by a whole host of environmental factors which affect it in varying degrees; they include the economic circumstances of the country, the state of domestic and foreign trade, the rate of technological advance of the country affecting the demand for its products, the presence of competition etc. It receives stimuli from its environment by way of sales demand, availability of materials, manpower and finance, and it responds in two ways: firstly, by trying to influence the local stimuli by such measures as advertising and marketing, wages and salary incentives, and dividend policy; secondly, by continuously adapting itself through the improvement of its products, the development

of new resources, and the more efficient use of its existing ones. The measures of effectiveness of these responses are its rate of growth, and its chances for ultimate survival. The link between stimulus and response is the management decision. At the highest level, broad far-reaching decisions are taken by the Board and top executives, and at lower levels of responsibility, decisions are narrower in scope and more technological in character. There are two main classes of decision: routine decisions and policy decisions. Routine decisions are part of the normal functioning of the firm, and concern production very largely; policy decisions are of a more fundamental kind and do not arise with the regularity of routine decisions. Two categories of policy decision, the tactical and strategic, can be distinguished. Like the routine decision, the tactical decision lies mainly in the field of production. The strategic decision concerns the adaptation and development of the firm. In modern industry there are various specialized aids to the taking of decisions, including accountancy, work study and, more recently, operational research. Many decisions are straightforward, but there are a number of problems whose background contains conflicting elements which make the "best" decision far from obvious. It is to this class of problems that operational research owes its existence. Certain of these problems with which operational research is concerned do not lend themselves to direct solution by ordinary methods and it is in these cases that simulation has been useful. In general, they have been concerned with problems of operation requiring routine and tactical decisions at what might be called the "production level" rather than the broad strategic decisions of board level. In recent years there has been another development, "business games", in the field of executive training which is closely related to our subject. It is in the case of the strategic decision that human judgement is an essential part of the simulation, while in the "production level" simulations the personal element of human discrimination is not so vital. We discuss these cases in the following sections.

The Production Field

(a) *Routine Decisions*

Within an existing framework, a scheme of routine decisions can be found which is "best" according to certain criteria. These routine decisions are recurrent, and are concerned for the most part with the questions: What should be made? How much should be made? When should it be made? And possibly, how should it be made?

The last question refers to a possible choice of production facility or route. There are records of stock, orders on hand and opinions on future trends of sales. On the basis of the current stock position and orders on hand and anticipated, decisions are made regarding these questions. The position described is shown diagrammatically in Fig. 2. The system as a whole has to resolve three factors: average stock levels, rate of production change and "customer satisfaction". "Customer satisfaction" is meant in the restricted sense of providing the goods required without delay, or with only a short delay. The first two factors are concerned with cost and they act in opposition to each other: a high rate of production change corresponds to low average stock, and conversely. The system operates according to certain rules in its attempt to resolve these factors. There are re-order levels in the production planning organization, and scheduling rules on the production side. These rules may be known numerical ones, or they may exist in someone's mind. How can a system of this kind operate in the optimum way? The question implies

a criterion. In this case it might be supposed to be that of cost subject to any technical restriction that may exist and a proviso that "lost sales" should occur with no greater than a prescribed frequency, a statistical property of the system. What is required is a set of decision rules (a policy) which will satisfy this criterion. (Ideally, the decision rules should be optimum with respect to the indefinitely extended sequence of decisions that constitutes the routine working of the system.)

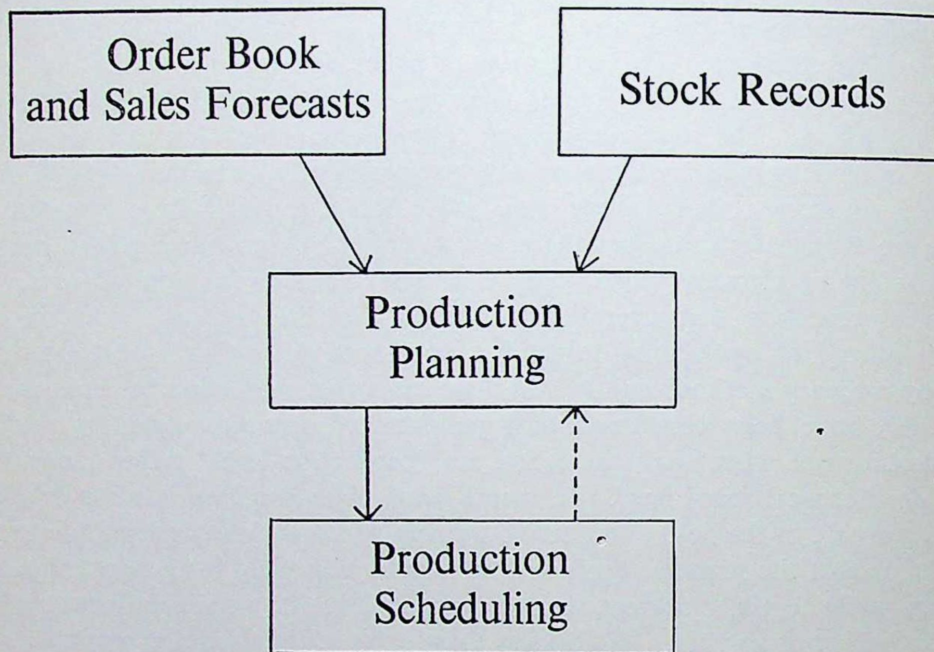

FIG. 2—Simplified production decision systems.

Any investigation of problems of this kind should begin with a theoretical approach; this helps to identify the elements of the problem which are critical, and it may even lead to an answer in some cases. In complex systems, however, any theoretical answer is subject to doubt, by virtue of the simplifying assumptions that are almost bound to be made. What can be done is to construct a theoretical model of the system, which may still be a simplification of reality but will nevertheless contain more detailed elements than could optimistically be handled by a purely mathematical attack. Thus, in approaching a problem of the type we have outlined, we may specify a policy which contains specific elements:

(*a*) Stock order levels;
(*b*) Rule for defining requirements in terms of stock and sales position;
(*c*) Machine scheduling restrictions.

These factors are controllable. We will identify them by the term "system logic". What this system has to act upon is an input which in the real case is uncontrollable or only partly controllable, the sales requirements. Although its real counterpart may be uncontrollable, the input in the simulation "experiment" is controllable. Thus, the experimenter may study the "response" of the system to various postulated inputs. This response will not be a single quantity; it will in general be a set of statistical estimates of various

quantities of interest. In this case it will be the stock variation patterns, machine utilization, production cost, "lost sales". From these quantities we may evaluate the criterion function, and then for the same data make a further experiment with different policy factors, find yet another value of this function, and so on. Although it has not as yet been tried, it is possible that the Box-Wilson steepest ascents approach may be useful in this as in genuine experimentation.

It will be apparent that simulation in the sense of the exploration of "policy space" will not in general be feasible without an electronic computer.

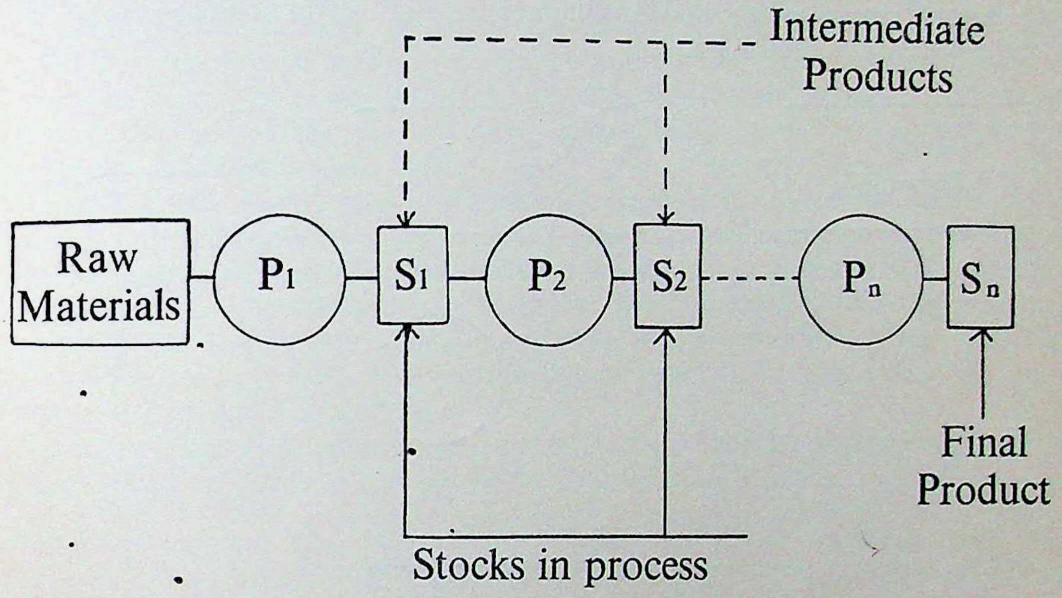

FIG. 3.—Multi-stage production process.

Theoretical work on stock control is not so far very helpful where it concerns a stock *and* production problem. A common model is a single product situation where an order is placed of amount $S - s$ when stock falls below a level s. If a distributed "lead time" (a delay between the placing and receipt of an order) is postulated the problem is more realistic. It is still a long way from most production problems, where there is a limited amount of productive capacity (machines), and where each of a number of stocks operating on an (s, S) scheme with, in general, different sales characteristics, has a lead time which is in fact a waiting time. This problem has superficial resemblances to a machine attention problem with N machines and n servers. The problem can be explored by simulation, and results from such experiments can be subjected to analysis to see whether they fit a mathematical hypothesis. If one is found this is a stimulus to theoretical work and also suggests a wider practical application.

The case described is a one-stage production process. Many manufacturing operations, however, are chains of processes, separated by stocks, where there may or may not be intermediate products. Fig. 3 illustrates this case.

Here there is a final product which is the result of a sequence of n processes. Each process (P_i) produces an intermediate product for which there may exist a market, and

there is a stock (S_i) between processes (P_i) and (P_{i+1}). A more general case exists where there may be branching at one or more stages in production, as is shown in Fig 4.

Further, each production process is likely to use raw materials in addition to the product of the previous stage, so that the general picture includes intermediate stocks of raw materials as well as possible branching. In principle, an optimal mode of working of such a system can be defined in terms of cost of stock holding, for final and intermediate products plus raw materials, and the cost of production fluctuations in each of the stages. The criterion will include the usual stock safety factors of postulated risks. Some theoretical work has been done on this subject (see for example Simpson, 1958), but the general problem would appear to have eluded solution so far. It is a type of problem very suitable for the application of simulation.

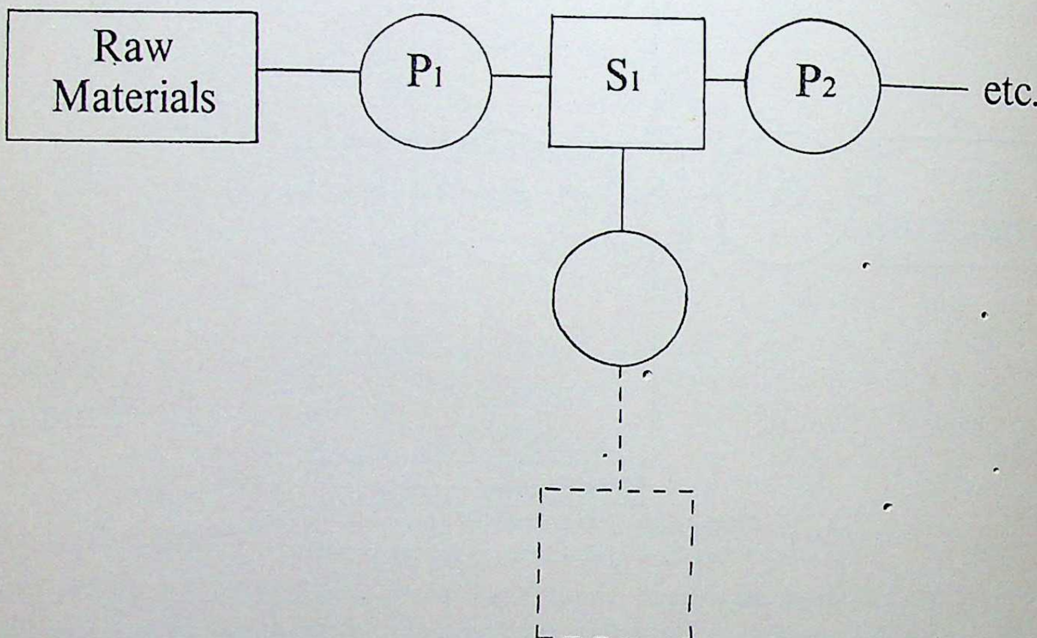

FIG. 4.—Multi-stage production process with branching.

(b) *Tactical Decisions*

Simulation may be used to evaluate the effects of projected changes of policy or changes of productive capacity, both of which may be costly.

As a hypothetical example of evaluating a policy change, we take the case of product rationalization: a production department has a certain set of machines which it uses for making a variety of products. The effect of this variety is lost capacity, due both to changes in machine settings and to the technical restrictions in the order in which one product may follow another. If a rationalization policy were carried out, similar products being replaced by one product, in groups, an overall gain in capacity could be expected.

The evaluation would be concerned with the reduced cost of manufacture caused by the prevalence of longer runs. This would allow for price reduction of the standardized products, leading to an expected net increase in sales to fill the increased capacity. The simulation would contain a subjective element, the estimated increase of sales. Otherwise, the evaluation depends upon (i) the "product mix", (ii) the production scheduling, (iii) the

particular grouping of products adopted in the rationalization scheme. If the first is regarded as virtually constant and the production scheduling routine is a reasonable one, then that leaves for comparison a limited number of choices under (iii). The simulation experiment now amounts to obtaining data relating to machine utilization and frequency of changeover under each of the hypothetical groupings. The basis is past data on the pattern of sales. These data can be transformed into the anticipated sales patterns under each of these groupings used as input. The input is scheduled in accordance with the established system which is part arithmetical and part logical (i.e. conditional factors are involved). From this scheduling, the machine utilization and changeover frequencies are calculated by production period (day, week or month). This information can be analysed as if it were genuine data, finding statistical parameters and using them as a basis for comparison. As a result, and with the subjective assessments mentioned above, the probable consequences of the proposed policy change can be assessed. A detailed description of such a simulation is given by Musk (1959).

A common field of application of simulation is that which seeks to evaluate the effects of alteration in the capacities of parts of the system under investigation. The type of system envisaged is one under fluctuating load, and alteration in the system will be directed to eliminating the bottlenecks that arise under these conditions. The frequency and magnitude of bottlenecks represents a cost deficit, and the problem reduces to finding whether a projected alteration in the production or handling facilities would lead to an improvement that would offset its cost.

It is not proposed to elaborate on this case since there are good examples in the literature (Banbury and Taylor, 1958; Clapham, 1958; Jones and Lee, 1955).

Simulations Involving Human Decisions

In the eighteenth and nineteenth centuries war games were developed that attempted to simulate the main features of actual warfare. The purpose of these games was (*a*) training, (*b*) to test the feasibility of operational plans before their application. The quite independent development of the von Neumann-Morgenstern theory with the mathematically precise notions of "game value", "optimal strategy", etc., although creating some confusion of thought, appears to have stimulated the ambition actually to "solve" games by this method. The development of high-speed computing machinery has encouraged this view. The dangers of this viewpoint are cogently argued in the paper by Thomas (1957) which also contains an excellent account of the historical development of the gaming technique and the various interacting trends of thought at the present time.

Where this does impinge on the theme of our discussion is in a recent development of war gaming, "business games". In this case the physical conflict between two sides is replaced by the commercial conflict of $N\,(\geq 2)$ firms. For details of two business games the reader is referred to the papers by Bellman *et al.* (1957), and Andlinger (1958). These games differ in detail, but not essentially. In the Bellman game, for example, each of the competing teams is presented at each "play" with a statement of its assets, income, stocks, etc. Each team has then to make decisions regarding the allocation of capital for the next period, its objective being to "improve its position", which means to increase its total assets and share of the market. It can invest money in research and development, marketing, additional plant capacity, and it can buy market research information. The

game has a hidden mathematical structure so that the money allocations of the teams can be used to compute the position of each "firm" at the beginning of the next play.

Business games were developed as a means of executive training. Their practical value would appear to be confined to that at the present time. Testing the feasibility of specific policies, by analogy with testing the feasibility of operational plans in war gaming, does not apply here because of ignorance of the nature of the conflict in the real world. If they encourage executives to promote the investigation of certain questions they will have served a useful purpose. They may, however, have a long-term value by encouraging fundamental research in mathematical, economic, statistical and psychological matters (see Bellman *et al.*, 1957).

Between the level of business games, that of top executives taking key strategic decisions in a firm, and that of the relatively impersonal decisions discussed previously, there is probably a class of problem where the subjective element is difficult to eliminate. The situation need not be competitive. In this case, it may be necessary to use the human agent in a simulation from which guidance regarding policy is required. The Rand Corporation has carried out such a man-machine simulation to compare two logistics systems. It would appear necessary in such cases to assess the variability of results from different persons or groups, if indeed decisions are made by a number of people in the actual case.

Validity

The test of any mathematical model is whether the results obtained from it are in good agreement with reality. Since a simulation model is simply a mathematical one from which results do not readily follow by direct manipulation and computation, the same criterion applies. There is a difficulty, however, in that at least part of a simulation experiment includes something for which there is no real counterpart, so that no direct check is possible. An indirect one may be possible, however. Since in many cases part of the experiment will concern a simulation based on a model of the real situation, there should be data from this which could be compared with actual data if they exist. If the correspondence is good, it may be assumed that any approximations or rejection of factors as second-order or irrelevant are valid, and we may assume with fair confidence that that is generally true.

We must admit reluctantly that there will be cases in which the experimenter has to rely on faith.

Conclusions

(*a*) The situations in which a simulation approach would appear most useful are those in which a complex system is subjected to variable inputs, and it is required either to define rules for the optimum working of this system, or to find how this system would function if modified in some way.

Such cases are of the "impersonal" type, where the laws of operation can be reduced to arithmetic and logic on known data.

(*b*) In approaching a problem of this kind, some theoretical formulation, however simplified, is better than none at all. If some selection of factors is necessary, this should help to decide which should be eliminated.

(*c*) Data obtained from simulation are analysed as if they were statistical data from

the real world. A validity check is made, if possible, by carrying out a simulation run to obtain data which are comparable with real data.

(*d*) Apart from answering an immediate problem, a simulation experiment should serve as a means of obtaining a deeper understanding of complex processes, so that in due course a basis of theoretical knowledge can be built-up, from which future problems are more readily solved.

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Mr. STAFFORD BEER: I should like to pose what seems to me a fundamental distinction between two possible approaches to this subject of simulation.

The first is the one to which this evening's speakers have primarily addressed themselves. It is to construct a stochastic model of a plant or a series of industrial operations. This is to be done by identifying the major variables in the system, and by linking them together to create a structure. The probabilism in such a system is to be treated by statistical methods. Thus simulation for such a model will consist in sampling probability distributions, which are known from either experiment, observation or hypothesis, and in inserting the sample values into the known structure—thereby creating an artificial history of the operation of the plant. Before any experimentation or prophecy is attempted with this technique, it will be advisable to operate the simulation for circumstances typical of actual conditions as they are at the moment. The answers obtained by simulation can then be compared with the answers that are currently obtained in practice from the plant. If the simulated answers look right, we shall declare that any variables which could have been considered in our model, but were not, are in fact nugatory.

This might be called by now the classic approach to simulation. It leaves us with a feeling that the method is not very respectable, because it lacks mathematical rigour. We use it, we are tempted to say, because we cannot solve the mathematics of so complex a system: we have no alternative. But we still feel unhappy, and speak as if the day will dawn when mathematics has advanced sufficiently far to provide the rigour that is missing.

I am not speaking against this first approach; on the contrary, it has been widely used in my department. Dr. Tocher has given you a sketch of a computer programme which has been used in just this way quite recently. Dr. Tocher did not mention that the "policy space" (as Mr. Jessop called it) in this case involved costs of the order of £1½ million per annum; and the decision on which strategy to use is decided on a scale each point of which is worth £7,000 a year. I quote these figures to indicate that whether we be unhappy about the absence of rigour or not, this first approach is well worth using; and also in order to distinguish it sharply from the second approach which seems to me quite different.

The second approach is distinguished from the first by the kind of system it tackles. Hitherto, large systems have been discussed (were they not large, they could probably be

handled by less cumbersome methods than simulation). But we may go on to consider *exceedingly* large systems, which I would define as systems that are so big that they cannot be defined in detail. Once it is accepted that such systems exist and require study, the first approach becomes absurd. For the task of identifying the important variables from which to construct a model is by definition impossible. It is true that some variables might be selected, but the test we used before to decide whether other variables in the system are nugatory or not will no longer work. The reason is that the span of behaviour in such a system is so great that "typical" behaviour is impossible to recognize objectively: behaviour that is alleged to be typical usually turns out to have been manufactured in some way by the observer—usually through the language he has used in order to describe the system.

Let me give a few examples of such systems. The brain is one; the economy is another; and the industrial company is a third. Some people speak as if small companies were quite simple to handle; but I would not accept this. Even a small company is likely to be so complex a system that it falls within the definition I have proposed. What should we do when confronted by a system so complicated that we could never hope for a rigorous methodology by which to attack it, never hope to identify the major variables? We are used to this problem in real life. Suppose we approach a baby, for example. It is true that we can identify some of the variables in the stimulus-response system consisting of ourselves as observers and the baby. But the essence of our approach is to consider the baby as an organic system. We do not try to make to ourselves a model of the way the baby works in detail; we consider instead our dynamic interactions with the organic system represented by the *operational* baby. The major parameters of the system may be considered as fixed within certain limits, and if we are wise we will keep an eye open for the operation of certain major variables; but in general our approach is to enter into an experimental situation with the baby.

This sort of situation can be simulated, and I think that it is quite different from the first kind—in theory as well as practice. There is no point any longer, for instance, in bemoaning the absence of a rigour which could treat the whole situation as a computable convolution of stochastic processes. It is accepted that, even in principle, such processes cannot be identified or stated. Instead our whole attack is of a different kind, which I would illustrate by the way in which we treat disorders of the brain. Only four major physical techniques are known: leucotomy, continuous narcosis, shock therapy, and drugs. These methods all depend on our inability to produce a detailed model of the operation of the brain, and they rely instead upon large-scale effects which will *absorb* a great deal of intricate variety generated by the disorder. In the economy, which is another large and complex system, the bank rate is used for the same purpose. When we get to industry, we have to use the same kind of treatment. But Dr. Tocher gave us the clue when he said that it is possible to generate a system that cannot be described in detail by choosing any complex of activities we like to be "the machine" in his computer programme. This is the approach by simulation to an organic system, where we are trying to absorb high and indefinable variety in the system by a homomorphic representation in the model. This does not require recognition of "major variables", but provides instead a series of transformations for the information which the system is generating.

That remark gives the clue to the treatment of the question of rigour in the second approach. For if we discuss a system, an industrial plant if you like, not as a machine for processing steel or some other product, but as a machine for processing information, and if we then adopt an information-theoretic model to describe it, we have at our disposal a great deal of rigour. For example, we know in general that the amount of control we can exert on this system that we are not going to describe, is proportional to the logarithm of the amount of information in it. We know from von Neumann's work that we can process a message through this system and get it out as correctly as we wish for any pre-designated degree of breakdown in the system—which is to say for any degree of probability in the system if we design the simulation properly. We have that kind of rigour. We know from Ashby's "Law of requisite variety" that we must rigorously have the right

amount of information in the control to balance the amount of information in the system to be controlled; and then again we have the whole of the theory which is known under the heading of the "black box". This list is by no means exhaustive, and this is not the place to offer explanations of the details; I merely seek to show that there is a wonderful range of rigorous science available by which to discuss simulations of this second kind.

May I suggest that we call this "second order rigour"? When a manager says to a statistician: "You will never succeed in handling this problem scientifically; we do not know what is going to happen, and although we make plans these are always falsified by mischance", the statistician laughs and says: "I have a technique for handling uncertainty; although I cannot predict individual events, I can produce a stochastic account of them which will be helpful". If this same statistician now says to me: "You cannot treat the kind of system you have proposed scientifically, because on your own showing you cannot identify its variables nor define its structure", I give him in turn a similar answer. That is, there is a second order level of rigour which will handle this kind of system despite its indefinability, just as you have a first order rigour which will handle a defined situation despite its uncertainty. The first level, with which we are already familiar, deals in stochastic methods which can specify probabilities without specifying events; the second deals in group theoretic models which can specify systems without specifying their contents.

I commend this second kind of simulation to the Society's study. People talk about simulation as if it were a "quick and dirty" method, which can only achieve scientific respectability when a rigorous mathematics has been provided for the first kind of simulation. This attitude I take to be a slander of simulation techniques, since the second kind is more important and already has this second order rigour of its own.

Finally, may I make a comment on another matter altogether. I should like to draw attention to Dr. Youle's remark that management should call upon these methods earlier than they do, namely at the time when new plant is being designed. Academic people present may find it surprising that this needs to be said: they may regard the industrial scientist as having an automatic influence on new developments of this kind. The fact is, however, that new developments tend to be driven forward by technical enthusiasm which leaves until later the kind of problem with which operational research is competent to deal. The result is that scientists in industry often share with the general public the privilege of reading about their company's new developments in the newspapers. Later on, of course, they may become separated from the general public when they are given the specific privilege of trying to alter the system (which by now has been determined in metal) without actually changing anything. I should like to reinforce with whatever emphasis I can command Dr. Youle's remark that simulation studies can help at the design stage.

Mr. R. Brech: I should like to develop Mr. Stafford Beer's second point. It is not so much the manufacturer's responsibility to call on the statistician for help; it is for the statistician to make certain that the manufacturer will want to call for his help. I am certain that simulation can help industry very much more than it has done, both in taking decisions and in training managers to take decisions without the costly experiment of allowing a man to learn from his experience by taking wrong decisions. But the more I hear and read about the progress of simulation, the more I am convinced that the information coming from the simulated operation is not the information that managers want.

From what I have heard this evening, the information discussed by the speakers was not the type of information that would enable me as a manager to take a right decision. A manager is concerned with maximizing profits; he is juggling with revenue and costs the whole time, and all management information must be given in terms of a relationship between revenue and costs. Management is an economic function. So far the manager has acted empirically and often apparently illogically but for very good reasons. He knows, and every accountant knows, that the information that the accountant gives to the manager is often misleading, and if the manager were to work logically from that information he would invariably take a wrong decision. In his mind he assesses the wrongness of

this information, acts in what seems to be an illogical manner, and comes to a right decision.

If we are going to simulate business operations successfully, we must first construct a sound *economic* model, secondly we must get our statistics right. In the first place, we have to understand the meaning of the word "cost". It is not the irrevocable price I have paid for something; it is rather the foregone opportunity—the economic concept of cost, which in accounting terms is replacement cost. For this reason when we apply statistical techniques, we have to take the existing accounting data and reconstruct them on the basis of economic principles before we can use them.

Industry is not primarily concerned with technical efficiency; this is secondary to its concern with economic efficiency. If technical efficiency increases costs and thereby reduces total profits, it should be of no interest to industry at all. The main consideration of business is maximizing profits. I am not saying that all of the profit goes to the shareholders. Maximizing profits is a test of economic efficiency. What the manufacturer does with the increase of profits—whether it goes to shareholder or labour or consumer—is a different problem. The essential problem of simulation as far as I see it—and we have worked on it ourselves primarily on the economic side—is to make certain that the input data are consistent with the true aims of the firm and that the output data are in a form which a manager can recognize and use.

One final point on this: we are not concerned with giving the manager the maximum of information to take the right decision. We are more concerned with giving him the minimum of information. Managers nowadays have too much information. The fact that the quality of their information is poor means that they want more information to give them security and confidence. If information is based on sound economic principles and properly processed with the mathematical and statistical techniques which we have at our disposal, it is possible to streamline management information so as to give a manager the minimum quantity to enable him to take the right decision and afterwards to test the rightness of the decision.

Dr. J. M. Hammersley: The kind of mathematics used by industry in operational research is quite different in character from the mathematics taught in academic circles. The emphasis is on building mathematical models and using digital computers to by-pass the need for and the shortcomings of theoretical techniques. The answers sought are not clear-cut as they are in examination questions. The underlying philosophy and standpoint of all this is not well understood by the majority of teachers in universities and technical colleges. If these teaching institutions are to play their part in training the people industry requires, there must be more feedback to them from industry and from those who are developing and using these recent methods. The papers we have heard this evening are therefore very welcome; but I would plead that teachers need to be given, besides generalities, more details of actual examples. Without such details they have little to bite on.

Mr. W. E. Duckworth (read in his absence by Mr. Robinson): The openers of this discussion have, quite properly for a meeting of this Society, concentrated on simulation studies for industrial operations in which there is uncertainty about the results of particular activities, and they have discussed the problems of sampling from the distributions which arise in these situations.

However, when an industrial operation is sufficiently complex, even if it is completely deterministic in nature or can be assumed to be so for purposes of analysis, it is certainly well worth while and often necessary to carry out a simulation.

We have recently carried out such a simulation into the operation of a machining unit which was set up to manufacture certain products which had previously been made in a general machine shop. We knew the kind of product required and the various ranges of size and complexity of the parts. We knew also, from time studies and a considerable amount of engineering experience, the average times taken to manufacture the articles, the times taken to set up for manufacture on various machines and the number and types of

machines for the production of each part. The problem was therefore deterministic in nature because the random variations from the expected manufacturing times, etc., were so small that they could be ignored.

The question to be answered was how many machines of what type and how many operators on how many shifts would be required in order to manufacture a certain output of this kind of product with a given average throughput time. Other questions were what would be the amount of work in progress associated with certain throughput times, what would be the distribution of throughput times around the average and how would this depend on the nature of the product, and what would be the labour utilization.

The situation in the machine shop where each product could go on several machines in any order and where one product might be made on as few as two machines or as many as sixteen was so complex that ordinary queueing theory was not adequate to provide a sufficient answer. A pencil and paper simulation was tried but even here the complexity of the problem was so great that progress was extremely slow and very painful for those taking part. It was decided, therefore, to simulate the entire operation on a computer and, with the co-operation of Messrs. Ferranti Ltd. to whom we are extremely grateful, this has been done. A programme for this simulation and the results are being reported elsewhere. Several runs on the simulation have now been made and in each case they have provided adequate information to enable appropriate decisions to be made.

The machine shop itself has been in existence now for some six months and so far the results of the simulation studies have been extremely accurate in predicting the performance of the shop.

Dr. S. Vajda: We have had some extremely interesting opening contributions on various topics, and from various levels, and it is up to us to try to survey their contents. At the lowest level, as it were, we have a simulation model where the human element does not enter at all, where everything is simulated by a machine.

At the next level we have the human element, a man-machine system, but there again we can distinguish two phases: one is where man comes into the picture because there is no machine to simulate him, although if a principle is available to tell the man what he should do one could programme it on a computer. On the next higher level this is not possible. This is the case mentioned by Dr. Tocher, where the manager has to make a decision with all his preconceived ideas, and he must be allowed all his usual errors and idiosyncrasies.

The next step, mentioned by Mr. Jessop and Mr. Musk, is where you have machines playing against one another. I think this is possible without involving the human element, but I am not sure. Management is always done by people; in fact, the machine seems to come into the picture only because one wants to know quickly the result of any decision, and the machine can give the answer most quickly.

One important aspect has not been discussed so far: that of training. It is true that in the contribution from Mr. Jessop and Mr. Musk there is a paragraph in which it is mentioned, but only in connection with business training, so that the manager gets an idea of what happens if he takes a certain decision. It could be extremely valuable to train a man, not to find out what happens if he does something, but to do the thing which he is told he ought to do. I should like to know whether this question of training was not considered because none of the authors believes in it, or whether it was overlooked.

Dr. K. D. Tocher: In reply to Dr. Vajda's question, I think that ultimately simulation models will be very useful for training but, at least in my own applications, before they can be used in this way we shall have to instil in management some confidence in the value of this technique, and this will take some little time. The proposal in the last part of my written contribution is a beginning of what could, in another connection, be a training method.

I should like to say how much I agree with Mr. Beer's attitude. Our simulation experiments are all designed with exactly this black-box approach to the actual control of the

plant, and the general simulation programme I have described is simply a means by which we can give an accurate physical description of the plant upon which is imposed the complex management control structure which leads to the situation he has described.

As a result of the ballot taken during the meeting, the candidates named below were elected Fellows of the Society:

William Eric Armstrong
Mindo Rustomji Batliwalla
Cecil James Brown
Robert Thomson Buchanan
Robert Croasdale
John Mandel
Paul Meier
Ingram Olkin
Dervin Manuel Rodrigo
Joan Raup Rosenblatt
Ludwik Joseph Stafford
Norman Carman Severo
Robert Turner

Corporate Representative

William Randolph Spencer *representing* Urwick, Orr and Partners, Ltd.

THE 125TH ANNIVERSARY OF THE ROYAL STATISTICAL SOCIETY

AT the invitation of the Right Hon. the Lord Mayor of London, Sir Harold Gillett, M.C., a Dinner preceded by a reception and attended by some 350 Fellows and guests of the Society was held at the Mansion House on Tuesday, March 17th 1959, to mark the occasion of the 125th anniversary of the foundation of the Society. The President, Sir Harry Campion, C.B., C.B.E., presided. The Prime Minister, the Right Hon. Harold Macmillan, P.C., M.P., was the principal guest. The Society was glad to welcome also as its guests:

The Right Hon. the Lord Mayor, Sir Harold Gillett, M.C.
Mr. Alderman and Sheriff R. E. Perring
Mr. Sheriff J. Evan Cook
The Lord Adrian, O.M., F.R.S., Vice-Chancellor, University of Cambridge
Mr. J. M. Bannerman, O.B.E., Rector, University of Aberdeen
Sir Hugh Beaver, K.B.E., Past President, Federation of British Industries
Mr. F. A. Bishop, C.V.O., Private Secretary to the Prime Minister
Sir Maurice Bowra, F.B.A., President, The British Academy
Monsieur Gabriel Chevry, President, Société de Statistique de Paris
Sir Thomas Creed, K.B.E., M.C., Q.C., Deputy Vice-Chancellor, University of London
Professor Ely Devons, President, Manchester Statistical Society
Dr. Churchill Eisenhart, Vice-President, American Statistical Association
The Right Hon. Hugh Gaitskell, P.C., C.B.E., M.P., Leader of the Opposition
Sir William Garrett, President, British Employers' Confederation
Dr. G. Goudswaard, Hon. Secretary, International Statistical Institute
Sir Robert Hall, K.C.M.G., President, Royal Economic Society
Professor W. V. D. Hodge, F.R.S., Vice-President, The Royal Society
The Earl of Limerick, G.B.E., K.C.B., D.S.O., T.D., Chairman, Medical Research Council
Mr. James Meenan, President, Statistical and Social Inquiry Society of Ireland
Mr. F. M. Redington, President, Institute of Actuaries
Mr. J. M. Ross, President, Faculty of Actuaries
Professor Walter Schlapp, Pro Vice-Chancellor, University of Manchester
Mr. Donald Tyerman, Editor, *The Economist*

The following Past Presidents of the Society attended:

Dr. David Heron
The Lord Heyworth
Professor A. Bradford Hill, C.B.E., F.R.S.
Mr. H. Leak, C.B.E.
Professor E. S. Pearson, C.B.E.
The Lord Piercy, C.B.E.
The Right Hon. The Viscount Samuel, O.M, P.C., G.C.B., G.B.E.
The Right Hon. The Earl of Woolton, P.C., C.H.

The Lord Piercy, C.B.E., a Past President of the Society, in a short speech proposed the Toast of the Lord Mayor and Corporation and Sheriffs of the City of London. The Lord Mayor responded.

The Toast of the Royal Statistical Society was then proposed by the Prime Minister in the following speech:

"I rise to propose the Toast of the Royal Statistical Society. This is the 125th anniversary of its foundation. It is, therefore, an opportunity to acknowledge how much the Society and its Fellows have contributed by their investigations to an understanding of the economic and social problems of this country and in developing methods of scientific

analysis which can now be so effectively used in many fields of enquiry—and sometimes, even of controversy!

"Curiously enough, my wife (though no statistician herself) has a kind of inherited interest in your Society. She is a direct descendant, through her mother, of your first President in 1834, the third Lord Lansdowne. His great-great-grandfather was Sir William Petty—himself a very eminent statistician and quite a practical one, too. Your first President, Lord Lansdowne, was also quite a character, being Chancellor of the Exchequer at the early age of 26. My own story was very different. I was a back bencher for 16 years before I held any office at all. However, things came out right in the end! In his first Presidential address Lord Lansdowne urged strongly 'the need for statistics and the co-operation of private persons with the Government' in the development of statistics—something which I have said myself, not so elegantly but perhaps more dramatically, on more than one occasion in recent years. It was, therefore, almost a personal duty for me to accept your gracious invitation tonight. I am no expert but, after your excellent dinner, I nevertheless feel rather the same about statistics as Sir Winston Churchill has recorded that he once felt about mathematics. He had a feeling once about mathematics—that he saw it all. Depth beyond depth was revealed to him—the byss and the abyss. He saw—as one might see the transit of Venus or even the Lord Mayor's Show—a quantity passing through infinity and changing its sign from plus to minus. He saw exactly how it happened and why the tergiversation was inevitable—but, says Sir Winston, 'It was after dinner and I let it go.' Well, it is after dinner now so I, too, will let it go.

"But I was brought up with many of the names which appear in your Journals. My father's and grandfather's firm started not many years after the establishment of your Society and published many of the books of Fellows, for example, those of Stanley Jevons, a Secretary of your Society, and of others after him. But it has been my own deep and lifelong interest in economic and social affairs which has made me very aware of the activities of your Society. Many of the subjects you discuss bear more directly on the lives of the ordinary citizen than those with which most learned societies are concerned. The Journals of your Society provide exhaustive and penetrating studies into the pattern of our economic and social development during the last 125 years. They shed light on many of the problems encountered in the period. But you have always held fast to the principles of scientific enquiry which inspired the founders of this Society. So we find at your headquarters a forum where subjects of day to day interest are discussed in the spirit of a learned society.

"Budget day will soon be with us and this is the time of year when the output of statistics reaches its peak with financial statements, economic surveys, National Income White Papers, and all the year-end reports on production, trade, prices and finance. Indeed, without statistics there could be little useful public activity of any kind. This may sound to some like a good reason for stopping all statistics. But to do so would not, alas, bring such activity to an end. It would merely cripple almost every form of social action. It was the invention of writing that rendered the existence of great States possible by enabling the central Government to make general and permanent laws and to keep in communication with outlying districts. It was the invention of arithmetic, and in modern times of statistics, which has made possible the elaborate organization of society in which we live. One hundred and twenty-five years ago here in London this was recognized. The importance of the need of statistics was realized. Many eminent men took part in starting the

collection of statistics and in this country of spontaneous, unofficial initiative, they came together and founded a Statistical Society.

"It has been said that if Englishmen were cast away on an uninhabited island the first thing they would do. would be to constitute a committee to run the island with a chairman, treasurer and honorary secretary. This is the first thing your own Society did. And they then authorized the Secretaries of the House Committee 'to purchase a carpet, a rug, floor cloth, candlesticks, snuffers and the necessary tea apparatus'. This suggests that they had some regard for creature comforts and were not solely concerned with man as a statistical animal. Of course it is not the subject under study but the way in which the study is pursued which entitles investigators to be called 'men of science'. No doubt it is partly vanity which leads us now to suppose that the economic and social problems of our day are more complex than those of our ancestors. Certainly each generation is apt at times to scoff at the inadequate methods used by its predecessors to tackle their problems. What your founders did was to introduce the methods of science into a field previously overgrown with weeds of prejudice and to introduce a scientific, a statistical approach to the problems of the day. They realized—as their successors have not always done—that economic and social questions could not be solved by the use of rhetoric, and perhaps this feeling accounts for their rather exaggerated dislike for what they called 'opinion'. Our business, they said, is not with figures of speech but with figures of arithmetic. They put right in the forefront of their activities the necessity to collect facts about the 'Conditions and Prospects of Society'. The use—or, rather, misuse—which some people (including, alas, members of my profession) make of some of your statistics must sometimes cause you pain. You may be tempted to echo the words of an American cynic: 'Figures can't lie but liars can figure'. A kind explanation, however,—and I trust a truer one—is that which Dr. Johnson gave to the lady who asked him at dinner why in his dictionary he had defined a horse's pastern as its knee: 'Ignorance, madam, pure ignorance'.

"I am delighted to see some of the ladies who are Fellows of the Society are present here this evening. They have a distinguished lineage in this Society for Florence Nightingale was one of its first members. I understand that on the occasion of the 50th anniversary of the Society 75 years ago, at a meeting in which the International Statistical Institute was formed, she offered overseas delegates 'a room, breakfast, dinner and a place to work at any time—a better dinner with notice'. There was clearly notice tonight!

"Since the early years of your Society the progress made in the collection of facts about economic and social conditions has, as everyone acknowledges, been very great. The use made of statistics and of statistical methods, both in business and in Government, is on a scale your predecessors could not have imagined. To have any hope of carrying out their policies, the Government of the day must have knowledge of the facts as they are, together with such information as will help it to deduce future trends. It is equally essential for the Opposition to have these facts at their command, in order to draw precisely opposite deductions. But, seriously, if the Government does not have this information it would, as your past President Lord Woolton said in his own Presidential Address to the Society some years ago, 'be like asking a doctor to be responsible for getting a patient back to health but denying him the right to take his temperature, check the pulse, test the condition of his heart, or to ask questions about his past health'. I have myself ventured to urge the need to get these statistics quickly as well as accurately—so that we can keep pace with changes in economic conditions—so that, as I put it, we do not always have to be looking

up our trains in last year's *Bradshaw*. Of course, statisticians are faced with difficult decisions in determining how much detail is really necessary for each exercise. But it has always been my belief that statistics must be available quickly if they are to be of real use in guiding those concerned with policy questions. Hence the importance of 'sampling' and the 'spot check': a sample is an instrument of policy when the whole record may merely be a piece of economic history.

"In recent years, there has been a marked widening of public interest in statistics of all kinds. Even the popular Press do not altogether despise them. Even in my youth, I was a student of two important collections—*Whitaker's Almanack* and *Ruff's Guide to the Turf*. But along with this increase in public recognition of the subject, there has been, among its practitioners, a considerable widening in the range of applications of statistical techniques. Originally, the first concern of your Society was with economic and social statistics. Now, statistical methods are being used in the physical sciences, in technology, in agriculture, in medicine and in the newer branches of sociology. Every year sees some new development where statistical methods can be usefully employed. The methods you use have themselves become more exact. Your Society has done well so far in avoiding the perils of extreme specialization which may occur in scientific societies. And it has tried to ensure that methods and practices employed successfully in one field of enquiry are made known to statisticians working on other topics.

"You have not, however, made things easier for Members of Parliament like myself. Years ago we could talk easily about the average man or the man in the street. Now we must be more careful and speak about the random listener or random viewer. You have invented expressions like 'degrees of freedom', 'Markoff chains', 'queueing problems', 'Monte Carlo methods' and 'Latin squares', which do not mean what the ordinary person might think they should mean. I was going to say that it's all Russian to me, but I now know a few words of Russian, whereas I am baffled by these expressions. But one of your phrases has caught my ear. The Member of Parliament often has to deal with irate questions on some point from one of his constituents and we may sometimes regret that we cannot dismiss them, as the statistician might do, as 'extreme values'.

"It is a very great pleasure indeed for me to be asked to propose this Toast. I have had for many years to deal with Government affairs for which exact statistical data were indispensable—as well as, for a short period, with the Foreign Office where they are impossible. I welcome this opportunity for expressing my gratitude to the statisticians. I know that it is often customary to say that they are only interested in the figures and not in the human beings they count. I do not think this is true; and if tonight's gathering is a representative sample, if that is the right word, it is certainly not true of Fellows of the Royal Statistical Society. The Society can well be proud of its past. But the widening scope of its activities in recent years makes its future even more exciting and full of promise."

Sir Harry Campion, C.B. C.B.E., President of the Royal Statistical Society, replied as follows:

"On behalf of the Fellows of the Society I should like to thank you, Mr. Prime Minister, most sincerely for honouring us with your presence here this evening and for the kind way you have proposed the continued well-being of our Society. We are all very much aware of the many calls on your time. I must confess that during the last few weeks we have been wondering whether your foreign travels might prevent you from being here. I am sure Fellows greatly appreciate the fact that on the very eve of another important journey

of state starting tonight you were prepared to find the hours to come and dine with us and to speak so encouragingly to us this evening. We are very grateful to you.

"I have received many messages of good wishes to the Society on this 125th birthday. The first is from Her Majesty the Queen, the Patron of the Society to whom I had sent our loyal greetings in these words:

> 'May it please Your Majesty
>
> On the occasion of the 125th Anniversary of the Royal Statistical Society I beg leave to offer to Your Majesty, on behalf of the Fellows of the Society, their most loyal greetings, with an assurance of their continued devotion to Your Majesty.'

"Her Majesty has most graciously sent this reply dated today:

> 'I send my sincere thanks to the Fellows of the Royal Statistical Society for the kind message of loyal greetings and assurances which you have sent me on their behalf.
>
> 'As Patron of the Society, I warmly congratulate it on its 125th Anniversary and send my very best wishes for its continued prosperity in the future.—Elizabeth R.'

"Many messages of good wishes have come in from other learned Societies and from many of our Honorary and Ordinary Fellows abroad unable to be with us this evening. I can mention only a few. From old friends, Herbert Marshall and R. H. Coats in Canada; from Mahalanobis in India; from Europe: Gini, Italy; Steffensen, Denmark; Cramer, Sweden; Tinbergen, Netherlands; a message from Fréchet in Paris, 'hoping the Society will continue as successfully as in the past, benefiting not only British statistics but also the world'. From South America a message from Dieulefait in Argentina speaking of his gratitude and admiration to the British Statistical School, a feeling, he says, that every statistician should have. There are messages also from our many friends and colleagues in the United States: Neyman, California; Stuart Rice, Washington; Kuznets, Harvard; Hotelling and Miss Cox in North Carolina, and one from E. B. Wilson in Massachusetts—'The Society has done, is doing and will continue to do great work and it is a great honour for me to be counted among its Honorary Fellows'. Perhaps I might read part of a most charming letter I received from that legendary figure, Walter Willcox of Cornell. 'It would give me great pleasure to attend the dinner on March 17th, but by that time I shall have just passed my 98th birthday and although my health, fortunately, continues reasonably good, it would not be safe for me to venture across the Atlantic, much as I would rejoice to attend. Please accept best wishes for the long life and growing influence of the Society we both love'.

"Supporting me in this reply to the Prime Minister for his good wishes are the Past Presidents sitting with me at this table—Professor Pearson, Lord Piercy, Professor Bradford Hill, Lord Heyworth, Dr. Heron, Lord Woolton and Mr. Leak. Lord Beveridge had hoped to be here, Dr. Snow unfortunately did not feel fit enough to travel from the West Country, and Sir Ronald Fisher is at present in Australia. But to our great satisfaction, Lord Samuel, our oldest former President, is with us this evening. This is the first occasion we have had of publicly congratulating him on the Order of Merit recently conferred on him.

"This is essentially a family gathering. We welcome the distinguished guests present, of whom Professor Bradford Hill will speak later. All the others present are Fellows of the

Society. Heading them is the sprightly Colonel Butler, our oldest member who, at the age of 90, still attends many Ordinary meetings of the Society—at least those followed by a meeting of the Dinner Club. The present company in number here is about as large as the Society was in its early years but now the Society has about 3,000 Fellows.

"This enlarging membership of the Society reflects the widening range of applications of statistical techniques in so many fields. Looking round this room, I see Fellows who use such techniques in their work in Government, in nationalized industries, in private industry, trade, insurance and finance, in universities and research bodies, in agriculture, in medicine, in production processes in the factories, in market research, in education, and I could go on. At times I must confess I wonder whether we can keep pace with the potential openings for the application of statistical methods. During recent months, for example, I have been asked about the use of statisticians in astronomy and the physical sciences and automatic data processing, and at the other extreme how best to train historians in using statistical data in historical research.

"As a Society we must do what we can to meet the increasing demands for statistical help and now that the universities are coming to our assistance by stepping up their facilities for teaching, training and research in statistics, our task may be easier than in the past.

We are a learned society and I hope we have kept, and will keep, steadfastly to the aims set for us 125 years ago. We are given no exclusive rights in the statistics we produce or use and therefore it is by our examples that our influence is brought to bear. As our predecessors insisted, we must seek first the facts and set a standard by the thorough and objective discussion of the facts.

"Perhaps, like other societies, we take ourselves too seriously, but turning over the pages of the past Journals of the Society makes one very proud of the initiative and enterprise of our predecessors, even though some of the statistical methods they used now seem inevitably strangely outmoded. The covers of the Journals hide the beginnings of so much we take for granted nowadays ('old dawns and new horizons').

"An occasion like this—our 125th Anniversary—makes us very conscious of the traditions of this Society—very humble—yet very resolved to carry on the work others started for us. As a Society we may seem to be getting on a bit, but in terms of what we have to do, we are young and vigorous and just beginning in many fields.

"It is very kind of you, Mr. Prime Minister, to join us on this, for us, an important occasion. May I thank you again for the kind way in which you proposed the Toast to the Society."

Professor A. Bradford Hill, C.B.E., F.R.S., a Past President of the Society, proposed the Toast of the Society's guests. Sir Maurice Bowra, F.B.A., replied on behalf of the guests.

Regulations of the Local Groups and of the Industrial Applications Section*

As a result of the recommendations of the Committee on the Structure of Sections, recorded in the Annual Report of the Council for the Session 1958–59 (*Journal*, Series A, **122** Part 4, 1959), revised regulations have been prepared for the Local Groups of the Society and for the Industrial Applications Section. These regulations have been approved by the Council, and are given below in full.

Local Groups

Organization

1. The Council of the Society may form or approve the formation of a Local Group in any district in which a sufficient number of persons signify their willingness to become members of such a Local Group.

2. The Local Groups shall be constituted and carried on subject to the Bye-Laws of the Society, and in accordance with these Regulations, made and subject to amendment by the Council of the Society. The Local Groups shall further the aims and interests of the Society in parts of the United Kingdom outside the London area, and each shall be called "The . . . Group of the Royal Statistical Society". Any Local Group may, if it so desires, attach to its title a sub-title to indicate a special field of interest.

3. The activities of the Local Groups shall be arranged so that the Session shall run from 1st July to 30th June, and the Financial Year from 1st January to 31st December.

Membership

4. Any Fellow of the Society or any other person who is approved by the Local Group Committee may become a member of the Local Group on making written application to the Local Group Secretary.

5. A Fellow of the Society shall not be required to make any payment for membership of the Local Group. Every other member shall pay an annual subscription of one guinea, which shall be due in advance on 1st January, provided that any person who becomes a member and pays his initial annual subscription in the last quarter of the year shall not be required to pay any further subscription in respect of the following year. Subscriptions shall be payable direct to the Honorary Treasurer of the Royal Statistical Society, 21 Bentinck Street, London, W.1.

6. If any member of the Local Group has not paid his annual subscription by 30th June, the Secretary of the Royal Statistical Society shall advise him and the Local Group Secretary in writing of the default, and if the arrears are not paid by 30th September the defaulter shall cease to be a member.

7. Payment of an annual subscription in respect of membership of the Local Group shall not confer on the member the status or the full privileges of Fellowship of the Society.

* These regulations supercede those published in this *Journal* **109**, 59–60 (1946).

8. Members who are not in arrear with their annual subscriptions may withdraw from membership by giving written notice to the Local Group Secretary.

The Local Group Committee

9. Subject to these Regulations, all the affairs of the Local Group shall be directed by a Local Group Committee consisting of at least four members elected by the members of the Local Group.

10. (*a*) At least one of the members shall retire each year, and shall not be eligible for re-election for the period of one year. The Committee shall decide on the members to retire and in doing so shall have regard to seniority on the Committee and degree of attendance.

(*b*) The Local Group Committee shall nominate members for election to the Committee for the following session, and these nominations shall be circulated to all members of the Local Group at least a month before the date of the last ordinary meeting of the Group in the current session. Members of the Group may, if they so desire, nominate further candidates (being members of the Group) for election, in which case a ballot shall be taken at the meeting. Candidates so nominated must be proposed and seconded by members of the Group and nominations must be received in writing by the Secretary of the Group at least a week before the date of the meeting.

11. The Local Group Committee shall elect a Chairman and a Secretary from amongst its members, unless these officers have been specified at the time of the election of the Committee. These two officers shall retire after a year of office but are to be eligible for re-election.

12. The Local Group Committee shall, if so requested, permit a representative from the Council of the Society and/or a representative from any of the Sections of the Society to attend its meetings, but such representatives shall not have voting rights.

13. The Local Group Committee shall arrange for periodical meetings or conferences of the Local Group for the reading of papers, discussions or demonstrations.

14. The Local Group Committee shall send to one of the Honorary Secretaries of the Society designated by the Council (i) not later than 30th November in each year a detailed estimate of the total expenditure of the Group in the ensuing year with a request for funds to meet such expenditure, and (ii) not later than 28th February a detailed statement of the expenditure of the Group in the past financial year and a report on the activities of the Group during the current session. The Group Committee may recommend that any papers read before the Group shall be published by the Society or filed in its Library for reference.

15. Every Local Group Secretary shall send to one of the Honorary Secretaries of the Society, designated by the Council, and to every Section Secretary, a copy of every notice issued in the name of the Local Group.

The Local Groups Co-ordinating Committee

16. The Local Groups Co-ordinating Committee shall consist of:

(*a*) One representative of each Local Group, appointed by the relevant Local Group Committee;

(*b*) One Fellow appointed by the Council of the Society;

(c) One representative of each Section, appointed by the relevant Section Committee;

(d) Two members nominated by the Local Groups Co-ordinating Committee of the previous session and approved by the Council;

(e) One of the Honorary Secretaries of the Society, designated by the Council, who shall act as the Secretary of the Local Groups Co-ordinating Committee.

17. The Local Groups Co-ordinating Committee shall elect a Chairman from amongst its members. The Chairman shall retire after a year of office but is to be eligible for re-election.

18. The Local Groups Co-ordinating Committee shall meet at least once in each year to consider the organization and finances of, and any other matters concerning the Local Groups.

19. Every Local Group Committee shall, not later than 31st October, nominate one representative to be a member of the Local Groups Co-ordinating Committee.

Industrial Applications Section

Organization

1. The Section shall be constituted and carried on subject to the Bye-Laws of the Society, and in accordance with these Regulations, made and subject to amendment by the Council of the Society. The Section shall be concerned with industrial applications of statistical methods.

2. The activities of the Section shall be arranged so that the Session of the Section shall run from 1st July to 30th June, and the Financial Year from 1st January to 31st December.

Membership

3. Any Fellow of the Society, any member of a Local Group of the Society, or any other person who is approved by the Section Committee, may become a member of the Section. Application for membership of the Section by a member of a Local Group shall be made through the Secretary of the Local Group. Any other person shall make application to the Secretary of the Section Committee.

4. A Fellow of the Society, or any member of a Local Group who is not a member of another Section shall not be required to make any payment for membership of the Section. Every other member shall pay an annual subscription of one guinea, which shall be due in advance on 1st January, provided that any person who becomes a member and pays his initial annual subscription in the last quarter of any year shall not be required to pay any further subscription in respect of the following year. Subscriptions shall be payable direct to the Honorary Treasurer of the Royal Statistical Society, 21 Bentinck Street, London, W.1.

5. If any member of the Section has not paid his annual subscription by 30th June, the Secretary of the Royal Statistical Society shall advise him and the Section Secretary in writing of the default, and if the arrears are not paid by 30th September, the defaulter shall cease to be a member.

6. Payment of an annual subscription in respect of membership of the Section shall not confer on the member the status or the full privileges of Fellowship of the Society.

7. Members who are not in arrear with their annual subscriptions may withdraw from membership by giving written notice to the Section Secretary.

The Section Committee

8. Subject to these regulations, all the affairs of the Section shall be directed by a Section Committee. The term of office of this Committee shall be the session. The Committee shall consist of:

(*a*) One representative of each Local Group, appointed by the relevant Local Group Committee;
(*b*) Two Fellows appointed by the Council of the Society;
(*c*) Nine members elected by the members of the Section;
(*d*) One of the Honorary Secretaries of the Society, designated by the Council, as an *ex officio* member of the Committee.

Casual vacancies in the Section Committee shall be filled respectively (*a*) by the relevant Local Group Committee, (*b*) by the Council of the Society, and (*c*) by the remaining members of the Section Committee.

9. (*a*) Three of the nine members of the Section Committee elected by the Section shall retire annually and shall not be eligible for re-election for the period of one year. The Committee shall decide on the members to retire and in doing so shall have regard to seniority on the Committee and degree of attendance.

(*b*) The Section Committee shall nominate members for election to the Committee for the following session, and these nominations shall be circulated to all members at least a month before the date of the last ordinary meeting of the Section in the London Area in the current session. Members of the Section may, if they so desire, nominate further candidates (being members of the Section) for election, in which case a ballot shall be taken. Candidates so nominated must be proposed and seconded by members of the Section and nominations must be received in writing by the Secretary of the Section Committee at least three weeks before the date of the meeting.

10. The Section Committee shall elect a Chairman and a Secretary from amongst its members. These two officers shall retire after a year of office but are to be eligible for re-election.

11. The Section Committee shall, if so requested, permit a representative from any of the other Sections of the Society to attend its meetings, but such representatives shall not have voting rights.

12. The Section Committee may, *inter alia*, arrange periodical meetings or conferences of the Section for the reading of papers, discussions or demonstrations, particulars of which shall be circulated to the Secretaries of the other Sections and of the Local Groups. These meetings shall be open to all Fellows of the Society, members of Local Groups, and members of Sections.

13. The Section Committee shall send to one of the Honorary Secretaries of the Society designated by the Council (i) not later than 30th November in each year a detailed estimate of the total expenditure of the Section in the ensuing year with a request for funds to meet such expenditure, and (ii) not later than 28th February a detailed statement of the expenditure of the Section in the past financial year and a report on the activities of the

Section during the current session. The Section Committee may recommend that any papers read before the Section shall be published by the Society or filed in its Library for reference.

The London Meetings Sub-Committee

14. As soon as possible after the last ordinary meeting of the session the Section Committee shall appoint a London Meetings Sub-Committee to arrange and conduct the ordinary meetings of the Section in the London Area. The London Meetings Sub-Committee may include members of the Section who are not members of the Section Committee. Members of the Section may nominate members for appointment to the Sub-Committee for the following session. Candidates so nominated must be proposed and seconded by members of the Section and nominations must be received in writing by the Secretary of the Section Committee at least a week before the date of the last ordinary meeting of the Section in the London Area in the current session.

15. The Sub-Committee shall elect a Chairman and a Secretary from amongst its members. These two officers shall retire after a year of office but are to be eligible for re-election.

16. The Sub-Committee shall send to the Secretary of the Section Committee (i) not later than 31st October an estimate of the total expenditure of the Sub-Committee in the ensuing year, and (ii) not later than 31st January a statement of its expenditure in the past financial year, and an account of the meetings arranged during the current session. The Sub-Committee may recommend that any papers read at these meetings shall be published by the Society or filed in its Library for reference.

ANNUAL REPORT OF THE COUNCIL

For the FINANCIAL YEAR *ended December* 31*st*, 1958, *and for the* SESSIONAL YEAR *ended June* 17*th*, 1959, *presented at the* ONE HUNDRED AND TWENTY-FIFTH ANNUAL GENERAL MEETING *of the* ROYAL STATISTICAL SOCIETY, *held at the London School of Hygiene and Tropical Medicine, W.C.*1, *on June* 17*th*, 1959.

Number of Fellows

THE number of Fellows continued to increase during 1958, the total at the end of the calendar year reaching 2,975. Of these, 112 represented corporate bodies. The net gain for the year was somewhat smaller than of late, amounting to 86. Losses by death, resignation or default totalled 102.

The total number on the roll at April 30th, 1959, was 2,982.

The number of Honorary Fellows at the end of 1958 was 35. Of these, 7 were Presidents of other Societies.

Number of Fellows (excluding Honorary Fellows)

	Losses			
Calendar Year	*By Death*	*By Resignation or Default*	*Elected or Restored to the Roll*	*On the Roll at December 31st*
1947	14	61	263	1,836
1948	17	79	238	1,978
1949	16	87	189	2,064
1950	10	83	222	2,193
1951	21	93	228	2,307
1952	15	89	221	2,424
1953	10	91	165	2,488
1954	14	106	171	2,539
1955	19	83	182	2,619
1956	16	85	254	2,772
1957	11	98	226	2,889
1958	12	90	188	2,975

Losses by Death

The Council records with regret the loss during the year ended April 30th, 1959, of the undermentioned Fellows:

	Date of Election
Bryce, W. D., F.C.W.A.	1923
Burchardt, F. A.	1942
Cole, Professor G. D. H.	1928
Craster, Sir Edmund, M.A., D.Litt.	1947
Farrar, Marmaduke, F.	1930
Fidler, Walter	1913
Goforth, William Wallace, O.B.E., M.A.	1930
*Gooch, Sir Henry, B.A., LL.B.	1901
Graham, Harold, A.S.A.A.	1931
Gunaratna, Llewelyn Frederick, L.M.S.(Ceylon), D.P.H.(Lond.)	1951
*Hitchcock, Sir Eldred, C.B.E.	1938
*Horwitz, Israel, B.Sc.(Econ.)	1913
Jarvis, C. H. R.	1947
Leggatt, Sidney, A.C.I.S.	1948
Nisbet, J. D. P., F.F.A., A.I.A.	1925
*Norman, Frank A., O.B.E.	1912
*Pigou, Professor A. C., M.A., F.B.A.	1900
Staples, Ronald	1927
Thomson, D. Halton, O.B.E., M.A., M.I.C.E.	1943

* Life Fellow.

Dr. F. A. Burchardt, Director of the Oxford University Institute of Statistics, represented that Institute in the Society from 1942 until his death. After gaining his doctorate at Kiel for a thesis on the Theory of the Stationary State, he was engaged in teaching and research work for some years at the Kiel Institut für Weltwirtschaft. A short period as economic adviser to a Berlin Bank was followed by a lectureship at the University of Frankfurt, whence he was dismissed in 1933 for "political unreliability". After two years as Economic and Financial Editor of the *Frankfurter Zeitung*, he saw he could no longer remain in Germany. Already he had enabled others to leave by his own refusal of offers of posts abroad. All Souls College, by a small research grant, gave him the opportunity to enter on an academic career at Oxford. In 1939 he joined the staff of the Oxford University Institute of Statistics and thenceforward played a leading part in its development. During the part-time Directorship of Professor A. L. Bowley the responsibility for the daily running of the Institute fell upon Dr. Burchardt. From 1945 to 1949 he served as Deputy Director under D. G. Champernowne and was appointed Director when the latter became Professor of Statistics. He was elected Fellow of Magdalen College in 1948 and University Reader in Economic and Social Statistics in 1950, becoming one of the outstanding applied economists of his generation. His early studies had been in theoretical economics; at Oxford he turned to statistical and empirical enquiries where these might illuminate economic doctrine or contribute to economic policy. The first sample surveys of personal incomes and savings in this country were initiated by him. Post-war years gave him the opportunity to visit the States and Europe, including Germany, and he was everywhere recognized as a distinguished scholar. He had also gifts which made him a particularly successful tutor, and his unselfish devotion alike to his work and his friends gained him affection as well as respect.

Professor G. D. H. Cole was elected a Fellow of the Society in 1928 and represented the Fabian Society from 1939 to 1945. Though many of the books for which he was famous were based on the collection of economic and social facts, he was less interested in the statistical treatment of such information than in its value as a basis for economic and political theory. A prolific author, mainly on various aspects of the Labour movement (with detective stories for contrast), he was also a capable administrator in research projects and outstanding in the academic world. A Fellow of All Souls College by virtue of his office as Professor of Social and Political Theory, he was also elected to Honorary Fellowships by University College and Balliol. On retiring in 1957, he became a Research Fellow of Nuffield College with which he had been closely concerned as Chairman of the Social Reconstruction Survey.

Sir Edmund Craster, Bodley's Librarian at Oxford from 1931 to 1945, represented All Souls College in the Society from 1947. A man of wide interests, he was an historian of note. He served the Bodleian Library from 1912, when he joined the staff as a sub-Librarian specially entrusted with the completion of a summary catalogue of Western Manuscripts, until 1945. During this period he became successively Keeper of the Western Manuscripts in 1927 and Bodley's Librarian in 1931. It was under his guidance that the Report of the Bodleian Commission was successfully carried into effect, a work of great complexity. In his retirement he rendered a final service to the great Library by writing his *History of the Bodleian Library* 1845–1945.

By the death of Sir Henry Gooch at the age of 87 the Society has lost one of its elder Fellows, whose election was as long ago as 1901. Trained as a lawyer, his lifelong interest

was in education. He sat on the London School Board from 1897 to 1904 and presided over various London County Council education sub-committees. He was a member of the original Secondary and Technical Burnham Committees and of the first Cambridge University Women's Appointments Board.

Professor A. C. Pigou, elected in 1900, was one of the oldest Fellows as well as one of those with the longest membership of the Society. Succeeding Alfred Marshall in the Chair of Political Economy at Cambridge, in 1908, he held that position with distinction until his retirement under the age-limit in 1943. His first professorial publication, which in its revised form became his most important one, was his *Economics of Welfare*, in 1912. Devoting himself to the systematic exploration of one department of economic doctrine after another, he produced a series of monumental works as well as smaller ones on specialized topics and expositions at a more popular level. Strongly critical at first of Keynes's theories, which he then believed to be unwarrantable departures from Marshallian theory, he came later to regard them more favourably. Though primarily academic and by nature extremely shy, Pigou was always ready to undertake any public duty demanded of him and his services on the Cunliffe Committee on the Currency and Foreign Exchanges (1918–1919), the Royal Commission on the Income Tax (1919–1920) and the Chamberlain Committee on the Currency and Bank of England Note Issues (1924–1925) made him known far outside academic circles. He was elected to the British Academy in 1925 and was, among other honorary appointments, honorary President of the International Economic Association.

Mr. D. Halton Thomson was a past President of the Institution of Water Engineers. He was a pioneer in the application of statistics to problems of water supply and conservation.

Distinctions

It gave the Council much pleasure to convey its congratulations to Lord Samuel, a past President of the Society, on his being awarded the Order of Merit.

Honorary Fellowship

There were elected to Honorary Fellowship at the Ordinary Meeting on June 18th, 1958, the following statisticians of world-wide reputation:

Professor Simon Kuznets
Professor Jerzy Neyman
Professor J. Tinbergen

Vice-Presidents

The President appointed as Vice-Presidents of the Society for the Session 1958–59, Professor G. A. Barnard, Miss Iris Douglas, Dr. J. O. Irwin and Sir Robert Shone.

Ordinary General Meetings of the Society

The subjects of the Ordinary General Meetings during the Session have been:

1958

November 19th	H. F. Lydall. The Long-Term Trend in the Size Distribution of Income.
December 3rd	A Discussion on the Report of the Committee on the Supply of and Demand for Statisticians, opened by Professor E. S. Pearson.

1958	
December 18th	L. A. Dicks-Mireaux and J. C. R. Dow. The Determinants of Wage Inflation: United Kingdom, 1946–56.
1959	
January 21st	K. S. Lomax. Production and Productivity Movements in the U.K. since 1900.
February 18th	P. Armitage. The Comparison of Survival Curves.
March 18th	W. E. Thomson. ERNIE—A Mathematical and Statistical Analysis.
April 15th	P. A. Stone. The Economics of Housing and Urban Development.
May 20th	A Discussion on Simulation Studies of Industrial Operations, opened by Dr. P. V. Youle, Dr. K. D. Tocher, Mr. W. N. Jessop and Mr. F. I. Musk.

Research Methods Meetings of the Society

Council decided at its meeting on November 6th, 1958, that meetings organized by the Research Section at which formal papers are read should be termed Research Methods Meetings of the Society. The following such meetings were held during the session, the change of name becoming effective from the date of the January meeting:

1958	
November 13th	C. B. Winsten. Geometric Distributions in the Theory of Queues.
1959	
January 7th	Miss V. R. Cane. Behaviour Sequences as Semi-Markov Chains.
March 4th	G. A. Barnard. Control Charts and Stochastic Processes.
May 27th	J. C. Kiefer. Optimum Experimental Designs.

The Research Section

Professor G. A. Barnard was re-elected Chairman of the Research Section for the Session and Mr. A. M. Walker was elected Honorary Secretary. Other members of the Committee were: Dr. P. Armitage, Dr. D. E. Barton, Mr. J. Durbin, Mr. E. C. Fieller, Mr. M. J. R. Healy, Mr. D. V. Lindley, Mr. W. N. Jessop, Professor E. S. Pearson, Mr. A. Stuart, and Dr. S. Vajda, with Dr. D. R. Cox and Professor H. E. Daniels appointed by Council. Dr. J. O. Irwin as Chief Editor, and Dr. N. T. J. Bailey as joint Associate Editor with Dr. D. R. Cox of the *Journal*, Series B, were *ex officio* members of the Committee.

During the Session, in addition to organizing the four meetings listed in the foregoing paragraph, the Research Section held a joint Conference with the Industrial Applications Section at St. Andrews from August 22nd–25th. The Conference was attended by 134 full members and 26 associate members, including visitors from the following countries: Finland, France, Holland, Hungary, New Zealand, Norway, Portugal, and the United States.

General Applications Section (formerly Study Section)

By a decision of the Council taken at its meeting on November 6th, 1958, the Study Section is now entitled General Applications Section.

The Section Committee for the Session has been: Mr. B. P. Emmett (Chairman), Mr. G. C. C. Chivers (Honorary Secretary), Mr. J. A. Bound, Mr. W. J. Corlett, Mr. J. Downham, and Mr. L. T. Wilkins. The Council representatives were Mr. B. Benjamin and Mr. J. Durbin. Mr. H. C. Mackenzie represented the Bristol Group at meetings.

Attendances at the meetings in London averaged 32.

The following papers were presented:

1958	
October 8th	Measuring Industrial Concentration—R. W. Evely.
December 5th	Recent Developments in American Economic Statistics—J. W. Knowles (Joint meeting with the Association of Incorporated Statisticians).
December 11th	Some Antecedents of Modern Experimental Design—C. Eisenhart.
1959	
January 14th	The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom 1861–1957—A. W. Phillips.
February 11th	An Introduction to Communication Theory—C. Cherry.
March 11th	Industrial Market Research—L. A. Williams.
April 8th	A Study of Retail Distribution Statistics—J. M. Lepere.
May 13th	Some Techniques of Educational Assessment—P. E. Vernon.

The Section were the guests of the Institute of Actuaries Students' Society at Staple Inn Hall on November 7th, 1958, when Mr. L. V. Martin presented a paper on "Mortality in Middle Age".

Bristol Group

The Bristol Group Committee for the Session has been: Mr. H. C. Mackenzie (Chairman), Miss M. E. Holgate (Honorary Secretary), Miss E. H. L. Duncan, Mr. E. S. Holdsworth, and Mr. W. S. Paige.

Six meetings have been held, and the following papers read and discussed:

1958	
October 16th	"Monte Carlo" for Statisticians—W. N. Jessop.
November 20th	A Study of Hospital Waiting Lists in Cardiff (1953–54)—E. Lewis Faning and R. A. N. Hitchens.
December 11th	The Risks of Acceptance Sampling Schemes—E. D. van Rest.
1959	
January 15th	An Economic Paradoxon—S. Vajda.
February 19th	Sensory Tests of Food—A. S. C. Ehrenberg.
March 20th	The Economics of Routine Testing in a Chemical Factory—O. L. Davies.

Attendances (ranging from 12 to 23) have averaged 17.

Medical Section

The Committee of the Section during the Session has been: Dr. W. R. S. Doll (Chairman), Mr. J. A. Heady (Honorary Secretary), Mr. M. P. Curwen, Mr. D. Hewitt, Dr. A. McKenzie, Dr. D. D. Reid, Dr. C. C. Spicer, Dr. L. Stein.

The following meetings were held:

1958	
September 30th	Prevalence of Mental Defect—J. Tizard.
October 28th	The Effect of Various Factors on the Incidence of the Common Cold—O. M. Lidwell and R. E. O. Williams.
November 25th	"Appendicitis" in Young Women—J. A. H. Lee.
1959	
January 27th	The School Progress of Prematurely Born Children—J. W. B. Douglas.
February 24th	The Development of National Health Service Statistics—J. Wrigley.
March 31st	Maternal Height and Childbearing Performance—W. Z. Billewicz.
April 28th	A Discussion on Matched Pairs—I. Sutherland, L. T. Wilkins and A. N. Oppenheim.
May 26th	The Value of Periodic Health Examinations—J. C. McDonald.

Attendances averaged 38.

A Day Conference was held at Bedford College, London, on April 17th, 1959. The subject of the morning session was "Accidents". Speakers were: Dr. F. Garwood on "Child Road Accidents", Dr. J. P. Bull on "Types and Severities of Accidental Illness", and Professor G. C. Drew on "Alcohol and Accidents". The subject of the afternoon session was "Medical Genetics". Speakers were: Dr. T. C. Carter on "The Genetical Aspects of Radiation", Dr. Eliot T. O. Slater on "The Genetics of Schizophrenia", and Dr. C. O. Carter on "Genetics and Congenital Malformations".

The Industrial Applications Section

The Section Committee which co-ordinates the functions of the various Local Groups and provides the link with the Society has consisted of: (*a*) appointed by Council: Professor E. S. Pearson and Mr. E. C. Fieller; (*b*) nominated by the 1957–58 Section Committee: Mr. D. A. Hopkins, Mr. A. N. James, Mr. W. N. Jessop, Miss J. Keen, Mr. D. J. Page, Mr. W. F. Pridmore, Mr. E. D. van Rest, Mr. G. W. Sears (retiring Chairman); (*c*) representing Local Groups: Mr. A. D. Airth, Dr. C. J. Anson, Mr. D. H. S. Forbes, Mr. D. Goldberg, Mr. J. V. Gregg, Mr. W. A. Hay, Mr. I. F. Hendry, Mr. J. Hepburn, Mr. P. M. Hooper, Mr. J. E. Le Gassick, Mr. J. S. Ogilvie, Mr. A. F. Purser, Mr. A. G. Simms, Mr. N. J. Squirrell, Dr. K. D. Tocher, Mr. I. H. Trueman, Mr. J. P. L. Truesdale, Mr. A. M. Whitehouse.

Mr. W. N. Jessop was elected Chairman and Mr. D. J. Page re-elected Honorary Secretary of the Committee.

The Officers of the Local Groups were:

Group	*Chairman*	*Honorary Secretary*
Birmingham and District	Dr. C. J. Anson	Mr. D. Goldberg
Edinburgh	Mr. A. F. Purser	Mr. D. H. S. Forbes
Glasgow	Mr. J. V. Gregg	Mr. W. R. Matheson
Leicester	Mr. A. G. Simms	Mr. P. M. Hooper
London	Mr. I. F. Hendry	Mr. I. H. Trueman
Merseyside	Mr. J. P. L. Truesdale	Mr. J. S. Ogilvie
North-Eastern	Mr. N. J. Squirrell	Mr. A. D. Airth
Sheffield	Dr. K. D. Tocher	Mr. J. E. Le Gassick
South Wales	Mr. A. M. Whitehouse	Mr. W. A. Hay

End-of-year membership and average attendances at meetings of the various Groups for the year ended December 31st, 1958, were as follows, the comparative figures for 1957 being shown in brackets:

	Fellows	*Non-Fellows*	*Total Membership*	*Average Attendance*
Birmingham	41 (41)	19 (23)	60 (64)	16 (22)
Edinburgh	10 (6)	12 (15)	22 (21)	10 (16)
Glasgow	10 (8)	8 (28)	18 (36)	15 (14)
Leicester	17 (16)	24 (24)	41 (40)	16 (18)
London	185 (176)	88 (96)	273 (272)	40 (50)
Merseyside	16 (17)	28 (30)	44 (47)	16 (17)
North-Eastern	34 (30)	6 (6)	40 (36)	19 (20)
Sheffield	34 (35)	56 (41)	90 (76)	22 (27)
South Wales	12 (12)	26 (21)	38 (33)	23 (27)
	359 (341)	267 (284)	626 (625)	

Fellows who are recorded as members of the Section are those who signify their wish to be attached to a particular group for the purposes of voting in its elections and being eligible to serve on its Committee.

The accounts of the Section and its Local Groups for the calendar year 1958, excluding expenditure by the Society in relation to the work of the Section, are summarized below:

Expenditure	£ s. d.	£ s. d.	*Income*	£ s. d.
Birmingham	36 13 9		Brought forward from 1957*	284 13 1
Edinburgh	7 11 1		Grants from Council	295 0 0
Glasgow	29 9 6		Other Income	54 5 6
Leicester	15 4 3			
London	50 7 1			
Merseyside	12 15 2			
North-Eastern	28 17 8			
Sheffield	20 6 1			
South Wales	23 12 7			
Section Committee	43 4 10			
		268 2 0		
Carried forward to 1959		365 16 7		
		£633 18 7		£633 18 7

* Corrected figure including Birmingham 1957 Conference transfer.

The total expenditure of £268 2*s.* 0*d.* compares with £230 14*s.* 9*d.* in 1957.

Another successful one-day conference was held during the Session by the Birmingham Group on May 7th at the Birmingham College of Technology, the subject under discussion being "Production Control". Contributions were made by Mr. E. D. van Rest, Mr. R. H. S. Lesser, Mr. F. Campbell and Mr. H. C. Mackenzie. A proportion of the surplus was allocated to subsidizing the publication of the proceedings.

Members of the Section Committee also assisted with the planning of the St. Andrews Conference (held jointly with the Research Section), and the Manchester Conference sponsored by the Manchester Statistical Society.

The meetings listed below were held during the Session by the various Local Groups:

Birmingham and District Group

1958
October Pseudo-random Elements for Computers—E. S. Page.
November New Developments in Linear Programming—Mrs. A. H. Land.
December Statistical Inference—D. V. Lindley.

1959
January Statistical Quodlibet—M. J. Moroney.
February How Long will a Whale Keep?—H. E. Daniels.
March Management's Search for the next Improvement—P. J. Stanley.
April A Catechism for Quality Control—C. J. Anson.

Edinburgh Group

1958
October Statistics in Sales—E. Jones.
November Bias in Industrial Sampling—R. C. Tomlinson.
December Electronic Computers for Administration and Business—C. M. Berners-Lee.

1959	
January	Open Discussion Meeting.
February	Eliminating Residuals—M. R. Sampford.
March 11th	Efficiency of Machines Subject to Regular Servicing—A. J. Howie.
March 14th	(Joint Meeting with the Institute of Biology.) Statistics, Their Cause and Cure—B. Woolf; Some Statistical Aspects of Dust Counting—J. R. Ashford.
April	Statistics in the Design of Aircraft Components—W. S. Smerdon.

Glasgow Group

1958	
October	Statistics in Sales—E. Jones.
November	Evolutionary Operation—A. Baines.
1959	
February	(Joint Meeting with the Scottish Economic Society, Glasgow Branch.) The Economist and the Statistician in Industry—J. E. Calvert and J. V. Gregg.
March	Statistical Testing without Tables—W. E. Duckworth.
April	Uses of Electronic Computers—C. M. Berners-Lee.

Leicester Group

1958	
October	Elementary Statistics—A. G. Simms.
December	Unfair Queues—H. F. Downton.
1959	
January	I.T.A. Audience Research—G. R. Dowson.
February	Statistical Test on the Accuracy of Weighing—D. H. Ward.
March	A Quiz Programme: Members' Questions answered by a Panel of Experts.

London Group

1958	
October	A Statistical Problem in Geochemical Prospecting—G. M. Jenkins.
November	Experimentation in a Commercial Breeding and Hatchery Organisation—J. D. H. Archibald.
December	Inspection of a Markov Process—S. R. Broadbent.
1959	
February	Some Problems of the Actuary in Industry—C. J. Cornwall.
March	A Case History in Operational Research—K. D. Tocher.
April	Road Transport Engineering—Some Statistical Problems—W. R. Buckland.
May	Fractional Mixed Factorials—R. C. Curnow.

Merseyside Group

1958	
October	Operational Research in Industry—K. D. Tocher.
November	Statistics and Operational Research—D. R. Read.
December	The Application of Linear Programming to the Design of Animal Feeding Stuffs—A. Muir.
1959	
January	Some New Developments in Quality Control—G. A. Barnard.
February	An Introduction to the Uses of Statistics—various local speakers. (Joint Meeting with the Liverpool Section of the Institution of Production Engineers.)
March	Non-Linear Regression Analysis—R. L. Plackett.
April	Bayesian Statistics—D. V. Lindley.

North-Eastern Group

1958	
October	Unfair Queues—H. F. Downton.
November	The Problem of Small Samples in Acceptance Sampling—G. Horsnell.
December	Design of Experiments—D. R. Cox.

1959	
January	Statistical Methods in Time-Study—C. J. Anson.
February	The Provision of Pseudo-random Elements for Computers—E. S. Page.
March	Some Statistical Problems of Business Forecasting—E. Shankleman.

Sheffield Group

1958	
October	Programming Problems in the Coal Industry—M. G. Simpson.
November	A New Approach to Control Chart Theory—G. A. Barnard.
1959	
January	Problems of Air Pollution—D. Kerridge.
February	The Analysis of Replicated Experiments on an Electronic Computer—M. J. R. Healy.
March	Recent Work on Birth and Death Processes—D. G. Kendall.
April	A General Simulation Programme—K. D. Tocher.

South Wales Group

1958	
October	A Simple Application of Queueing Theory to Railway Traffic—R. A. Acton.
November	Evolutionary Operation—G. A. Coutie.
December	Some Examples of the Effective Use of Simple Statistical Techniques—C. Eisenhart.
1959	
January	Cybernetics—F. G. George.
February	The Statistical Analysis of Experiments Using a Computer—D. H. S. Rees.
March	Discussion on the Analysis of Variance, opened by K. W. Kemp.

The Society's Examinations

The Certificate and Diploma examinations on March 18th, 19th and 20th, 1959, were the last to be held. Twelve candidates entered for final parts of the Certificate and ten candidates entered for the Diploma.

Certificates have been awarded to the following Fellows:

Adams, Alec Harold
Bishop, Bernard Cyril
Chalk, Dorothy Selina
Overton, Gloria Mary
Parker, John Michael
Pugh, Derek Salman
Rigby, Lucy
Roots, John Christopher Herbert
Scregg, Sidney
Tapp, Patricia Ann

The following were successful in the Diploma examination, the specialized field on which one section of the examination was based being given in brackets:

Davis, John Roberts (Industrial Experimentation, including Quality Control)
Gent, Michael (Biological and Agricultural Statistics)
Nair, C. V. Gopinathan (Medical Statistics)
Proctor, Jean Rosemary (Biological and Agricultural Statistics) (With Distinction)
Tate, Elizabeth Anne Markham (Biological and Agricultural Statistics)
Thomas, David Anthony Hayden (Industrial Experimentation including Quality Control) (With Distinction)

The examinations in London were held at the London School of Economics. Other centres were: The College of Technology, Birmingham (for the Certificate) and the Indian Council of Agricultural Research, New Delhi (for the Diploma), and the Council is much

indebted to Dr. C. J. Anson of Birmingham and Professor A. D. Roy of New Delhi for kindly taking charge of the local arrangements. It is also grateful to those who acted as invigilators and to the Examinations Committee which consisted of Mr. J. Durbin (Chairman), Professor R. G. D. Allen, Dr. N. T. J. Bailey, Mr. B. Benjamin, Mr. W. J. Corlett, Dr. D. R. Cox, Professor H. E. Daniels, Mr. B. P. Emmett, Dr. F. G. Foster, Mr. W. N. Jessop, Dr. N. L. Johnson, Miss R. J. Maurice, Mr. F. A. A. Menzler, Dr. P. G. Moore, Mr. H. D. Patterson, and Dr. C. C. Spicer.

The Journals

During the year consideration was given to the possibility of reducing printing costs without loss of quality. A number of printing firms, including the present printer, were invited to tender for the two journals and as a consequence the Council decided, starting with the 1960 volumes, to change to another printer. It is anticipated that a substantial saving will be effected.

The stock of past journal issues and the current print orders have been kept under review by the Journal Committee and, where necessary to secure continuous series in stock, certain issues have been reprinted. The current sizes and print orders of the 1958 volumes were:

	Pages	*Print Order*
Series A	521	4,500
Series B	416	3,200

Pressure on space in both Journals continues to be acute.

The Council gratefully acknowledges the work of the Editorial Panel for the *Journal*, Series B which has consisted of: Dr. J. O. Irwin (Editor), Dr. N. T. J. Bailey and Dr. D. R. Cox (Associate Editors), Dr. P. Armitage, Professor G. A. Barnard, Professor M. S. Bartlett, Professor H. E. Daniels, Dr. O. L. Davies, Mr. E. C. Fieller, Dr. D. J. Finney, Mr. J. M. Hammersley, Dr. N. L. Johnson, Mr. D. G. Kendall, Professor M. G. Kendall, Mr. D. V. Lindley, Mr. R. L. Plackett, Dr. C. A. B. Smith, Dr. K. D. Tocher, Dr. S. Vajda, Mr. A. M. Walker, and Dr. F. Yates.

Applied Statistics

The volume of *Applied Statistics* for 1958 contained 204 pages. There was a temporary shortage of material for the June issue owing to some articles being unsuitable. The supply of articles, however, has been fairly regular, though the varying times taken by authors for revision make it difficult to maintain issues of a constant size. Articles have covered a wide range, both in type and in field of application.

The numbers of subscribers in the last three years are given in the following table:

	Fellows	*Non-Fellows*	*Total*
1956	508	1,035	1,543
1957	494	1,079	1,573
1958	536	1,172	1,708

It is gratifying to note the increase in the number of subscribers, both of Fellows and non-Fellows.

Dr. D. G. Beech has continued as Editor and Mr. H. S. Booker as Assistant Editor.

Other members of the Committee have been: Dr. N. T. J. Bailey, Mr. A. Stafford Beer, Mr. J. A. Bound, Miss S. V. Cunliffe, Mr. B. P. Emmett, Mr. R. F. Fowler, Mr. A. N. James, Mr. P. D. Oldham, Mr. E. Shankleman, Mr. P. J. Stanley, Dr. Lilli Stein, Mr. L. H. C. Tippett, and Mr. M. Whyte. The Council expresses its thanks to the Committee for their work.

The Library

The Library Committee for the Session has been: Mr. W. F. Searle (Chairman), Dr. W. R. Buckland, Miss S. V. Cunliffe, Miss M. Deane, Miss J. I. Douglas, Dr. J. O. Irwin, Mr. A. Maizels, Mr. E. Shankleman, Mr. P. H. Smith, and Mr. A. Stuart.

A satisfactory solution to the problem of storage space for the Society's historical series of foreign official statistics was found when the British Museum gladly accepted them to add to another incomplete collection already in their possession. The combined collection should prove extremely valuable to research workers. In the sense that the Librarian can direct Fellows seeking such material to the Museum, the volumes deposited there are not lost to them.

The problem presented by part of the library being temporarily accommodated away from Bentinck Street owing to shortage of space has been both acute and difficult and the Council is grateful to Mr. Searle and the other members of the Committee for achieving this solution.

After considering a report from the Library Committee on the advantages of co-operating with other libraries in order to help Fellows to obtain books on subjects just beyond or doubtfully within the scope of the Society's Library, the Council agreed that a change of policy was desirable. As a first step the Library has become an "outlier library" in the National Central Library inter-loan scheme. The advantages of the new policy will be kept under review, and it will be of interest to note the number of requests received to lend books to other libraries.

The table below, which gives an analysis of requests made by Fellows to borrow books during each of the years 1954–1958, shows that the number of volumes borrowed was slightly lower in 1958 than in 1957 but well above the figures for the earlier years. The partial closing of the Library in January, 1958, explained in last year's Report, must have affected borrowing, and may be the main reason for the slight decline in 1958.

	1954	1955	1956	1957	1958
Number of Fellows who made requests to borrow books	339	380	450	492	494
Number of volumes borrowed	1,509	2,339	2,832	3,724	3,691
Number of volumes not possessed by the Society at time of request	137	266	333	295	286
Total number of volumes asked for	1,827	2,897	3,494	4,529	4,390
Non-serial works added to the Library	695	930	719	957	922

Finance

Abstracts of the Honorary Treasurer's Accounts, viz., the Statement of Income and Expenditure for the year 1958 and the Balance Sheet as at 31st December, 1958, together with the Auditor's Report therein are in Appendices B and C.

Total expenditure rose sharply from £14,375 in 1957 to £19,122. This was due mainly to the fact that three issues of the *Journal*, Series B appeared in 1958 compared with one issue in 1957. Publication and distribution expenses of Series B rose from £1,703 to £5,592

and of Series A from £5,056 to £5,432. Larger print orders and the reprinting of some out of stock numbers of the journals also contributed to the increase in expenditure. Total expenditure, other than publication and distribution expenses of the journals, rose from £7,616 to £8,098, the principal items contributing to this rise being salaries and wages, postage and telephone, stationery and miscellaneous printing. House expenses, office and library equipment, expenses of Sections and miscellaneous expenses were all lower in 1958.

Total income rose from £16,295 in 1957 to £20,021. Fellowship subscriptions were higher at £7,699 and dividends and interest at £1,574. Receipts from sales of Series A rose by £525 to £4,602 while receipts from Journal B rose by £1,964 to £4,793. The increasing success of *Applied Statistics* was reflected in a surplus in 1958 of £613. As a result of the year's operations total income of the Society exceeded total expenditure by £899.

The balance of £899 on the Statement of Income and Expenditure was added to the Accumulated Fund which stood at £11,961 at 31st December, 1958. Life Composition Fees received during the year fell from £332 to £240. After the transfer to the Income and Expenditure Account of the fees of compounders who died during the year or were presumed to have died in previous years (£33) the Fund, maintained at the total of the Composition Fees received from Fellows still living, rose slightly from £8,215 to £8,422.

During the year it has been found possible to invest a further sum of approximately £3,500, and in selecting the securities regard has been had to the policy laid down by the Council, recorded in the Annual Report for last year. This was that, as a general rule, the funds of the Society, taken at their book values, should be invested in preference and ordinary shares (35 per cent.); debenture and loan stocks (15 per cent.); and trustee securities (50 per cent.). By the end of the year, the effect of this policy had been to raise the rate of return on the Society's investments by ½ per cent. per annum. At the same time, it will also be observed that, as a consequence of the change in investment policy, the depreciation in the market value of the Society's investments shown in previous years had, on balance, been reduced to nominal dimensions by the end of 1958.

The Council has decided that the return on the Society's investments could be further improved in future, with practical safety, by increasing the proportion invested in well-secured debenture, loan and preference stocks, and the above mentioned proportions for the main types of securities have accordingly been revised as follows: ordinary shares (35 per cent.); debenture, loan and preference stocks (25 per cent.); trustee securities (40 per cent.). The intention is to achieve these revised proportions as and when further money becomes available for investment.

As the result of a resolution adopted at a Special General Meeting held on December 18th, 1958, at the London School of Hygiene and Tropical Medicine, London, W.C.1, byelaws 5 and 5(a) were amended as indicated in the report of the meeting. The effect is to abolish the privilege of commutation for Fellows under the age of 60 years and to increase the commutation rates for those aged between 60 and 70 years. A report of the meeting will appear in the *Journal*, Series A.

During the year the Society successfully applied to the Commissioners of Inland Revenue for approval under Section 16 of the Finance Act, 1958 so that the annual subscriptions of Fellows could, in certain circumstances, be allowed income tax relief.

The Supply of and Demand for Statisticians

In April, 1956 the Council set up a Committee under the chairmanship of Professor E. S. Pearson to study the shortage of statisticians and to consider what might be done to improve recruitment. Other members of the Committee were: Professor G. A. Barnard, Sir Harry Campion, Professor D. G. Champernowne, Professor H. E. Daniels, Professor M. G. Kendall, Mr. G. W. Sears, Sir Robert Shone, and Mr. L. H. C. Tippett. This Committee reported to the Council in October last and its report was discussed at an additional meeting of the Society on December 3rd, 1958. A careers memorandum has been drafted by the Committee and will be printed for circulation to schools. The Council is indebted to Professor Pearson on whom has fallen the major burden of the work of the Committee.

Structure of Sections

In April, 1957 the Council appointed a Committee under the chairmanship of Mr. F. A. A. Menzler to "consider the views expressed by the Society's Sections, having regard, as necessary, to the repercussions on the organisation and administration of the Society, and to report to Council." Others members of the Committee were: Professor G. A. Barnard, Mr. B. B. Bonner, Miss S. V. Cunliffe, Mr. R. F. George, Professor A. Bradford Hill, Professor E. S. Pearson, Mr. G. W. Sears, and Mr. A. Stuart.

The Committee has reported on a number of occasions and it submitted its final report to the Council on February 12th, 1959. In consequence the Council has adopted the following recommendations:

(1) The statement of "Objects" prefacing the List of Fellows can be accepted as an adequate and well-balanced account of the Society's purposes and activities in the present state of statistical development.

(2) As far as possible, the major interests of the Society should be represented in the Honorary Officers and on the Executive Committee and the Council itself.

(3) In order to ensure more effective contact between the Society and the Sections and Local Groups, one of the Honorary Secretaries should be specifically charged with duties of liaison on behalf of the Council.

(4) Meetings of the Research Section at which formal papers are read should be raised to the status of meetings of the Society, the President or his representative taking the Chair, such meetings to be publicly described as "Research Methods Meetings".

(5) The Study Section should be renamed "General Applications Section".

(6) Consideration should be given to a concession to Fellows to subscribe for *Applied Statistics* at half price (15*s*. per annum) instead of two-thirds (£1 per annum) as at present.

(7) All Provincial Groups (including the Bristol Group) should be known as "(e.g. Bristol) Group of the Royal Statistical Society", it being left to each Group to decide whether to incorporate in its title a sub-title to indicate its special field of interest.

(8) A *Local Groups Co-ordinating Committee* should be set up which would undertake for all the Local Groups (including Bristol) the same functions of finance,

reporting, etc., as the Industrial Applications Section Committee has hitherto discharged for the Local Groups (other than Bristol); each Local Group to be entitled to send one representative to the Co-ordinating Committee.

(9) (*a*) London Industrial Applications meetings should be dealt with by a *London Working Committee*, only a few of whose members would be on the main Industrial Applications Section Committee.

(*b*) In the transition to the new arrangements the London working committee should in the first place be nominated by the present Industrial Applications Section Committee.

(10) London meetings of the Sections should be co-ordinated by a Standing Committee to be styled the *Co-ordinating Committee* (*London*), consisting of the Secretaries of the Research, General Applications and Medical Sections and of the working committee for Industrial Applications London meetings and the Honorary Secretary designated for liaison purposes.

(11) The finance, reporting and other business associated with meetings in London (other than those of the main Society) should be channelled through the four Section Committees to the Honorary Secretary and Council, and not through the Local Groups Co-ordinating Committee.

(12) It would be desirable for Sections to convene meetings or conferences at least once a year to which the Local Groups would be invited to send representatives, if they were interested in the subjects set down for discussion. The Local Groups could be invited to suggest subjects.

(13) Each of the four Section Committees should, if so desired, appoint a representative to attend the meetings of the other Section Committees.

(14) The General Applications Section Committee might explore the response to the formation of a new "Business and Economic Statistics Group" within the General Applications Section, which might eventually evolve into a new Section in its own right.

(15) A Papers Committee should be set up to assist the Honorary Secretaries in securing a well-balanced programme of papers. The Committee should include representatives of all the Society's Sections, who should preferably (but not necessarily) be Council Members. The Research Section Committee will organize the four meetings of the Society entitled "Research Methods Meetings".

Of these recommendations, Nos. 3, 4, 5 and 15 have already been put into effect. Dr. Armitage has been appointed the Honorary Secretary charged with liaison duties on behalf of the Council, and the Papers Committee was set up as a standing Committee last July. With regard to Recommendation No. 6, the Council decided not to reduce the concessional price of *Applied Statistics* at present.

The Council wishes to express its deep appreciation of the importance of the work of this Committee and is particularly grateful to Mr. Menzler who has directed its difficult deliberations. As a result of the Committee's recommendations the Society will be more strongly cohesive while ensuring the maximum possible expression of sectional interests. It is worth commenting that it is the very multiplication of the Society's membership and fields of activity that led to this need for a rationalization of its structure of organization.

Census of Population

The Society received an invitation from the Registrar General of England and Wales to express its views on the scope and content of the population census which is due to be held in 1961. A small committee was appointed consisting of: Mr. B. P. Emmett (Chairman), representing the General Applications Section, Dr. J. Knowelden (Medical Section), Miss S. V. Cunliffe (Industrial Applications Section), and Mr. A. Stuart (Research Section).

Their memorandum has been forwarded to the General Register Office.

Anniversary Dinner

On March 15th, 1959, the Society reached the 125th anniversary of its foundation. The event was celebrated by a dinner held, by the kind invitation of the Lord Mayor, at the Mansion House on March 17th, 1959. Some 320 Fellows and guests attended and the Society was honoured by the presence of the Prime Minister, the Right Hon. Harold Macmillan, M.P., as the principal guest. A Loyal Address was sent to Her Majesty the Queen who graciously replied with her good wishes for the future of the Society. Greetings were also received from a number of other learned societies. A fuller report of the event will appear in the *Journal*, Series A.

Accommodation

The Council again expresses its gratitude to the Dean and Board of Management of the London School of Hygiene and Tropical Medicine for their generosity in allowing the use of their lecture theatres during the Session for the meetings of the Society and of the Medical Section; and to the University of Bristol for kindly providing a room for the meetings of the Bristol Group and for the service of tea at these meetings.

The Council also wishes to thank the Director of the London School of Economics and the Principal of the Birmingham College of Technology for providing accommodation for the holding of the Society's examinations.

The Council and Officers

Sir Harry Campion has completed two consecutive years of office as President and is not therefore eligible for re-election. The Society is especially indebted to him for his distinguished service as President. His wide experience, his international reputation, his standing as the principal official statistician, and his wise judgment on all occasions have brought added dignity to the office, and have enhanced the esteem in which the Society is held.

It was with much pleasure that the Council received the consent of Sir Hugh Beaver to allow his nomination as President for the coming Session. Sir Hugh Beaver is well known as a leading industrialist and as a past President of the Federation of British Industries.

At the close of the current Session Dr. David Heron retires from the Council of which he has been an outstanding member, continuously except for only two sessions, since 1925. He was Honorary Treasurer from 1939–47 and President from 1947–49. As Honorary Treasurer Dr. Heron was instrumental in securing the services of Lord Plender as the first professional auditor to the Society, and was also responsible for introducing the present form of the accounts. In his hands the resources of the Society were scrupu-

lously husbanded and his advice on financial matters has always been greatly valued. His long experience as an Honorary Officer of the Society has been invaluable at the Council table and on important committees. Dr. Heron's sagacity, his firmness on matters of principle, his obvious enthusiasm for and loyalty to the Society, have won not only the respect but the affection of all those who have been privileged to work with him.

The other ordinary members of Council who retire at the close of the Session are: Mr. W. F. Searle, Mr. L. H. C. Tippett, Dr. N. T. J. Bailey, Dr. L. Moss and Mr. M. P. Curwen. The Council gratefully acknowledges their services.

Having observed the requirements of the bye-laws and considered suggestions put forward by Fellows, the Council recommended that the six vacancies for ordinary members be filled by the election of Professor D. G. Champernowne, Mr. Colin Clark, Miss S. V. Cunliffe, Dr. F. N. David, Miss J. Keen, and Mr. D. V. Lindley. No alternative proposals were received and the Fellows named below will be announced at the Annual General Meeting on June 17th, 1959, as having been elected President, other officers and other members of Council for the Session 1959–60. (Those whose names are marked * were not members of Council during the previous Session; those marked † have not previously served on the Council.)

President
Sir Hugh Beaver, K.B.E.

Council

G. A. Barnard
D. G. Beech
W. R. Buckland
*D. G. Champernowne
†Colin Clark
D. R. Cox
*Stella V. Cunliffe
H. E. Daniels
*†Florence N. David
Marjorie Deane
J. Durbin
E. C. Fieller
R. F. George
W. N. Jessop
J. O. Irwin
*Joan Keen
M. G. Kendall.
†D. V. Lindley
F. A. A. Menzler
W. B. Reddaway
D. D. Reid
C. T. Saunders
Sir Robert Shone
C. A. B. Smith
A. Stuart
H. Tetley

Honorary Treasurer
R. F. Fowler

Honorary Secretaries
B. Benjamin
Iris Douglas
P. Armitage

On behalf of the Council,
H. Campion, *President*.
B. Benjamin
J. Iris Douglas
P. Armitage
} *Honorary Secretaries.*

APPENDIX A

From June, 1958 to May, 1959 inclusive, the candidates named below were elected Fellows of the Society:—

Ahmad, Munir
Alltoft, Joseph Alfred Frobisher
Alltoft, Leonard William
Alonzo, Domingo Cruz
Amadi-Emina, Israel
Arbis, Lionel Wolfe
Armstrong, William Eric

Babb, William
Ball, Anthony Charles
Batliwalla, Mindo
Bennett, Arthur Alderson
Berenblut, Israel I.
Berndt, Gerald Darwin
Bradley, John Richard
Braybrooke, Trevor Stratton
Brown, Cecil James
Buchanan, Robert Thomson

Carpenter, Ronald Charles
Chandler, Mary Cecilia
Chandler, Peter
Chelsom, John Vernon
Cherriman, Vivienne Florence Laura
Childs, Peter Michael
Chung, John K.
Coghill, Christine
Cole, Dorothy Enid
Conn, Gerald David
Conolly, Brian Wallace
Cox-Johnson, Richard Mark
Croasdale, Robert
Cumming, Ian Grant

Darroch, John Newton
Dartnell, Albert William
Dicks-Mireaux, Leslie Arthur
Doherty, John Joseph
Dollar, Lavender Violet
Domb, Cyril
Dworkin, Paul David

Elce, Ivan
Emerson, John Henry

Fortis, Edward Michael
Freeman, Barbara Jane

Garforth, Joan Margaret
Garside, Michael John
Gilbert, Michael
Goddard, Laurence Stanley
Gould, Sidney
Griffin, John I.
Griffiths, Brian
Groves, Paul Stuart

Hald, Anders
Hall, Michael Graham
Harding, Hilary
Hargreaves, John Allan
Harris, Theodore Herbert
Hay, Warren Alexander
Hayes, Rosemary Susan
Haynes, Ernest Harry
Heywood, Herbert
Hodgson, Vincent
Holloway, Douglas Wynne
Horne, Edgar
Hsu, Shing Kit
Hui, Che Shing
Hunt, Peter

Ilahi, Namet
Iyer, G. Chidambara

Jackson, Harold Barry
James, John Selwyn
Johnson, Frederick John
Jones, Terence I.

Kahn, Louis B.
Kalton, Gordon Graham William
Kidd, Edwin Mark
Kitton, John Herbert

Lal, Ratan
Langner, Allan John
Lawson, Henry Ballantine
Lee, Frank
Lewis, Robert Leonard
Lofthouse, Richard Wilden
Lofthouse, Valerie
Lowe, Cecil William
Lyons, Edward Ronald

MacDonald, Martin Bedingfeld
Macdonald, Peter
Mackay, Henry Munroe
Mackie, Michael John
Mandel, John
Martin, Alison Mary
Maskell, Roy Ernest Charles
Matthews, Sidney Ivor
Meier, Paul
Mellen, Edward Farquharson
Miles, Howard George
Muller, Mervin E.
Myers, Robert Julius

Nair, C. V. Gopinathan
Newell, Richard Raymond

Olkin, Ingram

Paine, Richard Murray
Paramaswaran, K.
Parzen, Emanuel
Pask, Andrew Gordon Speedie
Peel, David Atheling
Picken, Richard Neil
Piper, Frederic Charles

Poyser, Cyril Alexander
Prichard, Terence Aubrey
Pye, Barbara Gillian

Reichmann, William John
Reid, Alexander Robert
Rimmer, Alfred Geoffrey
Roberts, Gwilym Eddfrwd
Robinson, David
Robson, James
Robson, John Bridgford
Rodrigo, Dervin Manuel
Rosenblatt, Joan Raup
Rothman, Louis James
Russell, Albert Henry

Sandeman, Peter
Sandom, Kent
Scott, Norman Wilkinson Glendinning
Severo, Norman Carman
Shields, Ronald McGregor
Shoenberg, Mark Ely
Skillin, Joyce Rosalind
Smith, Norman Down Stewart
Speak, Beatrice Mary
Sprent, Peter
Stafford, Ludwig Joseph
Stark, James Edward Douglas

Stewart, Douglas Scott
Strachan, Andrew John Webster

Tampin, Ernest
Tanburn, Jennifer Jephcott
Taylor, Philip Albert Stretton
Tilak, Divakarla Reddeyya
Tostevin, Leonard George
Truman, Anthony Barrett
Tsui, Dennis Gim Wyn
Turner, Robert

Ulyatt, Colin Vinten
Utting, John Edward George

Walker, Geoffrey Alan
Watson, George Thomas
Webb, Peter Charles
Whitaker, Ronald
Wilde, Maurice Vincent
Wilkinson, James Clifford
Williams, John Arthur Charles
Wilson, Brian Edward
Wood, Arthur Leslie

Zachariades, Phoebus Victor
Zaludová, Agnes Herbert
Zelen, Marvin

Corporate Representatives

Boggis, Frederick Daniel, *representing* International Co-operative Alliance.
Brooks, Kenneth John, *representing* International Latex Corporation.
Mooney, Peter Brian, *representing* Kemsley Newspapers.
Murray, Lionel, *representing* The Trades Union Congress.
Smith, Richard Stanley, *representing* The Library, University of Nottingham.
Spencer, William, *representing* Urwick, Orr and Partners Ltd.

APPEN

STATEMENT OF INCOME AND EXPENDITURE

1957 £	1957 £	EXPENDITURE	1958 £	1958 £
	130	Rents payable *less* rent received		130
	794	House expenses		730
	3,683	Salaries and wages (including contribution to staff superannuation scheme)		4,110
	250	Pension		300
	42	Insurance		61
	89	Office and Library equipment		5
	201	Postage and telephone		301
	558	Stationery and miscellaneous printing		660
		Publication and distribution expenses:		
5,056		Journal, Series A (General) and reprints	5,432	
1,703		Journal, Series B (Methodological) and reprints	5,592	
——	6,759		——	11,024
		Library:		
335		Books	225	
104		Binding	220	
——	439		——	445
	216	General Meetings—ordinary and annual		194
	20	Council and committee travelling expenses		28
		Expenses of Sections:		
49		Research	39	
300		Industrial Applications	302	
63		General Applications (formerly Study)	26	
6		Medical	11	
——	418		——	378
	5	Guy Medal		5
		Examination expenses:		
70		Printing, stationery, postage, etc.	27	
101		Examiners' fees	182	
——	171		——	209
	53	Auditor's fee (1957)		53
	102	Miscellaneous expenses		40
	425	Amortisation of leasehold premises		425
	20	Income tax irrecoverable		24
	14,375			19,122
	1,920	Surplus carried to Accumulated Fund		899
	16,295			20,021
	332	Amount carried to Life Composition Fee Fund		240
	£16,627			£20,261

DIX B

FOR THE YEAR ENDED 31ST DECEMBER, 1958

1957 £	£	INCOME	1958 £	£
		Annual subscriptions:		
7,299		Fellowship	7,697	
13		Research Section	13	
92		Industrial Applications Section	90	
11		General Applications Section (formerly Study Section) ...	10	
9		Medical Section	8	
	7,424			7,818
	50	Contribution from Royal Economic Society		50
	1,342	Dividends and interest (gross)		1,574
		Sales of Journal:		
4,077		Journal, Series A (General) and reprints	4,682	
2,829		Journal, Series B (Methodological) and reprints	4,793	
	6,906			9,475
	271	*Applied Statistics*—Surplus for year		613
	22	Sales of other publications		14
	192	Examination receipts: fees and sale of papers		319
	49	Surplus from Conferences		125
	39	Transfer from Life Composition Fee Fund, being fees of compounders known to have died during year and of those presumed to have died in previous years		33
	16,295			20,021
	332	Life Composition Fees received during year		240
	£16,627			£20,261

APPEN

BALANCE SHEET AT

FUNDS AND LIABILITIES

1957 £	£			1958 £	£
		Accumulated Fund:			
11,653		Balance at 31st December, 1957		11,078	
136		*Add: Bequest by Mr. Udny Yule*		—	
11,789				11,078	
720		*Less:* Loss on Sale of Investments	16		
		Past service contribution to staff superannuation			
1,911		*scheme*	—		
			—	16	
2,631					
9,158				11,062	
1,920		*Add:* Balance per Statement of Income and Expenditure ...		899	
—	*11,078*	Balance (subject to unrealised depreciation of investments *per contra*)		—	11,961
		Life Composition Fee Fund:			
7,922		Balance at 31st December, 1957		8,215	
332		*Add:* Life Composition Fees received during year		240	
8,254				8,455	
39		*Less:* Transfer to Statement of Income and Expenditure of fees of compounders known to have died during year, and of those presumed to have died in previous years		33	
—	*8,215*			—	8,422
		Building Fund:			
	1,661	Balance at 31st December, 1957			1,661
		1950 Benefaction Fund:			
	30,971	Balance at 31st December, 1957			30,971
		Frances Wood Memorial Fund:			
617		Balance at 31st December, 1957		647	
30		Income for year		30	
647				677	
—		*Less:* Prize awarded in 1958		53	624
—	*647*			—	
		Liabilities and Income held in suspense:			
3,359		Sundry creditors		3,392	
3,196		Amounts received in advance		1,646	5,038
—	*6,555*			—	
	£59,127				£58,677

REPORT OF

I have examined the above balance sheet and annexed statement of income and expenditure
31st December, 1958, and of its income and expenditure for the year ended on that date according

5, London Wall Buildings,
London, E.C.2.
15th May, 1959

DIX C

31st DECEMBER, 1958

ASSETS

1957 £	1957 £		1958 £	1958 £
		Leasehold Premises:		
21,368		At Cost	21,368	
2,553		*Less:* Amortisation to 31st December, 1958	3,084	
	18,815			18,284
		Investment of Leasehold Sinking Fund:		
	2,553	At cost, per schedule attached		3,084
		(Market Value 31/12/57, £2,190; 31/12/58, £2,889)		
		General Investments (including investment of Frances Wood Memorial Fund):		
	30,747	At cost, per schedule attached		33,657
		(Market Value 31/12/57, £24,896; 31/12/58, £33,059)		
		Current Assets:		
1,492		Debtors and payments in advance	2,052	
100		Arrears of subscriptions recoverable (estimated)	100	
196		Income tax recoverable	302	
5,224		Balances at banks and cash in hand	1,198	
	7,012			3,652
	£59,127			£58,677

NOTE.—

No value is placed in the Accounts on—

(a) Journals and other publications in stock.

(b) Books in library.

(c) Pictures, furniture and equipment.

H. CAMPION, *President*.

R. F. FOWLER, *Honorary Treasurer*.

THE AUDITOR

which, in my opinion, respectively give a true and fair view of the state of the Society's affairs as at to the best of my information and the explanations given to me and as shown by the books.

W. G. DENSEM,
Chartered Accountant.
(*Deloitte, Plender, Griffiths & Co.*)

APPENDIX D

SCHEDULE OF INVESTMENTS—31ST DECEMBER, 1958

1957 Market Value £	*1957* Cost £		Nominal	1958 Cost £	1958 Market Value £
		General Investments:			
82	*101*	3½% Conversion Loan	—	—	—
2,094	*2,992*	3% Savings Bonds 1965–75	£2,970	2,992	2,228
6,859	*8,672*	British Electricity 3½% Stock 1976–79	£8,841	8,140	6,851
1,864	*2,020*	British Electricity 4¼% Stock 1974–79	£2,246	2,020	1,977
12	*10*	Newcastle-upon-Tyne Corporation 6% Redeemable Stock 1973-76	£1,000	1,012	1,050
—	—	New Zealand 6% Stock 1976–80	£1,000	985	1,045
890	*1,059*	Bowater Paper Corporation Ltd. 5¾% Conv. Unsecured Loan Stock 1978–82 ...	£1,000	1,059	1,045
		British Petroleum Co. Ltd.			
835	*939*	5% First Debenture Stock 1974-78	£1,000	939	915
12	*10*	6% Conv. Debenture Stock 1976-80	£500	519	540
925	*980*	Metal Box Co. Ltd. 5½% Unsecured Loan Stock 1977-80	£1,000	980	955
845	*975*	Peninsular & Oriental Steam Navigation Co. Ltd. 5% Debenture Stock 1975-80	£1,000	975	905
—	—	Albert E. Reed & Co. Ltd. 6% Debenture Stock 1979–84	£500	490	522
—	—	Richardsons Westgarth & Co. Ltd. 6% Conv. Unsecured Loan Stock 1976–81 ...	£500	520	545
890	*955*	Tube Investments Ltd. 5¼% Unsecured Loan Stock 1977-82	£1,000	955	940
903	*1,012*	Ferranti Ltd. 7% Cumulative Preference Stock ...	£850	1,012	988
712	*989*	Associated Portland Cement Manufacturers Ltd. Ordinary Stock	£430	989	1,142
745	*977*	British Insulated Callender's Cables Ltd. Ordinary Stock	£370	977	962
784	*957*	English Electric Co. Ltd. Ordinary Stock	£330	957	1,011
872	*1,006*	Foreign, American & General Investments Trust Ltd. Deferred Stock	£450	1,006	1,209
878	*984*	Globe Telegraph & Trust Co. Ltd. Ordinary Stock	£525	1,021	1,365
1,222	*1,286*	Guest, Keen & Nettlefolds Ltd. Ordinary Stock ...	£676	1,286	1,960
822	*978*	Imperial Chemical Industries Ltd. Ordinary Stock	£645	978	1,226
735	*948*	International Combustion (Holdings) Ltd. Ordinary Stock	£175	948	1,072
750	*985*	Plessey Co. Ltd. Ordinary 10*s*. Shares	250	985	891
625	*922*	Pye Ltd. "A" Deferred Ordinary Stock	£250	922	750
540	*990*	Albert E. Reed & Co. Ltd. "A" Ordinary £1 Shares	400	990	965
£24,896	*£30,747*			£33,657	£33,059
		Investment of Leasehold Sinking Fund:			
£2,190	*£2,553*	British Electricity 3½% Stock 1976–79	£3,727	£3,084	£2,889

PROCEEDINGS OF THE ONE HUNDRED AND TWENTY-FIFTH ANNUAL GENERAL MEETING OF THE ROYAL STATISTICAL SOCIETY, held at the London School of Hygiene and Tropical Medicine on Wednesday, June 17th, 1959, at 5.30 p.m.

The Chair was taken by the President, Sir Harry Campion, C.B., C.B.E.

The President presented the Report of the Council for the financial year 1958 and the session 1958–59, and moved that it be received.

During his submission of the Report the President presented Certificates and Diplomas to the successful candidates in the 1959 examinations who were able to be present at the meeting.

The Honorary Treasurer, Mr. R. F. Fowler, presented the accounts and seconded the motion that the Report be received.

On being put to the meeting the Report was formally received.

On the proposal of Dr. David Heron, seconded by Mr. H. Tetley, Mr. W. G. Densem was unimously re-elected Auditor of the Society for the session 1959–60 at a fee of fifty guineas.

The President announced that as no alternative nominations had been received, the President, Council and Officers for the session 1959–60, nominated as shown on the list already circulated, were duly elected.

The meeting then terminated.

The Annual General Meeting was preceded by an Ordinary Meeting at which the following statistician was elected an Honorary Fellow of the Society:

Professor William Gemmell Cochran (United States)

The candidates named below were elected Fellows of the Society:

Peter Michael Childs
John Newton Darroch
Cyril Domb
Michael Gilbert
Paul Stuart Groves
Michael Graham Hall
Theodore Herbert Harris
Edgar Horne
Allan John Langner
Frank Lee
Valerie Lofthouse
Alison Mary Martin
Edward Farquharson Mellen
Howard George Miles
Richard Neil Picken
Barbara Gillian Pye
Louis James Rothman
Leonard George Tostevin
Dennis Gim Wyn Tsui
Ronald Whitaker
James Clifford Wilkinson
Marvin Zelen

REVIEWS OF STATISTICAL AND ECONOMIC BOOKS

CONTENTS

1.—*Statistical Methods in Biology*. By N. T. J. Bailey. London, English Universities Press Ltd. ix, 200 pp. $8\frac{1}{2}''$. 25*s*.

This book is intended, in the author's words, "to provide workers in the biological and medical sciences with an elementary account of the chief statistical methods liable to be needed in practice".

The key word in this quotation is "methods", and these are illustrated in detail, with well-chosen, interesting examples drawn from various branches of biology. The author explains in detail the circumstances for which the various methods are appropriate, in a lucid and idiomatic (indeed, at times almost "chatty") style. The field covered is the usual one for books of this type: the standard tests based on the normal distribution, up to elementary analysis of variance (randomized block designs and 2^n factorials), approximate and exact methods for the Poisson and binomial distributions and contingency tables, correlation and regression (including multivariate linear regression). A useful final chapter discusses the use of calculating machines, and the organization and accuracy of calculations.

Little or none of the underlying mathematical theory is included. This approach, adopted to avoid confusing the mathematically inexperienced reader, is legitimate, but seems at times to have been carried to extremes. For example, the binomial probability is introduced as a term in the expansion of $(p + q)^n$, and written out in terms of factorials, but the connection between the binomial coefficient and the number of ways in which a particular result can occur is not explained.

The principle of least squares estimation is nowhere stated, with the result that the "missing value" formula for a randomized block design is introduced with neither preparation nor explanation, and appears (as it so often does) as a piece of statistician's magic. The general discussion of correlation and regression is excellent, however.

Symbolism is, for the most part, standard. However, the use of c to denote covariance is an unhappy choice, particularly as c_{ij} is later used, to denote the *sum* of products of deviations of x_i and x_j. The use of s^2 to denote the estimated residual variance about a regression line, with no suffix to distinguish it from the ordinary variance estimate, may be confusing. The symbol $x!$ is introduced on p. 13, in the discussion of the binomial theorem, but is not defined until p. 63. This is not the only example of a concept being

introduced in passing some pages before its formal definition, but it is the only one that is likely to annoy biologists more than statisticians.

The use of large sample methods is stressed, in my opinion too much so. These methods are often useful, and sometimes unavoidable, but here they seem to me to be given too much precedence over exact methods: there seems little point in calling t by a different letter, and presenting its use as though it were a different test, merely because the sample size exceeds 30.

The book ends with a rather unusual feature: a section of some 40 pages is printed on blue paper, and contains, besides the index and the usual tables, a 23-page summary of the formulae introduced in the text, with a short note on each. In an attempt to avoid the danger of indiscriminate use, the author has prefixed this summary with a sort of "field guide" for the identification of problems and methods: whether this laudable attempt will succeed only experience can show.

The book is attractively printed and presented, and misprints are few (there is one on p. 118, where "than" instead of "that" causes some confusion). The price is remarkably low, in view of the number of tables and formulae included. M. R. SAMPFORD.

2.—*Elementary Statistics with Applications in Medicine and the Biological Sciences.* By Frederick E. Croxton. New York, Dover Publications; [London, Constable]; 1959. (vii), 376 pp. 8". 16*s.*

This book was first published in 1953 under the title of "Elementary Statistics with applications in Medicine". No previous study of statistics is assumed, and only a very modest knowledge of mathematics is expected from the reader. The book is lucidly written and is available at a reasonable price.

The first three chapters are devoted to a consideration of rates, ratios, percentages, tables, graphs and frequency distributions. Chapter 4 deals with measures of central tendency, the mean, median and mode, and Chapter 5 with dispersion, skewness and kurtosis. The next two chapters consider, respectively, linear correlation of two variables and non-linear and multiple correlation. Chapter 8 gives an account of the normal curve and the binomial and Poisson distribution. The reliability and significance of arithmetic means and proportions are considered in Chapters 9 and 10. Chapter 11 is concerned with the χ^2 test and Chapter 12 introduces the analysis of variance and tests for correlation co-efficients and for measures of skewness and kurtosis. As an *aide-memoir* each chapter is preceded by a complete list of symbols used in that chaper—a device which will be wel-comed by the beginner. Several useful tables are given in the appendices.

As might be expected from the title, the text is well illustrated with examples drawn from actual survey data. These are used to demonstrate the systematic way in which the calcu-lations should be made, and the results obtained are discussed in detail. The reader, therefore, is provided with a sound practical knowledge of elementary statistics. There is always the inherent danger that such an approach may lead to the indiscriminate use of statistical methods. If any criticism can be made of this book it is, perhaps, that too little stress is laid upon the need for a critical examination of the data to determine which, if any, statistical procedure is appropriate. None the less, the book can be thoroughly recom-mended; the student will obtain an informed introduction to statistics while the professional statistician will find it a useful source of reference. A. BARR.

3.—*Elementary Decision Theory.* By H. Chernoff and L. E. Moses. New York, Wiley; London, Chapman & Hall; 1959. xv, 364 pp. $9\frac{1}{4}$". 60*s.*

It seems largely a historical accident that recent developments in statistical decision theory have been carried through with such emphasis on attaining a pure mathematician's standards of maximum generality. By contrast the present book sets out to present the essentials of decision theory in an entirely elementary way. The book takes the form of a first course in statistics, the authors maintaining that all statistical problems can and should be formulated from the decision-theory point of view.

The general plan is that after a brief introduction, there are chapters on the presentation of data (histograms, means and standard deviations, and so on) and on probability and random variables. Then three chapters discuss carefully the ideas of utility, Bayes strategy and minimax strategy. The final part of the book deals with simple forms of the problems of estimation and hypothesis testing, treated of course entirely by a decision-theory approach. There are some tables and numerous appendices on more detailed mathematical points. Throughout there are plenty of exercises and examples.

The book has been very carefully and clearly written, and great pains have been taken to explain from first principles the mathematical tools that are used. Thus, not only are simple ideas of set theory explained, but also graph paper is defined (p. 20). Much of the discussion is in terms of examples, such as the following: the annual rainfall on the MacDonald nylon farm in South Phiggins is used to exemplify a histogram; the case of Mrs. Smith who, before deciding what to cook for dinner, tests her husband's mood by pretending to have lost the evening paper is used to illustrate a simple decision problem. The entertainment value of the examples is likely to have a high dispersion, both between readers and between examples within readers. My personal median was negative. At most one or two of the examples give even the slightest idea of practical statistical work.

The authors follow von Neumann and Morgenstern in assigning a personal utility to every combination of a state of nature and a course of action. They do not, however, make the closely related step of following Savage in assigning every state of nature a personal prior probability. They restrict their use of probabilities almost entirely to objective frequencies. Their approach leads them to place some emphasis on minimax strategies; they explain the drawbacks very clearly though.

Who will find this book useful? I am very sceptical indeed of the value of the book as a first course in statistics. One strong argument against it, at any rate in the circumstances prevailing in British universities, is that a student who stopped his study of statistics after one course, would be left with practically no idea at all of the statistical problems of science and technology. On the other hand a statistician who wants to get a simple, clear and interesting introduction to decision theory could not do better than to read carefully pp. 68–194, and to skip through pp. 195–297.

D. R. Cox.

4.—*Quality Control and Industrial Statistics*. By Acheson J. Duncan. Revised ed. Homewood (Ill.), Irwin. 1959. xxxiii, 946 pp. 9″, $9.

It may be worth while for statisticians to contemplate why a book of nearly one thousand pages can be written with this title. Reference to the contents list shows that only about one-third of these pages falls under the two headings to which many statisticians in this country believe quality control to be confined, namely acceptance sampling and the control chart.

The book is, in fact, a book on statistical methods with the greater number of its examples from the manufacturing field. Though the book does not fully take advantage of the fact, it is evidence of the replacement in statistical thought and reasoning of the *population* by the *process*, and of the not so recent discovery that statistical methods have much to contribute to the control and improvement of manufacturing processes. From the statistician's point of view, manufacturing processes form an ideal subject because in them variability is tolerated for economic reasons, not always from lack of knowledge; the absence of incentive to remove the variability often calls for measurement and control of variability from known sources.

This is a revised edition of a book that has already had a long and popular career. The revision has greatly lengthened it, chiefly by the addition of the 384-page section entitled "Some statistics useful in industrial research". This sets out, in a series of fourteen commendably short chapters, all the more generally used methods. Two of the chapters deal with the design of experiments and a final one deals with the "mapping of response surfaces". None of these chapters is more than 47 pages long; without the three long chapters they average 23 pages of which about two are occupied by unworked examples.

Though all the mathematical proofs are relegated to appendices (43 of them), the remaining text is freely interspersed with formulae and statements of theorems. Because of this and its great length the book is not likely to appeal to the British reader. Those who are able to cope with the mathematics will use the more thoroughgoing and general texts which are available; the others will be put off by the semi-formal treatment. The treatment is, in outline, that of a conventional text in statistical methods. When the book is used, as no doubt the author intends, as a text to supplement a fairly long lecture course these drawbacks might disappear. Some points on which the student might expect help are either summarily dismissed ("we cannot say what we mean by random") or passed over (the use of $n - 1$ as a divisor for estimating the standard deviation from a sample appears only in the account of Student's t test and might appear to contradict an earlier paragraph in which n is used, correctly).

The book is well produced, gives references at the end of each chapter as well as collected in one of the appendices, has a glossary of terms as well as, separately, of symbols. There are 34 tables. The footnote on page 5 referring to early history of the application of statistical methods to industrial processes states, incorrectly, that British Standard 600 by Professor Pearson is out of print.

E. D. VAN REST.

5.—*Probability and Statistics for Business Decisions: an Introduction to Managerial Economics Under Uncertainty.* By Robert Schlaifer. New York & London, McGraw-Hill, 1959. xii, 732 pp. Fold. diagr. 9″. 89*s.*

This very wordy book is a text for a one-year course for students of business administration and contains in its first three chapters an introduction to probability, expectation and utility. It is then divided into five parts, proceeding from a naive use of expectation to distributions, Bayes's theorem, sampling, and risk. It contains much arithmetic, though very little mathematics.

Inventory problems, queueing theory, and Monte Carlo techniques are described. There are also such expressions to be found as "pay-off", "game-tree", and so on, but without any reference to those branches of operational research where these concepts had a specific technical meaning. A chapter on "Min-Max Inventory Control", defined as a method in which the order quantity is fixed and the date of re-ordering has to be determined to be optimal, does not give any indication of why this system should be so called.

The author was developing his manuscript for about four-and-a-half years and this accounts for some unevenness of presentation. A reader must surely have some maturity to find his way through the difficulties of decision making under uncertainty, and yet he is told (pp. 66–67) that "the minus signs must not be neglected: they must be entered in the table and observed in all subsequent computations".

There are many exercises for the reader; for solutions we are referred to a "Student's Manual". The book contains a great amount of information and might be useful to teachers of statistics in search of examples.

S. VAJDA.

6.—*Operations Research: Methods and Problems.* By M. Sasieni, A. Yaspan and L. Friedman. New York, Wiley; London, Chapman & Hall; 1959. xi, 316 pp. 9″. 82*s.*

So many new books are being written on methods of operational research, that it is pertinent to ask for whom any one of them is meant. This question must also be asked by a reviewer who does not wish to judge a book by a standard which was not aimed at by its author.

"This work can serve as a textbook for a one-semester introductory techniques course, at either the graduate or the advanced undergraduate level" (p. vi), presumably at such schools as the Case Institute of Technology in Cleveland, Ohio, with which all three authors were at some time or other connected. "Much of the subject matter of this book arose out of a problems course in operations research, designed as a companion course to the general methods course offered" (p.v.). This book, then, does not contain complete case histories, but neither does it contain a systematic exposition of the mathematical

foundations of operational research techniques. It refers for proofs to other sources (mentioned at the end of the chapters), but explains clearly the procedures for solving problems of Inventory, Replacement, Waiting Lines, Competitive Strategies, Allocation, Sequencing, and Dynamic Programming—to quote the titles of Chapers 4 to 10. They are preceded by an Introduction, and by chapters on Probability and Sampling.

When one has accepted the self-imposed restrictions of the authors, one finds this a book which serves its purpose excellently. Each chapter contains some theory, a number of completely solved problems, and exercises for the reader. A wide variety of problems is tackled and anybody looking for illustrative examples, whether for the purpose of a talk, or just to see the scope of operational research techniques, will find the volume of great help. It might be useful to mention that the chapter on competitive strategies contains examples of game theory, but also of other competitive situations, and that the chapter on allocation is one on Linear Programming.

This book is a very useful addition to the shelves of any library which wants to keep up-to-date with the relevant literature on operational research. S. VAJDA.

7.—*Punched Cards: Their Applications to Science and Industry*. 2nd ed. Edited by Robert S. Casey, James W. Perry, Madeline M. Berry and Allen Kent. New York, Reinhold Publishing Corpn.; London, Chapman & Hall; 1958. x, 697 pp. 9″. 120*s.*

Many older Fellows will remember Eckart's book on the scientific applications of punched cards and the fascination of seeing the ingenuities to which one is put to use commercial punched card equipment for scientific calculation. Anyone who looks forward eagerly to any further instalments of that kind will be bitterly disappointed with this book.

The book starts, as one would expect, by describing punched card machinery, although one is puzzled by the emphasis given to hand punched cards. The succeeding parts of the book show that it is concerned with how to punch cards by hand and what coding systems to use. It is quite astonishing to find the variety of hand punched card systems now on the market and it is very unfair to give, as we tend to in England, the generic name of Cope Chatsworth to hand systems. In addition to edge punched cards, there are body punched cards and cards with holes which are enlarged to slots in both one and in two directions. There is even quite elaborate equipment for punching the holes, and in one system holes are actually drilled, using an elaborate coordinatograph. There are schemes in which a punched card record is reproduced on film and sensed by light rather than needles. There is even a punched card system with the delightful name "peek-a-boo" in which the light sensing technique is used on the cards themselves instead of the more usual needles. The first part of the book concludes with elementary considerations of coding problems.

The latter part of the book completely changes in character, and becomes a series of more or less unrelated articles on the applications of punched cards to library problems, technical indices, chemical glossaries, qualitative analyses by spectral methods, linguistic analyses and transcription service for plant reading and genetics.

Each of these applications is interesting enough in itself, but their individual authors, being specialists in their own field, have felt it necessary to add comments generalizing and theorizing about the problems of coding information. As the sum total to be said about this is quite small, the book becomes very repetitious. The theory having been repeated many times already, the third part of the book proceeds to do this formally. The principal discovery which is emphasized is that the dictionary type of classification exemplified by the Dewey decimal type of classification is very inflexible and a far wider range of classes can be generated by classifying each object according to whether it satisfies some test or not. In this way, the number of classes varies exponentially with the number of tests, and can always be expanded indefinitely by adding tests. This technique can be applied either by assigning a card to each item and coding the answers to the tests on the card, or by assigning a test to each card and coding the names of the items satisfying the test on the card. This is the basis of Zato coding, one of the main post-war developments in this field.

A method of random coding has been developed which suffers the disadvantage that in a search for an item satisfying certain conditions, items not satisfying all the conditions

may also be found. However, the simplification of this technique offsets the comparatively small secondary search through the selected items which is necessary.

The book concludes by looking forward to the future developments in search processes, using data processing machines. In spite of the surplus of books on this subject, it comes as a relief after hand punched cards.

K. D. TOCHER.

8.—*Handbook of Automation, Computation and Control*. Vol. 1. *Control Fundamentals*. Edited by Eugene M. Grabbe, Simon Ramo and Dean E. Wooldridge. New York, Wiley; London, Chapman & Hall; 1958. Various paging. 9″. 136*s*.

This amazing book is the first of three volumes which are being produced under the editorship of Grabbe and Wolldridge; if the other two reach the standard of the first, these will make a major technical encyclopaedia.

This volume concerns fundamentals and is divided into five parts: General Mathematics, Numerical Analysis, Operations Research, Information Theory and Transmission, and Feedback Control.

The first 200 pages are devoted to a statement of useful mathematics and cover set theory, the theory of algebraic equations, finite difference equations, differential equations, integral equations, matrix theory, complex variables, the theory of operators and in particular of Laplace transforms, conformal mapping and Boolean algebra. In such a sweeping enterprise, it is hardly surprising to find that probability theory is condensed to 18 pages and statistics to 17. Numerical Analysis receives the same condensed treatment in 88 pages.

By the time one reaches the third section, on Operations Research, one has become accustomed to the mode of treatment. After a very sound introduction about motivation of operational research and mathematical models and their relation to scientific method, the section discusses inventory models, allocation models, queueing theory, linear programming and the theory of games and rounds off with a discussion of the problems of implementation. The section on Information Theory is rather shorter, but is written in the same terse style.

It is rather interesting to see how, when we reach the last section of the book, on Feedback Control (which occupies nearly half the book), the terse style relaxes somewhat and reads rather more like the many other American books on automatic control which are now available.

One cannot but admire the energy and enthusiasm which must have gone into compressing such an amount of material between two covers, but it is very difficult to see what purpose such compression serves. I doubt if anybody could read this book without learning something he did not know, and I equally doubt if anybody would ever read enough to discover what it was he did not know. It will certainly be dangerous to try and learn any of these subjects from a handbook of this kind and, if it is used to act as a cook book, it should be frowned upon, because of the well known danger of employing unskilled cooks. However, for those who disagree with this verdict about the value of cook books, this is a very good buy.

K. D. TOCHER.

9.—*Strategy and Market Structure: Competition, Oligopoly, and the Theory of Games*. By Martin Shubik. New York, Wiley; London, Chapman & Hall; 1959. xix, 387 pp. 9″. 64*s*.

It has often been asked whether the theory of games could provide any guidance for either the theoretical or the practical economist. Dr. Shubik's book gives an answer. Here we have, for the first time, a unifying approach to that part of economic theory which deals with monopolistic, oligopolistic, and pure competition. The first and third of these are understood as limiting cases of the second, and the latter can be treated as an *n*-person (in duopoly: two-person) game. It becomes also clear why we must appeal to the more recondite parts of game theory for a more than trivial introduction to the intricacies of market structure.

The two parts of the book, of seven and six chapters respectively, deal with the static theory (under the title : The Background to Competition), and with The Dynamics of Oligopoly: Mathematical Institutional Economics.

For the static theory, the appropriate model is the non-zero-sum game, either co-operative with its coalitions and side payments, or non-co-operative. This supplies the concept of an equilibrium point solution, already implied in some of the classical equilibrium theories of price formation, in particular in the Cournot duopoly analysis. In the second part, the model is that of an "economic game of survival". This is a "semi-co-operative" model of a succession of games, the pay-off being a discounted value of the results of the several games, together with a bonus for survival. The role of asset structure, financial factors and corporate features of a firm are studied in this light in Chapter 10, by far the longest (65 pages) of the book.

Chapter 8, the first in Part Two, investigates the information conditions of a market, in particular incomplete information, i.e. a situation where not only the opponents' strategies, but even the "rules of the game" are partly unknown.

Throughout the text we find comparisons with and a critique of various economic theories of competition (Bertrand, Cournot, Edgeworth, Chamberlin, Mrs. Robinson, and others) from the point of view of the new theory. The whole volume is eminently readable. The necessary mathematics is painlessly introduced where needed and the claim that this approach adds unification and deeper understanding to the body of earlier theories is sustained. In the reviewer's opinion Martin Shubik has started a new epoch in economic theory.

A few misprints might be listed: References 1 and 2 on p. 179 (correct: Ricardo, von Neumann) and on p. 373 (correct: Mathematical Psychics, as on p. 57), the footnote references 9–20 on pp. 169–173 should be to 8–19, and in the graph on p. 194 the figure 7 should be replaced by 9.

There seems to be something vague about the description of behaviour strategies on p. 196, but otherwise the game theoretic descriptions are precise and clear. The book must be recommended to economists, operational research workers, mathematicians, and even, as the author claims, lawyers (Chapter 13 is on Economic Analysis, Social Policy, and the Law). S. VAJDA.

10.—*A Time Series Analysis of Interindustry Demands.* By K. J. Arrow and M. Hoffenberg. Amsterdam, North Holland Publishing Co., 1959. [ix], 292 pp. $8\frac{3}{4}''$. 55*s*. (Contributions to Economic Analysis XVII.)

This book presents the results of research on a dynamic version of the Leontief input-output matrix. Interest lay in determining whether plausible estimates of the input-output coefficients could be obtained for different years, using time series data, and whether those estimates provide more satisfactory predictions of output in each industry group than the fixed coefficients model.

The model adopted assumes that input-output coefficients are linear functions of variables which "explain" the effect of changes in technology, and in the relative importance of different products within each industry group: the so called product-mix. The starting point of the investigation was a 66×66 input-output matrix obtained from the results of the 1947 interindustry relations study of the U.S. Bureau of Labour. For various reasons 48 of these industries were considered unsuitable for analysis, and of the remainder only four were finally analysed. The residual deviations between estimates of output from the fixed coefficients model and actual values for each industry in each year, were related linearly to composite variables; the coefficients of which were identical with those in the "explanatory" functions mentioned above. These parameters were estimated by linear programming techniques and a variant of Theil's method of two round estimation. For the latter the composite variables and the residual were treated as dependent variables in a larger system of simultaneous equations.

In the course of the study a number of problems were met which resulted in change of plans and a curtailment of the scope of the study. A major problem was the lack of suitable

data on all variables in the model. A frequent response to this problem, and the one adopted here, is to compile proxy data to represent the measurements needed. These might be described as imperfect copies of images of reality. The authors rightly express doubt on the usefulness of such data in a study of this kind ; especially when there is incomplete knowledge of the primary sources employed. It is evidence of the importance given to this problem that over half the book is devoted to an explanation of the time series and input-output matrix used. This information is, undoubtedly, useful in assessing the results of the study and as an aid to future workers in the same field ; although some may jib at the price asked for it.

It was discovered after the inversion was begun that the product matrix for the 19 predetermined variables selected was nearly singular. Multicollinearity among economic variables is such a well known phenomenon that, with so many, it might have been expected, and the matrix examined for singularity before starting the calculations. The authors suggest that the rank of the matrix is probably six or seven. Nevertheless, they decided to use twelve predetermined variables in the final analysis in order that as many as four industries should have identifiable structural parameters. Understandably, the authors have little confidence in the point estimates obtained.

This work is essentially an exploratory study and it would be unwise to expect too much from the results. Indeed the authors claim no more than that the evidence suggests the existence of some model "explaining" changes in input-output coefficients, although the model that they have adopted is not the one. The latter statement can be accepted, although it must be remembered that only four industries were analysed. The former statement appears to be based mainly on the values of the multiple correlation coefficients for the reduced form equations of the residuals, which are mostly about 0·8 or 0·9. This might be indicative of a true structural relation, but is not a complete proof. The existence of an underlying structural relation would have been more evident if predictions outside the period of study had been found to be noticeably better than ones obtained from a fixed coefficients model. It is a pity that this was not done. The good fit over the period of study provides no evidence of the predictive value of the model as is claimed by the authors.

The authors have given a full description of the development of their research and explain frankly the difficulties encountered. Future research workers using similar methods should benefit from the lessons learnt in this study.

P. R. Fisk.

11.—*Social Theories of Fertility and the Malthusian Debate.* By D. E. C. Eversley. Oxford, Clarendon Press, 1959. ix, 313 pp. $8\frac{1}{2}''$. 35*s.*

"Malthus stands out as a man who scorned all statistics, and never even made use of the four census reports which came out in his lifetime." The author passes this judgment, quite appropriately, upon his main focus of interest. He could have said much the same of some of the other writers with whom he is concerned. Population theorists have often tended not to be statistically-minded, either because no data were available in their day or because such information as had been published did not seem to be particularly relevant to their thesis. This is not, then, a statistical book and it contains no figures at all—not even the results of inquiries in which direct attempts have been made to ascertain what factors influence the size of families. Such efforts as have been made to base theory on scientifically observed facts are thus, by implication, discounted and, indeed, it is stated that in the matter of reasons why one should not enlarge one's family "very little is so far known".

The Malthusian debate could hardly have continued until the present day, it is true, if statistical arguments had been capable of proving conclusively that one view or another was correct. Nevertheless, the data that have been collected from time to time, and the accounts of the findings made from them, must have influenced the development of theory in considerable degree. Indeed, it is probably true also that theory has influenced the choice of the type of information that has been sought in census and registration procedures and in sample inquiries. An interesting account could be written of the inter-relationships of fact

and fancy. By divorcing the history of ideas about population from their material background, the author has denied himself the chance of telling the complete story.

Mr. Eversley discusses the contention that social theory should be appraised for its correspondence with facts and predictive powers, but he comes to the conclusion that this requirement asks too much. "One looks instead for intelligent and consistent explanations of observed phenomena, for internally consistent hypotheses, and for a statement of remaining gaps and uncertainties which so far prevent prediction". It is on this basis that he examines the history of writings on population, starting in the classical period and ending mainly in the nineteenth century. He says relatively little about present-day opinions, in part, no doubt, because today theorising has largely given place to active fact-seeking.

Within the framework that he has chosen, however, Mr. Eversley has written a well-constructed and readable work. He shows that there is a fascination in logically analysing population philosophies, and he demonstrates his capability for finding gaps and flaws in the arguments of the principal protagonists in the Malthusian debate. His treatise starts by tracing the origins of ideas—often buried deep in antiquity and superstition—about population and fertility and the influence upon them of poverty, wealth and social position. Views about the direct and indirect effects of particular elements in social and political life were succeeded by more generalized theories, and attention is devoted to these in the central part of the book, including the author's own appraisal of Malthus. So far as modern writings are considered, interest is directed chiefly towards Marxist ideas and in particular Coontz's recent presentation of them in his *Population Theory and the Economic Interpretation.*

The concluding chapter, in which theories are classified and finally weighed up, makes a strong ending to the book. Mr. Eversley has gathered together an astonishing variety of notions: some interested persons have thought (before censuses began) that population declines, others that it grows; some have argued in favour of growth as a good thing, others have condemned it; some have believed in natural checks, others in the need for special action; and so on. Much of this is self-cancelling, but one thing at least seems to stand out. There is "a consensus of modern opinion that the decline of fertility in Western countries in the last century has been caused by developments which we can summarize under the headings of material conditions, political institutions, and social organization, or more traditionally, the standard of living, democracy, and civilization." But "we must now ask ourselves", he says, "does this widely-held opinion amount to anything at all? In other words, is this only a convenient first approximation to an explanation of past behaviour, or is this the core of a population theory as such? . . . Reluctantly, the historian must at this stage decide that he is not competent to judge these present issues". After such an outcome, one imagines that general philosophies of population, in spite of their intellectual attractions, will continue to follow many other former aspects of philosophy in yielding place to empirical and more specific analyses.

P. R. Cox.

12.—*Smoking: The Cancer Controversy. Some Attempts to Assess the Evidence.* By Sir Ronald A. Fisher, Sc.D., F.R.S. Edinburgh and London, Oliver & Boyd, 1959. 47 pp. 8¾". 2*s.* 6*d.*

In this pamphlet Sir Ronald Fisher has collected together his published criticisms of the hypothesis that cancer of the lung is caused by cigarette smoking. He has also added an essay on the philosophical foundations of statistical inference and a hitherto unpublished analysis of Doll & Bradford Hill's original data on the incidence of lung cancer in those who do and do not inhale.

The production of this pamphlet is agreeable, the price is small, and it should be read by every statistician who takes the social function of his profession seriously. One hopes the opposite side will produce a similar summary of their case.

To a medical statistician the controversy is strongly reminiscent of that which centred around smallpox vaccination at the beginning of this century. In both cases much of the evidence cited is statistical and difficult to interpret and the biological background is not

fully understood; and there is the same conflict between the public health worker seeking any means to eradicate a dangerous disease and the more academic mind which would support no strong measures of control until the pathology of the condition was clear.

Professor Fisher does not attempt any single explanation of the observed association between smoking and lung cancer. He is mainly concerned to point out the anomalies and limitations of the present findings and to provide alternative explanations of some of them. The main criticism he makes is that rigorous statistical argument can be based only on a properly randomized and replicated experiment and he devotes a special chapter to the philosophy of this type of inference. He also insists that the statistical studies that have followed the original papers of Hammond & Horn, and Doll & Hill, are corroborative and throw no fresh light on the existence or otherwise of a causal relation between smoking and lung cancer.

This would, I think, be admitted by those who believe in a causal relationship. There is evidence of the same kind for other diseases, notably coronary thrombosis, but there is much less willingness to accept a causal relationship with smoking for them since it is either much easier to see alternative explanations, or much more difficult to visualize a direct connection.

The main strength of the statistical evidence that smoking causes cancer of the lung comes from the quantitative relationship between the number of cigarettes smoked and the liability to develop cancer. Fisher does not advance any detailed criticism of this except to cite the curious anomaly that, in Doll & Hill's original survey, smokers who inhaled were about 10 per cent. less liable to lung cancer than those who did not, and the fact that no quantitative relation exists with the amount of cigar or pipe tobacco smoked.

Neither of these two objections is fatal to the hypothesis of a causal connection between cigarette smoking and lung cancer, though they certainly imply that there are factors in the relationship that are not properly understood and are worth special study.

Other factors that Fisher puts forward for consideration concern the existence of a hereditary disposition to become a smoker, and the greater effect of a given level of smoking in the town than in the country. He suggests that the predisposition to lung cancer and to excessive smoking may both be aspects of the same hereditary make-up and not linked as cause and effect, but he admits that this would not explain the large increases in lung cancer in the last 30 years. He points to the mixing of the rural and urban populations in recent years as a possible factor of this increase.

All these arguments seem to me to carry some weight though they provide no satisfactory alternative to the hypothesis that cigarette smoking directly causes lung cancer. However, in view of them, Sir Ronald Fisher fears that there is danger of advocating an erroneous theory based on statistical arguments which may bring discredit on the science of statistics in medicine.

I am not sure that this is right nor that we are justified in doing nothing if it is right. Many branches of science, both pure and applied, can be allowed to follow a natural line of development in relation to the knowledge and techniques of their time. In medicine, on the other hand, one is confronted all the time with problems that demand, in human terms, some attempt at a solution regardless of its theoretical feasibility.

The advance of medicine in the past has been a painful process of trial and error in which people have again and again accepted the risk of making mistakes to introduce a new idea of possible usefulness. In treatment and prophylaxis we now have, largely through the efforts of statisticians, rigorous and objective methods of research. No one need make mistakes provided he observes the rules. But in the investigation of the causes of disease such methods are rarely available and we are driven back on doing the best we can in the light of what we know.

It is difficult to see how, on the evidence, any physician could ignore the possibility that cigarette smoking causes cancer of the lung, and it seems entirely reasonable to take some pains to put the evidence before the public. However, smokers cling fiercely to their habit enormous financial interests are involved for the tobacco manufactuerers and the state, and on the other side many people have strong puritanical objections to smoking. If any-

thing is to be done to stop people smoking cigarettes it must overcome a habit of addiction and powerful vested interests. An attitude of uncommitted scientific detachment is utterly ineffective as a basis for propaganda and Sir Ronald Fisher maintains that the facts as they stand do not justify a campaign of the violence required. It is for the reader to decide, on the basis of this pamphlet, and the other evidence available, whether or not he agrees, and whether the campaign in this country is being prosecuted more, or less, violently than is justifiable.

There appear to be one or two misprints in the tables on pages 46 and 47 of the pamphlet. If I have followed Sir Ronald Fisher's reasoning correctly, the expected values at 5–14 and >49 cigarettes *per diem* in Table 2 should read 139·2 and 31·3 respectively. In Table 3 the "reduced deficiency" at 5–14 *per diem* should be −0·947. The sampling variance of the reduced deficiency at >49 cigarettes *per diem* also seems to be in error.

C. C. SPICER.

13.—*Tobacco Manufacturers' Standing Committee Research Papers, No.* 1 *Statistics of Smoking*. 2nd ed., Edited by G. F. Todd. London, 1959. 81 pp. 13″. No price.

The second edition of this publication represents a complete revision of the first edition which was published in 1957. Not only have the various tables in the first edition been brought up to date, but a number of additional tables have been included. The paper consists almost entirely of tables with a minimum of explanatory notes and a complete absence of comment; the reader is left to draw his own conclusions from the data presented. The aim has been solely to present a detailed picture of the pattern of smoking in the United Kingdom as well as a more general comparison of tobacco consumption in various countries.

Part 1 is concerned with the United Kingdom. The first five tables give figures of the total amount of tobacco consumed in various forms in each year from 1870 onwards. Then come seventeen further tables based, for the most part, on a 1958 survey using quota sampling in which about 10,000 persons were interviewed. From this and other surveys estimates have been made of the total numbers smoking different types of tobacco goods, the proportion of smokers in the various age groups, consumption levels including variations between urban and rural districts, the incidence of inhaling and the ages at which men and women began to smoke. The two final tables in this part relate to smoking among boys and young men aged 12 to 19, and are based on the results of some 1,200 interviews. The editor remarks that sample surveys are likely to produce under-estimates of individual tobacco consumption. He has therefore adjusted the figures in various tables so that they will be consistent with the actual total United Kingdom consumption. It would have been more satisfactory if the actual figures obtained from the surveys had been given and a note had been appended setting out the extent of the differences from the national figures; at the very least the magnitude of the adjustments should have been stated.

Part 2 comprises tables of tobacco consumption year by year for some thirty countries, in most cases from 1920 onwards; they bring out some interesting differences in smoking habits between various nationalities. It will prove most useful to all those studying various aspects of smoking to have all this information from a variety of sources brought together in one publication.

LESLIE V. MARTIN.

14.—*Economics for Pleasure*. By G. L. S. Shackle. Cambridge, University Press, 1959. x, 269 pp. 8½″. 21*s*.

The reviewer is not an economist and therefore regarded himself as a good subject for a self-controlled clinical trial of "sinister pleasure". For those who have an interest in the world around them this book undoubtedly can be (and was) read with enjoyment. The dust cover claims that "in good plain English, the essence of economics is laid bare". There must be some hesitancy over accepting the "plain" English, for sometimes the words are too many and too long, but the rest of the claim is undoubtedly justified.

The book ranges over the whole stuff of our daily lives—value, production, income, distribution, employment, finance, government and trade. The plan is first to give a full

(mostly pleasant but sometimes laboured) exposition of a particular concept and then to give a shorter summary of the same idea. Undoubtedly this is a good method of lecturing. It may not be quite such a good method of writing a book for it presents the tantalizing choice between reading for pleasure (and profit) and the quicker satisfaction of demand for information. This perhaps is a hazard only for the impatient.

Statisticians who stray into the field of economic statistics without the advantage of formal instruction in economics will find this book a valuable introduction to the basic concepts. The ordinary man who may often criticize economics as an inexact science will get a better understanding of what the economist, by his system of abstract models, is trying to do. The title is incorrect. Your reviewer read the book not "for" but "with" pleasure, and so it is hoped will many others.

B. BENJAMIN.

STATISTICAL NOTES

(1) British Official Statistics

The index of retail prices compiled by the Ministry of Labour, which was 109 in August 1959 (prices at 17th January, 1956 = 100) remained unchanged in September and October. Calculated to one place of decimals the figures were 109·3, 108·7 and 109·2. The detailed figures of the weights used in calculating the index and the indices for different commodity groups were as follows:

	Food	*Alco-holic Drinks*	*Tobacco*	*Housing*	*Fuel and Light*	*Durable House-hold Goods*	*Clothing and Foot-wear*	*Trans-port and Vehicles*	*Miscel-laneous*	*Services*	*All Items*
Weights:	350	71	80	87	55	66	106	68	59	58	1,000
Aug. 18th, 1959	108·1	98·1	107·8	128·5	111·4	97·9	102·5	114·8	113·5	116·5	109·3
Sept. 15th, 1959	106·1	98·1	108·2	128·7	111·7	97·9	102·8	114·9	113·6	116·6	108·7
Oct. 13th, 1959	107·4	98·0	108·2	128·8	112·2	97·8	102·9	115·0	113·7	116·8	109·2

The Ministry of Labour index of weekly wage rates, calculated on the basis of January 31st, 1956 = 100, showed that in August, 1959 the level was 117·1. It rose to 117·2 in September and to 117·3 in October. In manufacturing industries alone the figures were 116·6 in August and 116·9 in September and October.

The total working population and the numbers in civil employment in the three months ended September, 1959, were as follows:

(*Thousands*)

Date	*Total Working Population*			*Numbers in Civil Employment*		
	Males	*Females*	*Total*	*Males*	*Females*	*Total*
July, 1959	16,078	7,932	24,010	15,245	7,809	23,054
August, 1959	16,138	7,987	24,125	15,304	7,860	23,164
September, 1959	16,145	8,011	24,156	15,318	7,884	23,202

The numbers of persons on the unemployment registers of the Employment Exchanges fell by 21,900 in September and rose by 14,000 in October. The total for October represented 1·9 per cent. of the number of employees in Great Britain. The percentages in the different Regions ranged from 1·1 in London and the South-East and the Midlands to 3·4 in Wales and 4·0 in Scotland.

The following is the sex analysis of the figures:

Numbers of Unemployed Persons on the Registers of Employment Exchanges

Date	*Men and Boys*	*Women and Girls*	*Total*
August 17th, 1959	307,366	119,596	426,962
September 14th, 1959	292,525	112,538	405,063
October 12th, 1959	301,522	117,514	419,036

Of the total of 419,036 in October, 93,594 had been unemployed for not more than two weeks, 105,873 for 2 to 8 weeks and 208,034 for over 8 weeks, while 11,535 were temporarily stopped. In the four weeks ended October 7th, 1959, 158,913 vacancies were filled by the Employment Exchanges and the number unfilled at that date was 246,497.

In the week ended September 26th it is estimated that in manufacturing industries 38,100 manual workers were on short time.

The number of insured workers absent from work owing to illness, including self-employed as well as employed persons, was 806,500 on one day in August, 835,600 in September and 897,000 in October. The numbers of employed persons absent owing to industrial injuries were 58,600, 64,200 and 65,600.

A review appears in the November, 1959 issue of the *Ministry of Labour Gazette* of the number, membership, transactions, etc. of Co-operative Societies in 1958. There were 1,015 retail Societies with a membership of nearly 12½ million. Sales amounted to £976 million and salaries and wages to £124 million. Total assets were £460 million. The number of employees in retail distribution was 215,354. Wholesale Societies numbered 166 with sales of £600 million. Assets amounted to £214 million. These figures for both the retail and wholesale Societies cover both their distributive and productive activities. On the production sides of the retail and wholesale Societies there were 97,645 employees and the wholesale value of production was £325 million.

(2) Other Statistics

The *World Economic Survey* 1958 (U.N., 298 pp., 22*s.* 6*d.*) is in two parts; Part I focuses attention on international commodity problems and policies and Part II examines recent events in the world economy and gives the outlook for 1959. The Survey treats separately, industrial countries, primary exporting countries and those with centrally planned economies. These parts are preceded by an Introduction which considers the implications of the fears of inflation for the economic growth of industrial countries and examines the significance of the trends in these countries for commodity trade and international balance.

The Survey points out that recent developments have emphasized the vulnerability of under-developed countries to even moderate shifts in world economic activity, that the centrally planned economies continued their high rates of industrial expansion while experiencing a shortage of primary commodities and that the under-utilization of equipment and labour seems likely to persist for some time especially if fears of price increases lead to new restrictions on demand.

International Trade 1957–58 (G.A.T.T., Geneva, 1959, 220 pp., $2) contains a comprehensive survey of recent developments in the structure and pattern of foreign trade in 1957 and 1958 followed by a chapter on recent commercial policy and one on the activities of the contracting parties to the General Agreement on Trade and Tariffs. A general table gives index numbers of the total value, total volume, and unit value of world exports. Volume of exports is estimated to be about 170 in 1957, 165 in the first half of 1958 and 171 in the second half (1938 = 100). Appendix tables show the network of international trade by eleven regions (of which the principal are the North Americas, the European sterling area, the European Economic Community area, the overseas sterling area, East Europe and China, Latin America, and Japan) and the percentage share of major industrial countries in various export markets.

The *International Flow of Private Capital* 1956–58 (U.N., New York, 1959, 109 pp., 5*s.*) is an attempt to survey private foreign investments and assess their changes in recent years. These movements are first examined within the content of international payments and reserve positions in relation to official financing. The available information on private capital movements and the financing of the current external position in 1955 to 1958 are given in an annex and shows how capital movements, both public and private

have in recent years financed current surpluses and defects in international payments. "The most significant development of recent years", the report states, "is the fact that west European countries are again becoming important suppliers of "private capital"."

The second issue of the *Yearbook of National Accounts Statistics*, 1958 (U.N. New York, 1959, 257 pp., 18*s*.) contains analyses of the national accounts of 78 countries (70 in the first issue). A summary table gives for these countries the principal aggregates and their relation to one another at current prices, viz. gross national product, gross domestic product at market prices and at factor cost, net domestic product at factor cost, and national income. Introductory chapters define these concepts of national accounts and give definitions of the terms used.

CURRENT NOTE

Readers of this *Journal* will notice a number of changes in its appearance and contents which are being introduced with the first issue of 1960. Changes are to be made in the *Journal* cover and in the style of printing for Series B as well as for Series A, and it is hoped that these will improve the legibility and general appearance of the *Journal*.

A change in the editorial arrangements for Series A is to be made at the same time. Until now the editing has been one of the tasks of the Honorary Secretaries, but with the growth in the activities of the Society, Council has decided to make independent arrangements for the editing of Series A. Beginning with the first issue of 1960, Series A will have its own Editor (Mr. C. A. Moser), two Assistant Editors (Dr. P. Armitage and Miss S. V. Cunliffe) and an Editorial Board (Professor D. G. Champernowne, Miss M. Deane Professor E. Grebenik, Dr. M. R. Healy, Mr. J. L. Nicholson, Mr. C. T. Saunders and Dr. S. Vajda). Some familiar features of the Journal are likely to disappear or to undergo changes, the aim being to make more space available for contributed papers. The Editor will be glad to have papers submitted to him for publication, or to correspond with authors about papers they are working on or planning. Papers should be kept reasonably short and should in any case not exceed about 12,000 words.

STATISTICAL AND ECONOMIC ARTICLES IN RECENT PERIODICALS

UNITED KINGDOM—

Annals of Human Genetics—

Vol. 23 (1959), Part 3—On the effect of maternal age and parity on the birth weight of the offspring (Indian infants); *N. K. Namboodiri* and *V. Balakrishnan.* Non-random mating, and its effect on the rate of approach to homozygosity; *G. A. Watterson.* A new genetic population model, and its approach to homozygosity; *G. A. Watterson.* Some comments on Schützenberger's analysis of data on the human sex ratio; *A. W. F. Edwards.* A note on the effects of method of ascertainment on segregation ratios; *C. A. B. Smith.*

British Journal of Preventive and Social Medicine—

Vol. 13 (1959), No. 2—Incidence of disease and disability in elderly men; *F. Edwards, T. McKeown* and *A. G. W. Whitfield.* Contributions and demands of elderly men; *F. Edwards, T. McKeown* and *A. G. W. Whitfield.* Incapacity for work among the insured population of Great Britain; *G. M. Jones.*

British Journal of Statistical Psychology—

Vol. 11 (1958), Part 1—Estimation in factor analysis under various initial assumptions; *D. N. Lawley.* The relation between coefficients of correlation and difficulty factors; *H. F. Dingman.* Definition and scientific method in psychology; *C. Burt.*

Vol. 11 (1958), Part 2—Scientific method in psychology; *T. R. Willis, C. Burt, W. L. Gregory, G. W. Pilkington* and *J. G. Taylor.* The inheritance of intelligence and its social implications; *J. Conway.* The rescaling of teachers' estimates; *A. E. G. Pilliner.* The conversion of scores to a uniform scale; *M. Howard.*

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Vol. 21 (1959), No. 2—Finance of the nationalized industries; *D. L. Munby.* The growth of manufacturing firms; *H. F. Lydall.* Can the level of unemployment explain changes in wages? *K. G. J. C. Knowles* and *C. B. Winsten.* Inexpensive nutrition before and after the war; *T. Schulz.*

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Vol. 2 (1959), No. 2—A simulation of melting shop operations; *R. Neate* and *W. J. Dacey.* A Monte Carlo simulation of a production planning problem; *F. I. Musk.*

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Vol. 69 (1959), No. 274—Asset revaluation and terms-of-trade effects in the framework of the national accounts; *G. Stuvel.*

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Vol. 9 (1959), No. 3—The operating characteristic of attribute sampling schemes; *D. J. Farlie.* Sampling by variables; *A. N. James.* Tendering for materials of uncertain quality; *J. Keen* and *E. A. Fielding.* Statistics as an aid to management in engineering workshops; *A. G. Pearce.*

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Vol. 13 (1959), No. 3—Seasonality and elasticity of the demand for food in Great Britain since derationing; *J. A. C. Brown.* A theoretical approach to the profit maximization problems in farm management; *G. E. Clarke* and *I. G. Simpson.* Some further results of production function analysis of the farm management survey data; *R. W. M. Johnson.* Economic structure of the food trades in the United Kingdom; *I. O. H. Lepper.*

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Vol. 85 (1959), No. 369—Provision for old age: the future development of the national insurance scheme; [discussion]. A note on the measurement of digital preference in age recordings; *N. H. Carrier.*

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Vol. 27 (1959), No. 2—World exports of manufactures: 1956 vs. 1937; *S. Spiegelglas*. The periodicity and stability of inventory cycles in the U.S.A. Part I; *D. J. Coppock*. Employment fluctuations, labour supply and bargaining power; *H. A. Turner*. A monthly index of wage-rates by industries; *J. R. Crossley*.

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Vol. 55 (1959), Part 3—Cumulants of some distributions arising from a two-state Markoff chain; *P. V. Krishna Iyer* and *N. S. Shakuntala*.

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Vol. 26 (1959), No. 71—Cardinal utility for even-chance mixtures of pairs of sure prospects; *G. Debreu*.

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Vol. 35 (1959), No. 70—The Australian economy, February 1959; *G. G. Firth* and *A. J. Hagger*. The Australian male work force; *H. P. Brown*. Depreciation and stock appreciation adjustments in the national income accounts; *J. McB. Grant* and *R. L. Mathews*.

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Vol. 8 (1959), No. 32—On an extension of Stein's two-sample procedure to the multi-normal problem; *S. K. Chatterjee*. Construction of index numbers of foreign trade: some comments; *J. B. D. Derksen*. On some simplification in the construction of similar regions; *K. C. Chanda*. On conditions for the forms of the type XAX' to be distributed independently or to obey Wishart distribution; *C. G. Khatri*.

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Vol. 38 (1959), No. 4—Equilibrium delay distribution for one channel with constant holding time, Poisson input and random service; *P. J. Burke.*

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Vol. 15 (1959), No. 2—A simple method for constructing orthogonal polynomials when the independent variable is unequally spaced; *D. S. Robson.* The estimation of environmental and genetic trends from records subject to culling; *C. R. Henderson, O. Kempthorne, S. R. Searle* and *C. M. von Krosigk.* Experimental design in the evaluation of genetic parameters; *A. Robertson.* A distribution-free asymptotic method of estimating, testing and setting confidence limits for heritability; *L. Schwartz* and *S. Wearden.* The regression analysis of causal paths; *M. E. Turner* and *C. D. Stevens.* A class of two replicate incomplete block designs; *J. Roy.* The centric systematic area-sample treated as a random sample; *A. Milne.* Sensory item sorting; *N. T. Gridgeman.* Orthogonal contrasts in slope ratio investigations; *P. J. Claringbold.*

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Vol. 27 (1959), No. 2—A complete scheme for computing all direct and cross demand elasticities in a model with many sectors; *R. Frisch.* The validity of cross-sectionally estimated behavior equations in time series applications; *E. Kuh.* A model of seasonal inventories; *M. J. Brennan.* Simultaneous equations and canonical correlation theory; *J. W. Hooper.*

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Vol. 5 (1959), No. 4—Competitive coefficients; *L. Friedman.* Production lines and internal storage: a review; *E. Koenigsberg.* The use of an economic lot range in scheduling production; *M. J. Solomon.* Some analysis of purchasing policy; *W. T. Morris.* A simple allocation problem; *C. Derman.* Increasing the capacity of a network: the parametric budget problem; *D. R. Fulkerson.*

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Vol. 13 (1959), No. 66—On the inversion of certain matrices; *S. Schechter.* A sieve method for factoring numbers of the form $n^2 + 1$; *D. Shanks.* Numerical quadrature of Fourier transform integrals. II; *H. Hurwitz, Jr., R. A. Pfeiffer* and *P. F. Zweifel.* Note on bivariate linear interpolation for analytic functions; *W. Gautschi.* Gaussian quadrature of some integrals involving Airy functions; *J. A. Riley* and *C. Billings.*
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Vol. 7 (1959), No. 3—A report on some organizational experiments; *D. F. Clark* and *R. L. Ackoff.* Coefficient perturbation of a constrained extremum; *T. L. Saaty.* Inference from tests with continuously increasing stress; *W. R. Allen.* An operations-research evaluation technique of the use of sales-research information; *G. K. Chacko.* On optimum target assignments; *G. G. den Broeder, Jr., R. E. Ellison* and *L. Emerling.* Parametric programming and the primal-dual algorithm; *J. E. Kelley, Jr.* On the allocation of effort

among deterrent systems; *J. M. Dobbie.* Traffic analysis of small telephone switchboards; *J. C. Coe.* Dynamics of two classes of continuous-review inventory systems; *H. P. Galliher, P. M. Morse* and *M. Simond.*

Vol. 7 (1959), No. 4—A comparison of maximum-seeking methods; *S. H. Brooks.* Stationary single-server queuing processes with a finite number of sources; *G. Harrison.* An optimum policy for detecting a fault in a complex system; *B. Gluss.* Optimal inventory location and control in production and distribution networks; *F. Hanssmann.* Car-following theory of steady-state traffic flow; *D. C. Gazis, R. Herman* and *R. B. Potts.*

Psychometrika—

Vol. 24 (1959), No. 3—Item selection procedures for item variables with a known factor structure; *G. Elfving, R. Sitgreaves* and *H. Solomon.* Maximum likelihood estimates of item parameters using the logistic function; *A. E. Maxwell.* Three multivariate models: factor analysis, latent structure analysis and latent profile analysis; *W. A. Gibson.* Strategies and learning models; *S. J. Bryant* and *J. G. Marica.*

Quarterly Journal of Economics—

Vol. 73 (1959), No. 3—Waiting and the period of production; *R. Dorfman.*

Review of Economics and Statistics—

Vol. 41 (1959), No. 3—For the abandonment of symmetry in game theory; *T. C. Schelling.* The utilization of agricultural land: a theoretical and empirical inquiry; *J. M. Henderson.* On concepts and measures of changes in productivity; *L. L. Pasinetti.* The relationship between tangible investment and consumer saving; *E. Scott Maynes.*

Technometrics—

Vol. 1 (1959), No. 2—Measurements made by matching with known standards; *W. J. Youden, W. S. Connor* and *N. C. Severo.* Random balance experimentation; *F. E. Satterthwaite.* The application of random balance designs; *T. A. Budne.* Discussion on the papers of Messrs. Satterthwaite and Budne; *W. J. Youden, O. Kempthorne, J. W. Tukey, G. E. P. Box* and *J. S. Hunter.* Quick analysis methods for random balance screening experiments; *F. J. Anscombe.*

BELGIUM—

Annales de Sciences Economiques Appliquées—

Vol. 17 (1959), No. 2—L'entreprise de travaux publics devant le programme de construction d'autoroutes; *R. Latinne.*

Bulletin de l'Institut de Recherches Economiques et Sociales—

Vol. 25 (1959), No. 4—La Belgique en 1958.

FRANCE—

Journal de la Société de Statistique de Paris—

Vol. 100 (1959), Nos. 4-5-6—Une enquête par sondage sur les dépenses d'entretien de la propriété bâtie; *J. de Portzamparc.* L'investissement en France; *P. Cauboue.* Méthodes d'analyse des observations par "tout ou rien"; *E. Morice.* L'analyse hiérarchique; *J. Desabie.*

Revue d'Economie Politique—

Vol. 69 (1959), Nos. 4-5—La France économique en 1958.

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Vol. 7 (1959), No. 2—Un programme destiné à contrôler et à assurer la qualité des produits; *J. J. Riordan* and *J. A. Greenwood.* Contrôle de réception quantitatif ou par mesures: *Tables military standard 414, sampling procedures and tables for inspection by variables for percent defective; P. Férignac.* Relation entre loi de Poisson et loi de χ^2. La qualité des fournitures de produits ouvrés et d'accessoires dans une usine d'assemblage; *P. Durlach.* L'analyse des expériences répétées sur calculatrice électronique; *F. Yates, M. J. R. Healy* and *S. Lipton.* Equipement d'une station de pompage; *J. Delcourt.* Le test des intervalles intermaximaux; *R. Meigniez.* Etude et comparaison des niveaux de vie des travail-

leurs des industries de la Communauté Européenne du Charbon et de l'Acier; *R. Wagenführ*. Les plans d'échantillonnage: un moyen pratique d'utilisation; *M. Costesèque* and *G. Dubasque*.

GERMANY—

Allgemeines statistisches Archiv—

Vol. 43 (1959), Heft 1—Die erste Volkszählung im Sudan, 1955/56; *R. H. Rotach*. Gedanken über den Brauch und Missbrauch der Statistik in der Politik; *W. Maier*. Masszahlen in der Wanderungsstatistik; *K. Schwarz*. Einfluss der Kinderzahl auf das Lebensniveau der Familien; *H. Schmucker*. Ein Stichprobenverfahren zur Auswahl von Haushaltungen; *W. Heyn*.

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Vol. 2 (1959), Heft 2—Ein einfacher Beweis für das dem χ^2-Verfahren zugrundliegende Theorem; *J. Rybarz*. A problem of maximum likelihood estimation from a grouped sample; *G. Kulldorff*. Über die Parameterschätzung bei Kontingenztafeln, deren Randsummen vorgegeben sind (I); *K. Weichselberger*. Regressions- und Kausalanalyse in der Biologie; *R. Wette*. Techniques for discriminant analysis with discrete variables; *H. Linhart*. Der maximale Kontingenzkoeffizient im Falle nichtquadratischer Kontingenztafeln; *K. Pawlik*.

Weltwirtschaftliches Archiv—

Vol. 82 (1959), Heft 2—Entrepreneurial typology; *F. Redlich*. Zur empirischen Bestimmung der Einkommensabhängigkeit der Nachfrage im Bereich des privaten Verbrauchs; *H. Schmucker*. Produktivitätsmessung auf der Grundlage von Input-Output-Tabellen; *A. Stobbe*.

HUNGARY—

Demográfia—

Vol. 2 (1959), Nos. 2–3 (Papers in Hungarian, with Russian and English summaries)—Principles of the 1960 Czechoslovak census of population and housing; *F. Fajfr*. Some problems of fertility, family planning and birth control: on basis of the experimental processing of the first phase of a sampling enquiry; *G. Acsádi* and *A. Klinger*. The demegraphic basis of manpower analysis; *J. Tamásy*.

ITALY—

Annali della Facoltà di Economia e Commercio, Università di Palermo—

Vol. 13 (1959), No. 1—Il metodo statistico nella rilevazione dei fenomeni sociali: recenti contributi a talune tecniche di campionamento; *F. Galantino*. Di alcuni indici non parametrici; *C. Ferreri*. Sulla stima dei parametri della logistica semplice; *G. Lunetta*.

Atti della Associazione Italiana per il Controllo della Qualità—

No. 8 (1959)—Come si realizzano in un'azienda moderna i rapporti fra compratore e venditore; *L. Bono*. Il controllo statistico della qualità nelle filature del cotone: controllo del titolo medio al filatoio; *S. Firrao*. Su di una regola per controllare statisticamente un processo continuo di produzione; *P. Fungo* and *A. Longo*. Come diffondere la statistica nell'industria; *U. T.*

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No. 7–8 (1958)—A special class of stochastic problems; *F. Grabiel*. Application case example in O.R.; *W. A. Lipowec*. Un metodo diretto per la soluzione di sistemi lineari e l'inversione di matrici, particolarmente adatto per macchine calcolatrici superautomatiche da tavolo; *G. Pozzi* and *A. Gazzano*.

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Vol. 18 (1959), Nos. 1–2—Il coefficiente di correlazione lineare multipla generalizzato; *G. Panizzon*.

Statistica—

Vol. 19 (1959), No. 2—Possibilità di applicazione della "analisi discriminante" ai problemi della clinica; *G. Barbensi*. Estimating the parameters of a censored or truncated dis-

tribution; *D. P. Banerjee.* Il metodo di Montecarlo e le sue applicazioni nella statistica industriale; *G. Cusimano.* Le variazioni della fecondità italiana dal 1936 al 1951; *P. Zangarelli.* Bibliografie: E. Alanís Patiño, M. Boldrini, A. L. Bowley, C. Bresciani-Turroni, M. da Silva Rodriguez, P. Fortunati, L. Henry, A. Loveday, J. Rueff, J. R. H. Shaul and A. Tizzano.

JAPAN—

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Vol. 10 (1959), No. 2—On a method for generalizations of Tchebycheff's inequality; *K. Isii.* Note on linear programming; *Y. Suzuki.* Acceptance inspection by variables when the measurements are subject to error; *H. T. David, E. A. Fay* and *J. E. Walsh.* Note on the numerical computation in the discrimination problem; *T. Uematu.* Stochastic methods of solving partial integro-differential equations and their application to non-stationary Markoff process. I; *T. Yokota.* On the evaluation of the risk index of the railroad crossing; *H. Aoyama.*

Vol. 10 (1959), No. 3—The extreme value of the generalized distances of the individual points in the multivariate normal sample; *M. Siotani.* Orthogonal polynomials without constant term; *M. Sibuya* and *T. Haga.* Large sample non-parametric rejection of outlying observations; *J. E. Walsh.* On the statistical control of the gap process; *H. Akaike.* On the analysis of epidemic model. I. (Theoretical approach); *S. Sakino* and *C. Hayashi.* Multiple confidence procedures; *M. Dwass.* Some factors related to social mobility in Japan; *K. Odaka* and *S. Nisihira.*

Reports of Statistical Application Research, Union of Japanese Scientists and Engineers—

Vol. 6 (1959), No. 2—Inequalities for Mills' ratio; *A. V. Boyd.* On cyclic difference sets which generate orthogonal arrays; *M. Masuyama.* Notes on statistical applications of information theory. IV; *M. Sakaguchi.* A probabilistic approach to economic maintenance; *S. Senju.* An illustration of experimental formula for simultaneous determination; *M. Iwanaga* and *R. Onishi.* A significance test for the fraction defective using the compressed limit; *Y. Akao.*

NETHERLANDS—

Statistica neerlandica—

Vol. 13 (1959), No. 2—Produktiviteitsnummer.

SPAIN—

Estadística Española—

No. 3 (1959)—Ciencia económica y realidad (el problema de las fuentes del conocimiento económico); *E. Fuentes Quintana.* Actividades de los estadísticos; *S. Ríos.* Sobre los potenciales de vida; *M. García Alvarez.*

Trabajos de Estadística—

Vol. 10 (1959), No. 1—Matriz de silla en juegos rectangulares; *P. Zoroa.* Haemocytometer counts and occupancy theory; *D. E. Barton* and *F. N. David.* Special methods of inventory by sampling if the population sets have approximately negative exponential distribution; *J. Talacko.* Teorías científicas y modelos matemáticos de los fenómenos naturales; *S. Ríos.* Ensayo para un proyecto de política estadística; *F. Norte Ramón.*

SWEDEN—

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Vol. 41 (1958), Häft 3–4—Non-parametric mean and variance estimation from truncated data; *J. E. Walsh.* On the distribution of the extreme values in a sample from a normal distribution; *G. Borenius.* Dirichlet transforms applied to interest functions; *A. Pistoia.* The distribution of the Student ratio in the case of serially correlated normal variables; *C. Weibull.* Some sampling problems when a stratification variable follows a logarithmic normal distribution; *C.-E. Quensel.* Numerical considerations for the stratification of variables following a logarithmic normal distribution; *E. Block.*

Statistisk Tidskrift—

Vol. 8 (1959), No. 9—The use of an EDB-machine in the production of statistics; *S. Nordbotten* (Swedish text, English summary).

SWITZERLAND—

Schweizerische Zeitschrift für Volkswirtschaft und Statistik—

Vol. 95 (1959), No. 2—Wohnungsbedarf und Wohnungsproduktion in Vergangenheit und Zukunft; *W. Bickel.*

U.S.S.R.—

Teoriya Veroyatnosteĭ i ee Primeneniya—

Vol. 4 (1959), No. 2—Transient phenomena in branching stochastic processes; *B. A. Sevast'yanov* (Russian text, English summary). On the transformations of random variables; *L. N. Bol'shev* (Russian text, English summary). General theorems on the factorization of infinitely divisible laws. III; *Yu. V. Linnik* (Russian text, English summary). On lateral conditions for multi-dimensional diffusion processes; *A. D. Wentzell* (Russian text, English summary). Some limit theorems for random functions. I; *V. A. Volkonskiĭ* and *Yu. A. Rozanov* (Russian text, English summary). Continuous one-dimensional Markov processes and additive functionals derived from them; *V. A. Volkonskiĭ* (Russian text, English summary). An extremal problem in probability theory; *Yu. V. Prokhorov* (Russian text, English summary). Some remarks on the strong law of large numbers; *Yu. V. Prokhorov* (Russian text, English summary). Asymptotic expansions for distributions of sums of independent random variables; *V. V. Petrov* (Russian text, English summary). On a class of limit theorems for independent random variables; *V. V. Petrov* (Russian text, English summary). On exact estimation in a local limit theorem; *S. Kh. Sirazhdinov* (Russian text, English summary).

Vol. 4 (1959), No. 3—The entropy of an automorphism of a solenoidal group; *L. M. Abramov* (Russian text, English summary). Probability distributions on bicompact topological groups; *B. M. Kloss* (Russian text, English summary). Spectral analysis of abstract functions; *Yu. A. Rozanov* (Russian text, English summary). An information theoretic proof of the central limit theorem on Lindeberg conditions; *Yu. V. Linnik* (Russian text, English summary). On bivariate stationary processes and the factorization of matrix-valued functions; *P. Masani* and *N. Wiener.* On positive solutions of the equation $Au + Vu = 0$; *R. Z. Khas'minskiĭ* (Russian text, English summary). Sur le calcul des semi-invariants; *V. P. Leonov* and *A. N. Shiryaev* (Russian text, French summary). Über eine Eigenschaft des nicht-singulären Diffusionsprozesses; *I. V. Girsanov* (Russian text, German summary). On a statistical estimate for the entropy of a sequence of independent random variables; *G. P. Basharin* (Russian text, English summary).

INTERNATIONAL—

International Monetary Fund Staff Papers—

Vol. 7 (1959), No. 1—Statistical estimates of elasticities and propensities in international trade: a survey of published studies; *Hang Sheng Cheng.*

Review of the International Statistical Institute—

Vol. 26 (1958),, Nos. 1–3—A case study of interdependent versus causal chain systems; *H. O. A. Wold.* Sur la portée pratique de la convergence presque certaine; *M. Fréchet.* Consumer price indexes and demand functions; *A. A. Konüs.* The extension of Wald's method of fitting straight lines to multiple regression; *J. W. Hooper* and *H. Theil.* An experiment in the theory of a census with control rounds; *A. Y. Boyarsky.*

W.H.O. Chronicle—

Vol. 13 (1959), No. 6—International work in health statistics, 1948–1958. 3. Securing international comparability.

LIST OF ADDITIONS TO THE LIBRARY

Since the issue of Part 3, 1959, the Society has received the publications enumerated below.

I.—OFFICIAL PUBLICATIONS

United Kingdom. *Board of Trade.* Migration of Commonwealth citizens between the United Kingdom and other countries by the long sea routes in 1952. [London, 1953.] [9]p. 11¼″. 2nd issue.

The report on the censuses of production for 1955, 1956 and 1957. London, H.M.S.O., 1959. viii, 101p. 11″. 14*s.*

Report on the census of distribution and other services, 1957. London, H.M.S.O., 1959. 83p. 12″. 5*s.*

Central Statistical Office. New contributions to economic statistics; reprinted from *Economic trends, 1957–1958.* London, H.M.S.O., 1959. ii, 53p. 11″. 4*s.* 6*d.*

General Register Office. Matters of life and death. [4th ed.] London, H.M.S.O., 1958. 28p. 8½″. 1*s.* 6*d.*

Meteorological Office. Tables of temperature, relative humidity and precipitation for the world. Part II. Central and South America, the West Indies and Bermuda. London, H.M.S.O., 1958. viii, 53p. maps. 9¾″. 5*s.* 6*d.* (M.O. 617b)

Ministry of Health. Medical records and secretarial services. London, H.M.S.O., 1959. 32p. 8½″. 2*s.* (Hosp. O. & M. Serv. Rep. 2)

Standards of normal weight in infancy. London, H.M.S.O., 1959. iv, 35p. 9¾″. 2*s.* 6*d.* (Rep. Publ. Hlth. Med. Subj. 99)

Ministry of Housing and Local Government. Report of the Committee on the Rating of Charities and Kindred Bodies. London, H.M.S.O., 1959. iv, 86p. 9¾″. 5*s.* (Cmnd. 831)

Ministry of Transport and Civil Aviation. The British Transport Commission: re-appraisal of the plan for the modernization and re-equipment of British Railways. London, H.M.S.O., 1959. ii, 52p. 9¾″. 3*s.* (Cmnd. 813)

Report of the Committee on London Roads. London, H.M.S.O., 1959. iv, 38p. 3 fold. maps (2 in pocket). 9¾″. 4*s.* (Cmnd. 812)

The transport of goods by road: report of a sample survey made in April, 1958. London, H.M.S.O., 1959. 28p. 9¾″. 1*s.* 9*d.*

Northern Ireland. General Register Office. The Ulster year book: the official year book of Northern Ireland, 1957–1959. Belfast, H.M.S.O., 1959. xlvii, 345p. photos. fold. map. diagrs. 9¾″. 6*s.* 6*d.*

Treasury. Committee on the Working of the Monetary System: report. London, H.M.S.O., 1959. viii, 375p. 3 fold. tables. 9¾″. 15*s.* (Cmnd. 827)

Australia. *Commonwealth Bureau of Census and Statistics.* Census of retail establishments, year ended 30th June, 1953. [Canberra, 1955–56.] 14 pts. 13–13¼″.

Cuba. *Dirección General de Estadística.* Resúmenes estadísticos seleccionadas (síntesis cronológica). La Habana, 1959. 143p. 9¼″.

Czechoslovakia. *Státní Uřad Statistický.* Krajská města v československu (statistický přehled). [Prague], 1958. 366p. photos. 9½″. *Kčs.* 33.

Denmark. *Statistiske Departement.* Boligtaellingen, Oktober 1955. Housing census. København, 1959. 138p. 9″. *kr.*4. (Statist. Medd. 4.170.3)

East Africa. *East African Statistical Department.* The balance of payments of East Africa, 1956 and 1957. [Nairobi], 1959. [v], 18p. 11¼″. 5*s.*

Eire. *Central Statistics Office.* Censuses of population of Ireland, 1946 and 1951: general report. Dublin, Stationery Office, 1958. vii, 274p. maps (2 fold.). fold. table. 12½″. 8*s.*

Finland. *Transport Council.* Suomen liikennetilastollinen vuosikirja. Yearbook of Finnish transport statistics, 1958. Helsinki, 1958. 139 l. 11¾″.

France. *Institut National de la Statistique et des Etudes Economiques.* Densité de la population d'après le recensement général de 1954. Paris, [1959]. fold. col. map. 29¼″ × 31½″. Scale 1 : 1,400,000.

Germany (Western). *Statistisches Bundesamt.* Handwerkszählung, 1956. Sonderheft: Struk-

turquoten des Handwerks. Wiesbaden, 1959. 303p. 11¾". *DM*.20. (Statistik der Bundesrepublik Deutschland 203, Sonderheft)

Die Jugend im wirtschaftlichen und sozialen Leben der Bundesrepublik Deutschland. Wiesbaden, 1959. 123p. 11¾". *DM*.8. (S.B.D. 220)

Wohnungsstatistik, 1956/57. Heft 2. Struktur der Wohnungen und Wohnparteien, Wohnungsmieten nach der 10 v.H.-Repräsentativerhebung am 25.9.1956. Wiesbaden, 1959. 170p. 11¾". *DM*.10. (S.B.D. 201, Heft 2)

Hungary. *Központi Statisztikai Hivatal.* Magyarország népesedése, 1957. [The population of Hungary]. Budapest, 1959. 384p. 11½". (Statiszt. Idöszaki Közl. 21)

International Labour Office. The cost of medical care. Geneva, 1959. xiii, 216p. 9½". 9*s*. (Stud. Rep. N.S. 51)

Labour costs in European industry. Geneva, 1959. vii, 170p. 9½". 7*s*. 6*d*. (Stud. Rep. N.S. 52)

Italy. *Istituto Centrale di Statistica.* IX censimento generale della popolazione, III censimento generale dell'industria e del commercio, 4–5 novembre, 1951: caratteristiche demografiche ed economiche dei grandi comuni. Roma, 1959. 3v. 12". *L*.9,000.

Tavole di mortalità della popolazione italiana, 1950–53 e 1954–57. Roma, 1959. vii, 221p. 16 col. diagrs. 10¼". *L*.1,200. (Ann. Statist. Ser. VIII, Vol. 10)

Japan. *Bureau of Statistics.* 1955 population census of Japan. Vol. V. Tokyo, 1957–59. 46 pts. fold. maps. 10¼".

Kenya. *East African Statistical Department.* The patterns of income, expenditure and consumption of Africans in Nairobi, 1957/58. [Nairobi], 1959. [ii], 48p. 13". 3*s*.

Reported employment and wages in Kenya, 1958. [Nairobi], 1959. [iii], 10, [8]p. 11¼". 3*s*.

Malta. *Central Office of Statistics.* Census 1957. The Maltese Islands: report on population and housing. [Valletta, 1959.] [v], xcix, 276p. maps. 12½". 10*s*.

Netherlands. *Centraal Bureau voor de Statistiek.* Nationale rekeningen, 1958. Zeist, W. de Haan, 1959. 123p. 11½". *fl*. 6.95.

New Zealand. *Department of Statistics.* Census of distribution, 1958. Wellington, 1959. 78p. fold. table. 12¼". 10*s*. 6*d*.

Population census, 1956. Vol. V. Incomes. 5*s*. 6*d*. Vol. VI. Birthplaces and duration of residence of overseas-born. 4*s*. 6*d*. Wellington, 1959. 2pts. 12¼".

Report on the inter-industry study of the New Zealand economy for the year 1954–55. Wellington, 1959. 40p. 12¼". 5*s*. 6*d*.

Organisation for European Economic Co-operation. Manpower population, 1900–1958. Paris, 1959. [v], vii, 20p. 9½" × 12½". 4*s*. (O.E.E.C. Statist. Bull.). English and French.

Poland. *Glówny Urzad Statystyczny.* Ludność wedlug plci i wieku [Population by sex and age]: zameldowana na zamieszkanie w dniu 31.XII.1955 r.; wyniki jednorazowego badania stanu i struktury ludności, przeprowadzonego w oparciu o ksiegi meldunkowe w maju 1956 r. Warszawa, 1959. xii, 72p. 11¼". *zl*.15. (Statyst. Polski F 5)

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Portugal. *Instituto Nacional de Estatistica.* Inquérito industrial: distrito de Evora, 1957. $20. IV. Distrito de Portalegre, 1957. $20. Lisboa, [1959]. 2 pts. maps. 10¼". Portuguese and French.

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Spain. *Instituto Nacional de Estadística.* Estadística de préstamos hipotecarios correspondiente a los años 1955, 1956, 1957 y 1958. Madrid, 1959. 91p. 10½".

Uganda. *East African Statistical Department.* Enumeration of employees, June 1958. [Nairobi], 1959. 25p. 13". 2*s*. 50*c*.

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Educational, Scientific and Cultural Organization. Basic facts and figures: international statistics relating to education, culture and mass communication, 1958. Paris, 1959. 142p. $9\frac{1}{2}''$. 10*s*.

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II.—AUTHORS AND MISCELLANEOUS

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Avondo Bodino, G. & Brambilla, F. Teoria delle code. Milano & Varese, Istituto Editoriale Cisalpino, 1959. 219p. $8\frac{3}{4}''$. *L.*2,500.

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Centraal Planbureau, *The Hague.* De methodiek van het Centraal Economisch Plan; door P. de Wolff. *Econ.-statist. Ber.* (1959), **44**, 266–271. 15p. $9\frac{1}{2}''$. (Overdrukken 57)

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REVENUE OF THE UNITED KINGDOM

Net Product in Quarters of 1958, *and the Financial Years ended March* 31, 1958/59, 1957/58, 1956/57, 1955/56

£ thousand

	Quarters ended				Total for Calendar Year 1958
	March 31, 1958	June 30, 1958	Sept. 30, 1958	Dec. 31, 1958	
Ordinary Revenue—	£	£	£	£	£
Income tax	1,341,193	327,527	338,629	269,438	2,276,787
Surtax	104,300	24,600	12,900	18,100	159,900
Death duties	44,400	40,600	47,800	46,600	179,400
Stamps	13,600	13,400	13,500	17,600	58,100
Profits tax and excess profits tax / Excess profits levy	59,950	49,900	91,400	79,500	280,750
Other Inland Revenue duties	400	70	50	30	550
Total Inland Revenue	1,563,843	456,097	504,279	431,268	2,955,487
Customs	288,751	324,448	316,034	327,472	1,256,705
Excise	238,235	216,925	236,440	241,275	932,875
Total Customs and Excise	526,986	541,373	552,474	568,747	2,189,580
Motor vehicles duties	70,583	11,857	10,357	9,433	102,230
Post Office (net receipt)	7,185	4,700	−15,800*	8,000	4,085
Broadcast receiving licences	9,500	6,000	5,400	11,500	32,400
Receipts from sundry loans	4,537	10,494	11,782	3,850	30,663
Miscellaneous receipts	42,867	13,266	22,092	29,167	107,392
Total Ordinary Revenue	2,225,501	1,043,787	1,090,584	1,061,965	5,421,837
Self-balancing Revenue—					
Post Office	87,415	85,100	93,100	97,800	363,415
Income tax deducted from excess profits tax, post-war refunds	305	44	201	70	620

* Net issue from Exchequer.

£ thousand

	Year 1958/59	Year 1957/58	1958/59 (compared with 1957/58)		Corresponding Years	
			Increase	Decrease	1956/57	1955/56
Ordinary Revenue—	£	£	£	£	£	£
Income tax	2,321,773	2,208,266	113,507	—	2,114,173	1,942,902
Surtax	166,600	157,400	9,200	—	158,000	138,600
Death duties	186,900	170,600	16,300	—	169,000	175,700
Stamps	65,400	63,700	1,700	—	63,000	70,600
Profits tax and excess profits tax	274,800	255,150	19,650	—	195,000	192,500
Excess profits levy					5,000	18,450
Other Inland Revenue duties	550	550	—	—	640	770
Total Inland Revenue	3,016,023	2,855,666	160,357	—	2,704,813	2,539,522
Customs	1,261,536	1,207,452	54,084	—	1,198,882	1,148,598
Excise	929,775	942,360	—	12,585	901,735	864,511
Total Customs and Excise	2,191,311	2,149,812	41,499	—	2,100,617	2,013,109
Motor vehicles duties	106,751	100,734	6,017	—	90,610	86,987
Post Office (net receipts)	150	7,785	—	7,635	4,464	—
Broadcast receiving licences	33,700	30,700	3,000	—	28,400	25,800
Receipts from sundry loans	27,167	32,168	—	5,001	32,714	29,858
Miscellaneous receipts	104,607	165,720	—	61,113	196,195	197,867
Total Ordinary Revenue	5,479,709	5,342,585	137,124	—	5,157,813	4,893,143
Self-balancing Revenue—						
Post Office	370,050	335,515	34,535	—	304,036	264,900
Income tax deducted from excess profits tax, post-war refunds	358	575	—	217	599	2,365

INDEX TO VOL. 122.

YEAR 1959